Lineares Gleichungssystem
Aufgaben und Lösungen

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25. April 2013

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1 Einsetzverfahren (2)

$I \qquad a1 \cdot x + b1 \cdot y = c1$
$II \qquad a2 \cdot x + b2 \cdot y = c42$

- Gleichung I oder II nach x oder y auflösen
- Term in die andere Gleichung einsetzen
- Gleichung nach der Unbekannten auflösen
- zweite Unbekannte berechnen

$I \qquad 3x + 5y = 19$
$II \qquad 7x + 5y = 31$
I nach x auflösen
$3x + 5y = 19$
$3x + 5y = 19 \qquad / - 5y$
$3x = 19 - 5y \qquad / : 3$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
I in II
$7(6\frac{1}{3} - 1\frac{2}{3}y) + 5y = 31$
$44\frac{1}{3} - 11\frac{2}{3}y + 5y = 31 \qquad / - 44\frac{1}{3}$
$-11\frac{2}{3}y + 5y = 31 - 44\frac{1}{3}$
$-6\frac{2}{3}y = -13\frac{1}{3} \qquad / : \left(-6\frac{2}{3}\right)$
$y = \frac{-13\frac{1}{3}}{-6\frac{2}{3}}$
$y = 2$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
$x = 6\frac{1}{3} - 1\frac{2}{3} \cdot 2$
$x = 3$
$L = \{3/2\}$

$I \qquad 3x + 5y = 19$
$II \qquad 7x + 5y = 31$
I nach y auflösen
$3x + 5y = 19$
$3x + 5y = 19 \qquad / - 3x$
$5y = 19 - 3x \qquad / : 5$
$y = 3\frac{4}{5} - \frac{3}{5}x$
I in II
$7x + 5(3\frac{4}{5} - \frac{3}{5}x) = 31$
$19 - 3x + 5x = 31 \qquad / - 19$
$-3x + 5x = 31 - 19$
$4x = 12 \qquad / : 4$
$x = \frac{12}{4}$
$x = 3$
$y = 3\frac{4}{5} - \frac{3}{5}x$
$y = 3\frac{4}{5} - \frac{3}{5} \cdot 3$
$y = 2$
$L = \{3/2\}$

1.1 *Matrix*

1.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
dfgdfgdf

(1)
(2)

1.1.2 Lösungen

Aufgabe (1)

$a + 2b + 3c = 4$
$5a + 6b + 7c = 8$
$9a + 10b + 11c = 12$

a	b	c	
1	2	3	4
5	6	7	8
9	10	11	12

Zeile2 = Zeile2-Zeile1 $\cdot \frac{5}{1}$
$z2s1 = 5 - 1 \cdot \frac{5}{1} = 0$
$z2s2 = 6 - 2 \cdot \frac{5}{1} = -4$
$z2s3 = 7 - 3 \cdot \frac{5}{1} = -8$
$z2s4 = 8 - 4 \cdot \frac{5}{1} = -12$

a	b	c	
1	2	3	4
0	-4	-8	-12
9	10	11	12

Zeile3 = Zeile3-Zeile1 $\cdot \frac{9}{1}$
$z3s1 = 9 - 1 \cdot \frac{9}{1} = 0$
$z3s2 = 10 - 2 \cdot \frac{9}{1} = -8$
$z3s3 = 11 - 3 \cdot \frac{9}{1} = -16$
$z3s4 = 12 - 4 \cdot \frac{9}{1} = -24$

a	b	c	
1	2	3	4
0	-4	-8	-12
0	-8	-16	-24

Zeile1 = Zeile1-Zeile2 $\cdot \frac{2}{-4}$
$z1s2 = 2 - (-4) \cdot \frac{2}{-4} = 0$
$z1s3 = 3 - (-8) \cdot \frac{2}{-4} = -1$
$z1s4 = 4 - (-12) \cdot \frac{2}{-4} = -2$

a	b	c	
1	0	-1	-2
0	-4	-8	-12
0	-8	-16	-24

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-8}{-4}$
$z3s2 = -8 - (-4) \cdot \frac{-8}{-4} = 0$
$z3s3 = -16 - (-8) \cdot \frac{-8}{-4} = 0$
$z3s4 = -24 - (-12) \cdot \frac{-8}{-4} = 0$

a	b	c	
1	0	-1	-2
0	-4	-8	-12
0	0	0	0

$L = unendlich$

1
2
3
4
5
6
7
8
9
10
11
12

Aufgabe (2)

$a + 2b + 3c = 4$
$5a + 6b + 7c = 8$
$9a + 10b + 11c = 12$

a	b	c	
1	2	3	4
5	6	7	8
9	10	11	12

Zeile2 = Zeile2-Zeile1 $\cdot \frac{5}{1}$
$z2s1 = 5 - 1 \cdot \frac{5}{1} = 0$
$z2s2 = 6 - 2 \cdot \frac{5}{1} = -4$
$z2s3 = 7 - 3 \cdot \frac{5}{1} = -8$
$z2s4 = 8 - 4 \cdot \frac{5}{1} = -12$

a	b	c	
1	2	3	4
0	-4	-8	-12
9	10	11	12

Zeile3 = Zeile3-Zeile1 $\cdot \frac{9}{1}$
$z3s1 = 9 - 1 \cdot \frac{9}{1} = 0$
$z3s2 = 10 - 2 \cdot \frac{9}{1} = -8$
$z3s3 = 11 - 3 \cdot \frac{9}{1} = -16$
$z3s4 = 12 - 4 \cdot \frac{9}{1} = -24$

a	b	c	
1	2	3	4
0	-4	-8	-12
0	-8	-16	-24

Zeile1 = Zeile1-Zeile2 $\cdot \frac{2}{-4}$
$z1s2 = 2 - (-4) \cdot \frac{2}{-4} = 0$
$z1s3 = 3 - (-8) \cdot \frac{2}{-4} = -1$
$z1s4 = 4 - (-12) \cdot \frac{2}{-4} = -2$

a	b	c	
1	0	-1	-2
0	-4	-8	-12
0	-8	-16	-24

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-8}{-4}$
$z3s2 = -8 - (-4) \cdot \frac{-8}{-4} = 0$
$z3s3 = -16 - (-8) \cdot \frac{-8}{-4} = 0$
$z3s4 = -24 - (-12) \cdot \frac{-8}{-4} = 0$

a	b	c	
1	0	-1	-2
0	-4	-8	-12
0	0	0	0

$L = unendlich$

1
2
3
4
5
6
7
8
9
10
11
12

1.2 Interaktiv

1.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
$a1 \cdot x + b1 \cdot y = c1$
$a2 \cdot x + b2 \cdot y = c2$

Gesucht:
x und y

(1) $3x + 5y = 19$
$7x + 5y = 31$

(2) $1x + 1y = 10$
$1x - 1y = 4$

(3) $9x - 2y = 5$
$5x - 2y = 1$

(4) $9x - 2y = 1$
$-3x - 3y = -7$

(5) $1x + 3y = 9$
$3x - 2y = -6$

(6) $7x + 1\frac{1}{2}y = 1$
$2x - 5y = 22$

(7) $4x - 5y = 43$
$-3x - 7y = 0$

(8) $8x - 3y = 10$
$12x - 5y = 24$

(9) $-\frac{1}{2}x + 1y = 2$
$\frac{1}{2}x - 3y = -3$

(10) $-1x + 1y = 3$
$\frac{1}{2}x - 4y = 5$

(11) $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$

(12) $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$1x + 1y = 10\frac{2}{3}$

(13) $1\frac{1}{2}x - 2y = 9$
$\frac{2}{5}x + \frac{1}{3}y = 5$

(14) $2x + 3y = 4$
$\frac{1}{3}x - \frac{1}{5}y = 12$

(15) $1x + 2y = -4$
$-1x + 2y = 5$

(16) $2x + 4y = 4$
$-\frac{1}{2}x + 3y = 3$

(17) $-\frac{1}{2}x + 4y = 6$
$-2x - 8y = 2$

(18) $1x + 1y = 1$
$2x + 2y = 2$

(19) $1x + 1y = 2$
$3x + 3y = 3$

(20) $-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-\frac{2}{3}x + \frac{1}{9}y = 9$

(21) $2x - 7y = -8$
$7x - 1y = -9$

(22) $-7x + 9y = -3$
$5x - 6y = -4$

(23) $\begin{array}{l} 2x + 2y = 1\frac{7}{10} \\ 3x + 6y = 3 \end{array}$

(24) $\begin{array}{l} 3x + 5y = 8 \\ 9x + 0y = 7 \end{array}$

(25) $\begin{array}{l} 3x + 6y = 8 \\ 9x + 0y = 9 \end{array}$

(26) $\begin{array}{l} 3x + 6y = 8 \\ 9x + 0y = 9 \end{array}$

1.2.2 Lösungen

Aufgabe (1)

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach x auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 5y$
$3x = 19 - 5y \quad / : 3$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
I in II
$7(6\frac{1}{3} - 1\frac{2}{3}y) + 5y = 31$
$44\frac{1}{3} - 11\frac{2}{3}y + 5y = 31 \quad / - 44\frac{1}{3}$
$-11\frac{2}{3}y + 5y = 31 - 44\frac{1}{3}$
$-6\frac{2}{3}y = -13\frac{1}{3} \quad / : \left(-6\frac{2}{3}\right)$
$y = \frac{-13\frac{1}{3}}{-6\frac{2}{3}}$
$y = 2$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
$x = 6\frac{1}{3} - 1\frac{2}{3} \cdot 2$
$x = 3$
$L = \{3/2\}$

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach y auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 3x$
$5y = 19 - 3x \quad / : 5$
$y = 3\frac{4}{5} - \frac{3}{5}x$
I in II
$7x + 5(3\frac{4}{5} - \frac{3}{5}x) = 31$
$19 - 3x + 5x = 31 \quad / - 19$
$-3x + 5x = 31 - 19$
$4x = 12 \quad / : 4$
$x = \frac{12}{4}$
$x = 3$
$y = 3\frac{4}{5} - \frac{3}{5}x$
$y = 3\frac{4}{5} - \frac{3}{5} \cdot 3$
$y = 2$
$L = \{3/2\}$

Aufgabe (2)

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
I nach x auflösen
$1x + 1y = 10$
$1x + 1y = 10 \quad / - 1y$
$1x = 10 - 1y \quad / : 1$
$x = 10 - 1y$
I in II
$1(10 - 1y) + -1y = 4$
$10 - 1y - 1y = 4 \quad / - 10$
$-1y - 1y = 4 - 10$
$-2y = -6 \quad / : (-2)$
$y = \frac{-6}{-2}$
$y = 3$
$x = 10 - 1y$
$x = 10 - 1 \cdot 3$
$x = 7$
$L = \{7/3\}$

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
I nach y auflösen
$1x + 1y = 10$
$1x + 1y = 10 \quad / - 1x$
$1y = 10 - 1x \quad / : 1$
$y = 10 - 1x$
I in II
$1x + -1(10 - 1x) = 4$
$-10 + 1x - 1x = 4 \quad / - (-10)$
$+1x - 1x = 4 - (-10)$
$2x = 14 \quad / : 2$
$x = \frac{14}{2}$
$x = 7$
$y = 10 - 1x$
$y = 10 - 1 \cdot 7$
$y = 3$
$L = \{7/3\}$

Aufgabe (3)

$I \qquad 9x - 2y = 5$
$II \qquad 5x - 2y = 1$
I nach x auflösen
$9x - 2y = 5$
$9x - 2y = 5 \qquad / + 2y$
$9x = 5 + 2y \qquad / : 9$
$x = \frac{5}{9} + \frac{2}{9}y$
I in II
$5(\frac{5}{9} + \frac{2}{9}y) + -2y = 1$
$2\frac{7}{9} + 1\frac{1}{9}y - 2y = 1 \qquad / - 2\frac{7}{9}$
$+1\frac{1}{9}y - 2y = 1 - 2\frac{7}{9}$
$-\frac{8}{9}y = -1\frac{7}{9} \qquad / : \left(-\frac{8}{9}\right)$
$y = \frac{-1\frac{7}{9}}{-\frac{8}{9}}$
$y = 2$
$x = \frac{5}{9} + \frac{2}{9}y$
$x = \frac{5}{9} + \frac{2}{9} \cdot 2$
$x = 1$
$L = \{1/2\}$

$I \qquad 9x - 2y = 5$
$II \qquad 5x - 2y = 1$
I nach y auflösen
$9x - 2y = 5$
$9x - 2y = 5 \qquad / - 9x$
$-2y = 5 - 9x \qquad / : (-2)$
$y = -2\frac{1}{2} + 4\frac{1}{2}x$
I in II
$5x + -2(-2\frac{1}{2} + 4\frac{1}{2}x) = 1$
$5 - 9x - 2x = 1 \qquad / - 5$
$-9x - 2x = 1 - 5$
$-4x = -4 \qquad / : (-4)$
$x = \frac{-4}{-4}$
$x = 1$
$y = -2\frac{1}{2} + 4\frac{1}{2}x$
$y = -2\frac{1}{2} + 4\frac{1}{2} \cdot 1$
$y = 2$
$L = \{1/2\}$

Aufgabe (4)

$I \qquad 9x - 2y = 1$
$II \qquad -3x - 3y = -7$
I nach x auflösen
$9x - 2y = 1$
$9x - 2y = 1 \qquad / + 2y$
$9x = 1 + 2y \qquad / : 9$
$x = \frac{1}{9} + \frac{2}{9}y$
I in II
$-3(\frac{1}{9} + \frac{2}{9}y) + -3y = -7$
$-\frac{1}{3} - \frac{2}{3}y - 3y = -7 \qquad / - \left(-\frac{1}{3}\right)$
$-\frac{2}{3}y - 3y = -7 - \left(-\frac{1}{3}\right)$
$-3\frac{2}{3}y = -6\frac{2}{3} \qquad / : \left(-3\frac{2}{3}\right)$
$y = \frac{-6\frac{2}{3}}{-3\frac{2}{3}}$
$y = 1\frac{9}{11}$
$x = \frac{1}{9} + \frac{2}{9}y$
$x = \frac{1}{9} + \frac{2}{9} \cdot 1\frac{9}{11}$
$x = \frac{17}{33}$
$L = \{\frac{17}{33}/1\frac{9}{11}\}$

$I \qquad 9x - 2y = 1$
$II \qquad -3x - 3y = -7$
I nach y auflösen
$9x - 2y = 1$
$9x - 2y = 1 \qquad / - 9x$
$-2y = 1 - 9x \qquad / : (-2)$
$y = -\frac{1}{2} + 4\frac{1}{2}x$
I in II
$-3x + -3(-\frac{1}{2} + 4\frac{1}{2}x) = -7$
$1\frac{1}{2} - 13\frac{1}{2}x - 3x = -7 \qquad / - 1\frac{1}{2}$
$-13\frac{1}{2}x - 3x = -7 - 1\frac{1}{2}$
$-16\frac{1}{2}x = -8\frac{1}{2} \qquad / : \left(-16\frac{1}{2}\right)$
$x = \frac{-8\frac{1}{2}}{-16\frac{1}{2}}$
$x = \frac{17}{33}$
$y = -\frac{1}{2} + 4\frac{1}{2}x$
$y = -\frac{1}{2} + 4\frac{1}{2} \cdot \frac{17}{33}$
$y = 1\frac{9}{11}$
$L = \{\frac{17}{33}/1\frac{9}{11}\}$

Aufgabe (5)

$I \qquad 1x + 3y = 9$
$II \qquad 3x - 2y = -6$
I nach x auflösen
$1x + 3y = 9$
$1x + 3y = 9 \qquad / - 3y$
$1x = 9 - 3y \qquad / : 1$
$x = 9 - 3y$
I in II
$3(9 - 3y) + -2y = -6$
$27 - 9y - 2y = -6 \qquad / - 27$
$-9y - 2y = -6 - 27$
$-11y = -33 \qquad / : (-11)$
$y = \frac{-33}{-11}$
$y = 3$
$x = 9 - 3y$
$x = 9 - 3 \cdot 3$
$x = 0$
$L = \{0/3\}$

$I \qquad 1x + 3y = 9$
$II \qquad 3x - 2y = -6$
I nach y auflösen
$1x + 3y = 9$
$1x + 3y = 9 \qquad / - 1x$
$3y = 9 - 1x \qquad / : 3$
$y = 3 - \frac{1}{3}x$
I in II
$3x + -2(3 - \frac{1}{3}x) = -6$
$-6 + \frac{2}{3}x - 2x = -6 \qquad / - (-6)$
$+\frac{2}{3}x - 2x = -6 - (-6)$
$3\frac{2}{3}x = 0 \qquad / : 3\frac{2}{3}$
$x = \frac{0}{3\frac{2}{3}}$
$x = 0$
$y = 3 - \frac{1}{3}x$
$y = 3 - \frac{1}{3} \cdot 0$
$y = 3$
$L = \{0/3\}$

Aufgabe (6)

$I \qquad 7x + 1\frac{1}{2}y = 1$
$II \qquad 2x - 5y = 22$
I nach x auflösen
$7x + 1\frac{1}{2}y = 1$
$7x + 1\frac{1}{2}y = 1 \qquad / - 1\frac{1}{2}y$
$7x = 1 - 1\frac{1}{2}y \qquad / : 7$
$x = \frac{1}{7} - \frac{3}{14}y$
I in II
$2(\frac{1}{7} - \frac{3}{14}y) + -5y = 22$
$\frac{2}{7} - \frac{3}{7}y - 5y = 22 \qquad / - \frac{2}{7}$
$-\frac{3}{7}y - 5y = 22 - \frac{2}{7}$
$-5\frac{3}{7}y = 21\frac{5}{7} \qquad / : \left(-5\frac{3}{7}\right)$
$y = \frac{21\frac{5}{7}}{-5\frac{3}{7}}$
$y = -4$
$x = \frac{1}{7} - \frac{3}{14}y$
$x = \frac{1}{7} - \frac{3}{14} \cdot (-4)$
$x = 1$
$L = \{1/-4\}$

$I \qquad 7x + 1\frac{1}{2}y = 1$
$II \qquad 2x - 5y = 22$
I nach y auflösen
$7x + 1\frac{1}{2}y = 1$
$7x + 1\frac{1}{2}y = 1 \qquad / - 7x$
$1\frac{1}{2}y = 1 - 7x \qquad / : 1\frac{1}{2}$
$y = \frac{2}{3} - 4\frac{2}{3}x$
I in II
$2x + -5(\frac{2}{3} - 4\frac{2}{3}x) = 22$
$-3\frac{1}{3} + 23\frac{1}{3}x - 5x = 22 \qquad / - \left(-3\frac{1}{3}\right)$
$+23\frac{1}{3}x - 5x = 22 - \left(-3\frac{1}{3}\right)$
$25\frac{1}{3}x = 25\frac{1}{3} \qquad / : 25\frac{1}{3}$
$x = \frac{25\frac{1}{3}}{25\frac{1}{3}}$
$x = 1$
$y = \frac{2}{3} - 4\frac{2}{3}x$
$y = \frac{2}{3} - 4\frac{2}{3} \cdot 1$
$y = -4$
$L = \{1/-4\}$

Aufgabe (7)

$I \qquad 4x - 5y = 43$
$II \qquad -3x - 7y = 0$
I nach x auflösen
$4x - 5y = 43$
$4x - 5y = 43 \qquad / + 5y$
$4x = 43 + 5y \qquad / : 4$
$x = 10\frac{3}{4} + 1\frac{1}{4}y$
I in II
$-3(10\frac{3}{4} + 1\frac{1}{4}y) + -7y = 0$
$-32\frac{1}{4} - 3\frac{3}{4}y - 7y = 0 \qquad / - \left(-32\frac{1}{4}\right)$
$-3\frac{3}{4}y - 7y = 0 - \left(-32\frac{1}{4}\right)$
$-10\frac{3}{4}y = 32\frac{1}{4} \qquad / : \left(-10\frac{3}{4}\right)$
$y = \frac{32\frac{1}{4}}{-10\frac{3}{4}}$
$y = -3$
$x = 10\frac{3}{4} + 1\frac{1}{4}y$
$x = 10\frac{3}{4} + 1\frac{1}{4} \cdot (-3)$
$x = 7$
$L = \{7/-3\}$

$I \qquad 4x - 5y = 43$
$II \qquad -3x - 7y = 0$
I nach y auflösen
$4x - 5y = 43$
$4x - 5y = 43 \qquad / - 4x$
$-5y = 43 - 4x \qquad / : (-5)$
$y = -8\frac{3}{5} + \frac{4}{5}x$
I in II
$-3x + -7(-8\frac{3}{5} + \frac{4}{5}x) = 0$
$60\frac{1}{5} - 5\frac{3}{5}x - 7x = 0 \qquad / - 60\frac{1}{5}$
$-5\frac{3}{5}x - 7x = 0 - 60\frac{1}{5}$
$-8\frac{3}{5}x = -60\frac{1}{5} \qquad / : \left(-8\frac{3}{5}\right)$
$x = \frac{-60\frac{1}{5}}{-8\frac{3}{5}}$
$x = 7$
$y = -8\frac{3}{5} + \frac{4}{5}x$
$y = -8\frac{3}{5} + \frac{4}{5} \cdot 7$
$y = -3$
$L = \{7/-3\}$

Aufgabe (8)

$I \qquad 8x - 3y = 10$
$II \qquad 12x - 5y = 24$
I nach x auflösen
$8x - 3y = 10$
$8x - 3y = 10 \qquad / + 3y$
$8x = 10 + 3y \qquad / : 8$
$x = 1\frac{1}{4} + \frac{3}{8}y$
I in II
$12(1\frac{1}{4} + \frac{3}{8}y) + -5y = 24$
$15 + 4\frac{1}{2}y - 5y = 24 \qquad / - 15$
$+4\frac{1}{2}y - 5y = 24 - 15$
$-\frac{1}{2}y = 9 \qquad / : \left(-\frac{1}{2}\right)$
$y = \frac{9}{-\frac{1}{2}}$
$y = -18$
$x = 1\frac{1}{4} + \frac{3}{8}y$
$x = 1\frac{1}{4} + \frac{3}{8} \cdot (-18)$
$x = -5\frac{1}{2}$
$L = \{-5\frac{1}{2}/-18\}$

$I \qquad 8x - 3y = 10$
$II \qquad 12x - 5y = 24$
I nach y auflösen
$8x - 3y = 10$
$8x - 3y = 10 \qquad / - 8x$
$-3y = 10 - 8x \qquad / : (-3)$
$y = -3\frac{1}{3} + 2\frac{2}{3}x$
I in II
$12x + -5(-3\frac{1}{3} + 2\frac{2}{3}x) = 24$
$16\frac{2}{3} - 13\frac{1}{3}x - 5x = 24 \qquad / - 16\frac{2}{3}$
$-13\frac{1}{3}x - 5x = 24 - 16\frac{2}{3}$
$-1\frac{1}{3}x = 7\frac{1}{3} \qquad / : \left(-1\frac{1}{3}\right)$
$x = \frac{7\frac{1}{3}}{-1\frac{1}{3}}$
$x = -5\frac{1}{2}$
$y = -3\frac{1}{3} + 2\frac{2}{3}x$
$y = -3\frac{1}{3} + 2\frac{2}{3} \cdot \left(-5\frac{1}{2}\right)$
$y = -18$
$L = \{-5\frac{1}{2}/-18\}$

Aufgabe (9)

$I \qquad -\frac{1}{2}x + 1y = 2$
$II \qquad \frac{1}{2}x - 3y = -3$
I nach x auflösen
$-\frac{1}{2}x + 1y = 2$
$-\frac{1}{2}x + 1y = 2 \qquad / -1y$
$-\frac{1}{2}x = 2 - 1y \qquad / : \left(-\frac{1}{2}\right)$
$x = -4 + 2y$
I in II
$\frac{1}{2}(-4 + 2y) + -3y = -3$
$-2 + 1y - 3y = -3 \qquad / -(-2)$
$+1y - 3y = -3 - (-2)$
$-2y = -1 \qquad / : (-2)$
$y = \frac{-1}{-2}$
$y = \frac{1}{2}$
$x = -4 + 2y$
$x = -4 + 2 \cdot \frac{1}{2}$
$x = -3$
$L = \{-3/\frac{1}{2}\}$

$I \qquad -\frac{1}{2}x + 1y = 2$
$II \qquad \frac{1}{2}x - 3y = -3$
I nach y auflösen
$-\frac{1}{2}x + 1y = 2$
$-\frac{1}{2}x + 1y = 2 \qquad / + \frac{1}{2}x$
$1y = 2 + \frac{1}{2}x \qquad / : 1$
$y = 2 + \frac{1}{2}x$
I in II
$\frac{1}{2}x + -3(2 + \frac{1}{2}x) = -3$
$-6 - 1\frac{1}{2}x - 3x = -3 \qquad / -(-6)$
$-1\frac{1}{2}x - 3x = -3 - (-6)$
$-1x = 3 \qquad / : (-1)$
$x = \frac{3}{-1}$
$x = -3$
$y = 2 + \frac{1}{2}x$
$y = 2 + \frac{1}{2} \cdot (-3)$
$y = \frac{1}{2}$
$L = \{-3/\frac{1}{2}\}$

Aufgabe (10)

$I \qquad -1x + 1y = 3$
$II \qquad \frac{1}{2}x - 4y = 5$
I nach x auflösen
$-1x + 1y = 3$
$-1x + 1y = 3 \qquad / -1y$
$-1x = 3 - 1y \qquad / : (-1)$
$x = -3 + 1y$
I in II
$\frac{1}{2}(-3 + 1y) + -4y = 5$
$-1\frac{1}{2} + \frac{1}{2}y - 4y = 5 \qquad / -\left(-1\frac{1}{2}\right)$
$+\frac{1}{2}y - 4y = 5 - \left(-1\frac{1}{2}\right)$
$-3\frac{1}{2}y = 6\frac{1}{2} \qquad / : \left(-3\frac{1}{2}\right)$
$y = \frac{6\frac{1}{2}}{-3\frac{1}{2}}$
$y = -1\frac{6}{7}$
$x = -3 + 1y$
$x = -3 + 1 \cdot \left(-1\frac{6}{7}\right)$
$x = -4\frac{6}{7}$
$L = \{-4\frac{6}{7}/-1\frac{6}{7}\}$

$I \qquad -1x + 1y = 3$
$II \qquad \frac{1}{2}x - 4y = 5$
I nach y auflösen
$-1x + 1y = 3$
$-1x + 1y = 3 \qquad / + 1x$
$1y = 3 + 1x \qquad / : 1$
$y = 3 + 1x$
I in II
$\frac{1}{2}x + -4(3 + 1x) = 5$
$-12 - 4x - 4x = 5 \qquad / -(-12)$
$-4x - 4x = 5 - (-12)$
$-3\frac{1}{2}x = 17 \qquad / : \left(-3\frac{1}{2}\right)$
$x = \frac{17}{-3\frac{1}{2}}$
$x = -4\frac{6}{7}$
$y = 3 + 1x$
$y = 3 + 1 \cdot \left(-4\frac{6}{7}\right)$
$y = -1\frac{6}{7}$
$L = \{-4\frac{6}{7}/-1\frac{6}{7}\}$

Aufgabe (11)

$I \qquad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \qquad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
I nach x auflösen
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \qquad / + 1\frac{1}{3}y$
$1\frac{1}{5}x = 5\frac{1}{3} + 1\frac{1}{3}y \qquad / : 1\frac{1}{5}$
$x = 4\frac{4}{9} + 1\frac{1}{9}y$
I in II
$2\frac{1}{2}(4\frac{4}{9} + 1\frac{1}{9}y) + -\frac{1}{4}y = 12\frac{3}{8}$
$11\frac{1}{9} + 2\frac{7}{9}y - \frac{1}{4}y = 12\frac{3}{8} \qquad / - 11\frac{1}{9}$
$+2\frac{7}{9}y - \frac{1}{4}y = 12\frac{3}{8} - 11\frac{1}{9}$
$2\frac{19}{36}y = 1\frac{19}{72} \qquad / : 2\frac{19}{36}$
$y = \frac{1\frac{19}{72}}{2\frac{19}{36}}$
$y = \frac{1}{2}$
$x = 4\frac{4}{9} + 1\frac{1}{9}y$
$x = 4\frac{4}{9} + 1\frac{1}{9} \cdot \frac{1}{2}$
$x = 5$
$L = \{5/\frac{1}{2}\}$

$I \qquad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \qquad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
I nach y auflösen
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \qquad / - 1\frac{1}{5}x$
$-1\frac{1}{3}y = 5\frac{1}{3} - 1\frac{1}{5}x \qquad / : \left(-1\frac{1}{3}\right)$
$y = -4 + \frac{9}{10}x$
I in II
$2\frac{1}{2}x + -\frac{1}{4}(-4 + \frac{9}{10}x) = 12\frac{3}{8}$
$1 - \frac{9}{40}x - \frac{1}{4}x = 12\frac{3}{8} \qquad / - 1$
$-\frac{9}{40}x - \frac{1}{4}x = 12\frac{3}{8} - 1$
$2\frac{11}{40}x = 11\frac{3}{8} \qquad / : 2\frac{11}{40}$
$x = \frac{11\frac{3}{8}}{2\frac{11}{40}}$
$x = 5$
$y = -4 + \frac{9}{10}x$
$y = -4 + \frac{9}{10} \cdot 5$
$y = \frac{1}{2}$
$L = \{5/\frac{1}{2}\}$

Aufgabe (12)

$I \qquad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \qquad 1x + 1y = 10\frac{2}{3}$
I nach x auflösen
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \qquad / + \frac{5}{7}y$
$\frac{2}{3}x = \frac{2}{3} + \frac{5}{7}y \qquad / : \frac{2}{3}$
$x = 1 + 1\frac{1}{14}y$
I in II
$1(1 + 1\frac{1}{14}y) + 1y = 10\frac{2}{3}$
$1 + 1\frac{1}{14}y + 1y = 10\frac{2}{3} \qquad / - 1$
$+1\frac{1}{14}y + 1y = 10\frac{2}{3} - 1$
$2\frac{1}{14}y = 9\frac{2}{3} \qquad / : 2\frac{1}{14}$
$y = \frac{9\frac{2}{3}}{2\frac{1}{14}}$
$y = 4\frac{2}{3}$
$x = 1 + 1\frac{1}{14}y$
$x = 1 + 1\frac{1}{14} \cdot 4\frac{2}{3}$
$x = 6$
$L = \{6/4\frac{2}{3}\}$

$I \qquad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \qquad 1x + 1y = 10\frac{2}{3}$
I nach y auflösen
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \qquad / - \frac{2}{3}x$
$-\frac{5}{7}y = \frac{2}{3} - \frac{2}{3}x \qquad / : \left(-\frac{5}{7}\right)$
$y = -\frac{14}{15} + \frac{14}{15}x$
I in II
$1x + 1(-\frac{14}{15} + \frac{14}{15}x) = 10\frac{2}{3}$
$-\frac{14}{15} + \frac{14}{15}x + 1x = 10\frac{2}{3} \qquad / - \left(-\frac{14}{15}\right)$
$+\frac{14}{15}x + 1x = 10\frac{2}{3} - \left(-\frac{14}{15}\right)$
$1\frac{14}{15}x = 11\frac{3}{5} \qquad / : 1\frac{14}{15}$
$x = \frac{11\frac{3}{5}}{1\frac{14}{15}}$
$x = 6$
$y = -\frac{14}{15} + \frac{14}{15}x$
$y = -\frac{14}{15} + \frac{14}{15} \cdot 6$
$y = 4\frac{2}{3}$
$L = \{6/4\frac{2}{3}\}$

Aufgabe (13)

$I \qquad 1\frac{1}{2}x - 2y = 9$
$II \qquad \frac{2}{5}x + \frac{1}{3}y = 5$
I nach x auflösen
$1\frac{1}{2}x - 2y = 9$
$1\frac{1}{2}x - 2y = 9 \qquad / + 2y$
$1\frac{1}{2}x = 9 + 2y \qquad / : 1\frac{1}{2}$
$x = 6 + 1\frac{1}{3}y$
I in II
$\frac{2}{5}(6 + 1\frac{1}{3}y) + \frac{1}{3}y = 5$
$2\frac{2}{5} + \frac{8}{15}y + \frac{1}{3}y = 5 \qquad / - 2\frac{2}{5}$
$+\frac{8}{15}y + \frac{1}{3}y = 5 - 2\frac{2}{5}$
$\frac{13}{15}y = 2\frac{3}{5} \qquad / : \frac{13}{15}$
$y = \frac{2\frac{3}{5}}{\frac{13}{15}}$
$y = 3$
$x = 6 + 1\frac{1}{3}y$
$x = 6 + 1\frac{1}{3} \cdot 3$
$x = 10$
$L = \{10/3\}$

$I \qquad 1\frac{1}{2}x - 2y = 9$
$II \qquad \frac{2}{5}x + \frac{1}{3}y = 5$
I nach y auflösen
$1\frac{1}{2}x - 2y = 9$
$1\frac{1}{2}x - 2y = 9 \qquad / - 1\frac{1}{2}x$
$-2y = 9 - 1\frac{1}{2}x \qquad / : (-2)$
$y = -4\frac{1}{2} + \frac{3}{4}x$
I in II
$\frac{2}{5}x + \frac{1}{3}(-4\frac{1}{2} + \frac{3}{4}x) = 5$
$-1\frac{1}{2} + \frac{1}{4}x + \frac{1}{3}x = 5 \qquad / - \left(-1\frac{1}{2}\right)$
$+\frac{1}{4}x + \frac{1}{3}x = 5 - \left(-1\frac{1}{2}\right)$
$\frac{13}{20}x = 6\frac{1}{2} \qquad / : \frac{13}{20}$
$x = \frac{6\frac{1}{2}}{\frac{13}{20}}$
$x = 10$
$y = -4\frac{1}{2} + \frac{3}{4}x$
$y = -4\frac{1}{2} + \frac{3}{4} \cdot 10$
$y = 3$
$L = \{10/3\}$

Aufgabe (14)

$I \qquad 2x + 3y = 4$
$II \qquad \frac{1}{3}x - \frac{1}{5}y = 12$
I nach x auflösen
$2x + 3y = 4$
$2x + 3y = 4 \qquad / - 3y$
$2x = 4 - 3y \qquad / : 2$
$x = 2 - 1\frac{1}{2}y$
I in II
$\frac{1}{3}(2 - 1\frac{1}{2}y) + -\frac{1}{5}y = 12$
$\frac{2}{3} - \frac{1}{2}y - \frac{1}{5}y = 12 \qquad / - \frac{2}{3}$
$-\frac{1}{2}y - \frac{1}{5}y = 12 - \frac{2}{3}$
$-\frac{7}{10}y = 11\frac{1}{3} \qquad / : \left(-\frac{7}{10}\right)$
$y = \frac{11\frac{1}{3}}{-\frac{7}{10}}$
$y = -16\frac{4}{21}$
$x = 2 - 1\frac{1}{2}y$
$x = 2 - 1\frac{1}{2} \cdot \left(-16\frac{4}{21}\right)$
$x = 26\frac{2}{7}$
$L = \{26\frac{2}{7} / - 16\frac{4}{21}\}$

$I \qquad 2x + 3y = 4$
$II \qquad \frac{1}{3}x - \frac{1}{5}y = 12$
I nach y auflösen
$2x + 3y = 4$
$2x + 3y = 4 \qquad / - 2x$
$3y = 4 - 2x \qquad / : 3$
$y = 1\frac{1}{3} - \frac{2}{3}x$
I in II
$\frac{1}{3}x + -\frac{1}{5}(1\frac{1}{3} - \frac{2}{3}x) = 12$
$-\frac{4}{15} + \frac{2}{15}x - \frac{1}{5}x = 12 \qquad / - \left(-\frac{4}{15}\right)$
$+\frac{2}{15}x - \frac{1}{5}x = 12 - \left(-\frac{4}{15}\right)$
$\frac{7}{15}x = 12\frac{4}{15} \qquad / : \frac{7}{15}$
$x = \frac{12\frac{4}{15}}{\frac{7}{15}}$
$x = 26\frac{2}{7}$
$y = 1\frac{1}{3} - \frac{2}{3}x$
$y = 1\frac{1}{3} - \frac{2}{3} \cdot 26\frac{2}{7}$
$y = -16\frac{4}{21}$
$L = \{26\frac{2}{7} / - 16\frac{4}{21}\}$

Aufgabe (15)

$I \qquad 1x + 2y = -4$
$II \qquad -1x + 2y = 5$
I nach x auflösen
$1x + 2y = -4$
$1x + 2y = -4 \qquad / -2y$
$1x = -4 - 2y \qquad / : 1$
$x = -4 - 2y$
I in II
$-1(-4 - 2y) + 2y = 5$
$4 + 2y + 2y = 5 \qquad / -4$
$+2y + 2y = 5 - 4$
$4y = 1 \qquad / : 4$
$y = \frac{1}{4}$
$y = \frac{1}{4}$
$x = -4 - 2y$
$x = -4 - 2 \cdot \frac{1}{4}$
$x = -4\frac{1}{2}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

$I \qquad 1x + 2y = -4$
$II \qquad -1x + 2y = 5$
I nach y auflösen
$1x + 2y = -4$
$1x + 2y = -4 \qquad / -1x$
$2y = -4 - 1x \qquad / : 2$
$y = -2 - \frac{1}{2}x$
I in II
$-1x + 2(-2 - \frac{1}{2}x) = 5$
$-4 - 1x + 2x = 5 \qquad / -(-4)$
$-1x + 2x = 5 - (-4)$
$-2x = 9 \qquad / : (-2)$
$x = \frac{9}{-2}$
$x = -4\frac{1}{2}$
$y = -2 - \frac{1}{2}x$
$y = -2 - \frac{1}{2} \cdot \left(-4\frac{1}{2}\right)$
$y = \frac{1}{4}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

Aufgabe (16)

$I \qquad 2x + 4y = 4$
$II \qquad -\frac{1}{2}x + 3y = 3$
I nach x auflösen
$2x + 4y = 4$
$2x + 4y = 4 \qquad / -4y$
$2x = 4 - 4y \qquad / : 2$
$x = 2 - 2y$
I in II
$-\frac{1}{2}(2 - 2y) + 3y = 3$
$-1 + 1y + 3y = 3 \qquad / -(-1)$
$+1y + 3y = 3 - (-1)$
$4y = 4 \qquad / : 4$
$y = \frac{4}{4}$
$y = 1$
$x = 2 - 2y$
$x = 2 - 2 \cdot 1$
$x = 0$
$L = \{0/1\}$

$I \qquad 2x + 4y = 4$
$II \qquad -\frac{1}{2}x + 3y = 3$
I nach y auflösen
$2x + 4y = 4$
$2x + 4y = 4 \qquad / -2x$
$4y = 4 - 2x \qquad / : 4$
$y = 1 - \frac{1}{2}x$
I in II
$-\frac{1}{2}x + 3(1 - \frac{1}{2}x) = 3$
$3 - 1\frac{1}{2}x + 3x = 3 \qquad / -3$
$-1\frac{1}{2}x + 3x = 3 - 3$
$-2x = 0 \qquad / : (-2)$
$x = \frac{0}{-2}$
$x = 0$
$y = 1 - \frac{1}{2}x$
$y = 1 - \frac{1}{2} \cdot 0$
$y = 1$
$L = \{0/1\}$

Aufgabe (17)

$I \qquad -\frac{1}{2}x + 4y = 6$
$II \qquad -2x - 8y = 2$
I nach x auflösen
$-\frac{1}{2}x + 4y = 6$
$-\frac{1}{2}x + 4y = 6 \qquad / -4y$
$-\frac{1}{2}x = 6 - 4y \qquad / : \left(-\frac{1}{2}\right)$
$x = -12 + 8y$
I in II
$-2(-12 + 8y) + -8y = 2$
$24 - 16y - 8y = 2 \qquad / -24$
$-16y - 8y = 2 - 24$
$-24y = -22 \qquad / : (-24)$
$y = \frac{-22}{-24}$
$y = \frac{11}{12}$
$x = -12 + 8y$
$x = -12 + 8 \cdot \frac{11}{12}$
$x = -4\frac{2}{3}$
$L = \{-4\frac{2}{3}/\frac{11}{12}\}$

$I \qquad -\frac{1}{2}x + 4y = 6$
$II \qquad -2x - 8y = 2$
I nach y auflösen
$-\frac{1}{2}x + 4y = 6$
$-\frac{1}{2}x + 4y = 6 \qquad / +\frac{1}{2}x$
$4y = 6 + \frac{1}{2}x \qquad / : 4$
$y = 1\frac{1}{2} + \frac{1}{8}x$
I in II
$-2x + -8(1\frac{1}{2} + \frac{1}{8}x) = 2$
$-12 - 1x - 8x = 2 \qquad / -(-12)$
$-1x - 8x = 2 - (-12)$
$-3x = 14 \qquad / : (-3)$
$x = \frac{14}{-3}$
$x = -4\frac{2}{3}$
$y = 1\frac{1}{2} + \frac{1}{8}x$
$y = 1\frac{1}{2} + \frac{1}{8} \cdot \left(-4\frac{2}{3}\right)$
$y = \frac{11}{12}$
$L = \{-4\frac{2}{3}/\frac{11}{12}\}$

Aufgabe (18)

$I \qquad 1x + 1y = 1$
$II \qquad 2x + 2y = 2$
I nach x auflösen
$1x + 1y = 1$
$1x + 1y = 1 \qquad / -1y$
$1x = 1 - 1y \qquad / : 1$
$x = 1 - 1y$
I in II
$2(1 - 1y) + 2y = 2$
$2 - 2y + 2y = 2 \qquad / -2$
$-2y + 2y = 2 - 2$

$L = unendlich$

$I \qquad 1x + 1y = 1$
$II \qquad 2x + 2y = 2$
I nach y auflösen
$1x + 1y = 1$
$1x + 1y = 1 \qquad / -1x$
$1y = 1 - 1x \qquad / : 1$
$y = 1 - 1x$
I in II
$2x + 2(1 - 1x) = 2$
$2 - 2x + 2x = 2 \qquad / -2$
$-2x + 2x = 2 - 2$

$L = unendlich$

Aufgabe (19)

$I \qquad 1x + 1y = 2$
$II \qquad 3x + 3y = 3$
I nach x auflösen
$1x + 1y = 2$
$1x + 1y = 2 \qquad / -1y$
$1x = 2 - 1y \qquad / : 1$
$x = 2 - 1y$
I in II
$3(2 - 1y) + 3y = 3$
$6 - 3y + 3y = 3 \qquad / -6$
$-3y + 3y = 3 - 6$

$L = \{\}$

$I \qquad 1x + 1y = 2$
$II \qquad 3x + 3y = 3$
I nach y auflösen
$1x + 1y = 2$
$1x + 1y = 2 \qquad / -1x$
$1y = 2 - 1x \qquad / : 1$
$y = 2 - 1x$
I in II
$3x + 3(2 - 1x) = 3$
$6 - 3x + 3x = 3 \qquad / -6$
$-3x + 3x = 3 - 6$

$L = \{\}$

Aufgabe (20)

$I \qquad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \qquad -\frac{2}{3}x + \frac{1}{9}y = 9$
I nach x auflösen
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \qquad / -1\frac{1}{3}y$
$-1\frac{4}{5}x = -1 - 1\frac{1}{3}y \qquad / : \left(-1\frac{4}{5}\right)$
$x = \frac{5}{9} + \frac{20}{27}y$
I in II
$-\frac{2}{3}(\frac{5}{9} + \frac{20}{27}y) + \frac{1}{9}y = 9$
$-\frac{10}{27} - \frac{40}{81}y + \frac{1}{9}y = 9 \qquad / - \left(-\frac{10}{27}\right)$
$-\frac{40}{81}y + \frac{1}{9}y = 9 - \left(-\frac{10}{27}\right)$
$-\frac{31}{81}y = 9\frac{10}{27} \qquad / : \left(-\frac{31}{81}\right)$
$y = \frac{9\frac{10}{27}}{-\frac{31}{81}}$
$y = -24\frac{15}{31}$
$x = \frac{5}{9} + \frac{20}{27}y$
$x = \frac{5}{9} + \frac{20}{27} \cdot \left(-24\frac{15}{31}\right)$
$x = -17\frac{18}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

$I \qquad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \qquad -\frac{2}{3}x + \frac{1}{9}y = 9$
I nach y auflösen
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \qquad / + 1\frac{4}{5}x$
$1\frac{1}{3}y = -1 + 1\frac{4}{5}x \qquad / : 1\frac{1}{3}$
$y = -\frac{3}{4} + 1\frac{7}{20}x$
I in II
$-\frac{2}{3}x + \frac{1}{9}(-\frac{3}{4} + 1\frac{7}{20}x) = 9$
$-\frac{1}{12} + \frac{3}{20}x + \frac{1}{9}x = 9 \qquad / - \left(-\frac{1}{12}\right)$
$+\frac{3}{20}x + \frac{1}{9}x = 9 - \left(-\frac{1}{12}\right)$
$-\frac{31}{60}x = 9\frac{1}{12} \qquad / : \left(-\frac{31}{60}\right)$
$x = \frac{9\frac{1}{12}}{-\frac{31}{60}}$
$x = -17\frac{18}{31}$
$y = -\frac{3}{4} + 1\frac{7}{20}x$
$y = -\frac{3}{4} + 1\frac{7}{20} \cdot \left(-17\frac{18}{31}\right)$
$y = -24\frac{15}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

Aufgabe (21)

$I \qquad 2x - 7y = -8$
$II \qquad 7x - 1y = -9$
I nach x auflösen
$2x - 7y = -8$
$2x - 7y = -8 \qquad / + 7y$
$2x = -8 + 7y \qquad / : 2$
$x = -4 + 3\frac{1}{2}y$
I in II
$7(-4 + 3\frac{1}{2}y) + -1y = -9$
$-28 + 24\frac{1}{2}y - 1y = -9 \qquad / - (-28)$
$+24\frac{1}{2}y - 1y = -9 - (-28)$
$23\frac{1}{2}y = 19 \qquad / : 23\frac{1}{2}$
$y = \frac{19}{23\frac{1}{2}}$
$y = \frac{38}{47}$
$x = -4 + 3\frac{1}{2}y$
$x = -4 + 3\frac{1}{2} \cdot \frac{38}{47}$
$x = -1\frac{8}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

$I \qquad 2x - 7y = -8$
$II \qquad 7x - 1y = -9$
I nach y auflösen
$2x - 7y = -8$
$2x - 7y = -8 \qquad / - 2x$
$-7y = -8 - 2x \qquad / : (-7)$
$y = 1\frac{1}{7} + \frac{2}{7}x$
I in II
$7x + -1(1\frac{1}{7} + \frac{2}{7}x) = -9$
$-1\frac{1}{7} - \frac{2}{7}x - 1x = -9 \qquad / - \left(-1\frac{1}{7}\right)$
$-\frac{2}{7}x - 1x = -9 - \left(-1\frac{1}{7}\right)$
$6\frac{5}{7}x = -7\frac{6}{7} \qquad / : 6\frac{5}{7}$
$x = \frac{-7\frac{6}{7}}{6\frac{5}{7}}$
$x = -1\frac{8}{47}$
$y = 1\frac{1}{7} + \frac{2}{7}x$
$y = 1\frac{1}{7} + \frac{2}{7} \cdot \left(-1\frac{8}{47}\right)$
$y = \frac{38}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

Aufgabe (22)

$I \qquad -7x+9y=-3$
$II \qquad 5x-6y=-4$
I nach x auflösen
$-7x+9y=-3$
$-7x+9y=-3 \qquad /-9y$
$-7x=-3-9y \qquad /:(-7)$
$x=\frac{3}{7}+1\frac{2}{7}y$
I in II
$5(\frac{3}{7}+1\frac{2}{7}y)+-6y=-4$
$2\frac{1}{7}+6\frac{3}{7}y-6y=-4 \qquad /-2\frac{1}{7}$
$+6\frac{3}{7}y-6y=-4-2\frac{1}{7}$
$\frac{3}{7}y=-6\frac{1}{7} \qquad /:\frac{3}{7}$
$y=\frac{-6\frac{1}{7}}{\frac{3}{7}}$
$y=-14\frac{1}{3}$
$x=\frac{3}{7}+1\frac{2}{7}y$
$x=\frac{3}{7}+1\frac{2}{7}\cdot\left(-14\frac{1}{3}\right)$
$x=-18$
$L=\{-18/-14\frac{1}{3}\}$

$I \qquad -7x+9y=-3$
$II \qquad 5x-6y=-4$
I nach y auflösen
$-7x+9y=-3$
$-7x+9y=-3 \qquad /+7x$
$9y=-3+7x \qquad /:9$
$y=-\frac{1}{3}+\frac{7}{9}x$
I in II
$5x+-6(-\frac{1}{3}+\frac{7}{9}x)=-4$
$2-4\frac{2}{3}x-6x=-4 \qquad /-2$
$-4\frac{2}{3}x-6x=-4-2$
$\frac{1}{3}x=-6 \qquad /:\frac{1}{3}$
$x=\frac{-6}{\frac{1}{3}}$
$x=-18$
$y=-\frac{1}{3}+\frac{7}{9}x$
$y=-\frac{1}{3}+\frac{7}{9}\cdot(-18)$
$y=-14\frac{1}{3}$
$L=\{-18/-14\frac{1}{3}\}$

Aufgabe (23)

$I \qquad 2x+2y=1\frac{7}{10}$
$II \qquad 3x+6y=3$
I nach x auflösen
$2x+2y=1\frac{7}{10}$
$2x+2y=1\frac{7}{10} \qquad /-2y$
$2x=1\frac{7}{10}-2y \qquad /:2$
$x=\frac{17}{20}-1y$
I in II
$3(\frac{17}{20}-1y)+6y=3$
$2\frac{11}{20}-3y+6y=3 \qquad /-2\frac{11}{20}$
$-3y+6y=3-2\frac{11}{20}$
$3y=\frac{9}{20} \qquad /:3$
$y=\frac{\frac{9}{20}}{3}$
$y=\frac{3}{20}$
$x=\frac{17}{20}-1y$
$x=\frac{17}{20}-1\cdot\frac{3}{20}$
$x=\frac{7}{10}$
$L=\{\frac{7}{10}/\frac{3}{20}\}$

$I \qquad 2x+2y=1\frac{7}{10}$
$II \qquad 3x+6y=3$
I nach y auflösen
$2x+2y=1\frac{7}{10}$
$2x+2y=1\frac{7}{10} \qquad /-2x$
$2y=1\frac{7}{10}-2x \qquad /:2$
$y=\frac{17}{20}-1x$
I in II
$3x+6(\frac{17}{20}-1x)=3$
$5\frac{1}{10}-6x+6x=3 \qquad /-5\frac{1}{10}$
$-6x+6x=3-5\frac{1}{10}$
$-3x=-2\frac{1}{10} \qquad /:(-3)$
$x=\frac{-2\frac{1}{10}}{-3}$
$x=\frac{7}{10}$
$y=\frac{17}{20}-1x$
$y=\frac{17}{20}-1\cdot\frac{7}{10}$
$y=\frac{3}{20}$
$L=\{\frac{7}{10}/\frac{3}{20}\}$

Aufgabe (24)

$I \qquad 3x + 5y = 8$
$II \qquad 9x + 0y = 7$
I nach x auflösen
$3x + 5y = 8$
$3x + 5y = 8 \qquad / - 5y$
$3x = 8 - 5y \qquad / : 3$
$x = 2\frac{2}{3} - 1\frac{2}{3}y$
I in II
$9(2\frac{2}{3} - 1\frac{2}{3}y) + 0y = 7$
$24 - 15y + 0y = 7 \qquad / - 24$
$-15y + 0y = 7 - 24$
$-15y = -17 \qquad / : (-15)$
$y = \frac{-17}{-15}$
$y = 1\frac{2}{15}$
$x = 2\frac{2}{3} - 1\frac{2}{3}y$
$x = 2\frac{2}{3} - 1\frac{2}{3} \cdot 1\frac{2}{15}$
$x = \frac{7}{9}$
$L = \{\frac{7}{9}/1\frac{2}{15}\}$

$I \qquad 3x + 5y = 8$
$II \qquad 9x + 0y = 7$
I nach y auflösen
$3x + 5y = 8$
$3x + 5y = 8 \qquad / - 3x$
$5y = 8 - 3x \qquad / : 5$
$y = 1\frac{3}{5} - \frac{3}{5}x$
I in II
$9x + 0(1\frac{3}{5} - \frac{3}{5}x) = 7$
$0 - 0x + 0x = 7 \qquad / - 0$
$-0x + 0x = 7 - 0$
$9x = 7 \qquad / : 9$
$x = \frac{7}{9}$
$x = \frac{7}{9}$
$y = 1\frac{3}{5} - \frac{3}{5}x$
$y = 1\frac{3}{5} - \frac{3}{5} \cdot \frac{7}{9}$
$y = 1\frac{2}{15}$
$L = \{\frac{7}{9}/1\frac{2}{15}\}$

Aufgabe (25)

$I \qquad 3x + 6y = 8$
$II \qquad 9x + 0y = 9$
I nach x auflösen
$3x + 6y = 8$
$3x + 6y = 8 \qquad / - 6y$
$3x = 8 - 6y \qquad / : 3$
$x = 2\frac{2}{3} - 2y$
I in II
$9(2\frac{2}{3} - 2y) + 0y = 9$
$24 - 18y + 0y = 9 \qquad / - 24$
$-18y + 0y = 9 - 24$
$-18y = -15 \qquad / : (-18)$
$y = \frac{-15}{-18}$
$y = \frac{5}{6}$
$x = 2\frac{2}{3} - 2y$
$x = 2\frac{2}{3} - 2 \cdot \frac{5}{6}$
$x = 1$
$L = \{1/\frac{5}{6}\}$

$I \qquad 3x + 6y = 8$
$II \qquad 9x + 0y = 9$
I nach y auflösen
$3x + 6y = 8$
$3x + 6y = 8 \qquad / - 3x$
$6y = 8 - 3x \qquad / : 6$
$y = 1\frac{1}{3} - \frac{1}{2}x$
I in II
$9x + 0(1\frac{1}{3} - \frac{1}{2}x) = 9$
$0 - 0x + 0x = 9 \qquad / - 0$
$-0x + 0x = 9 - 0$
$9x = 9 \qquad / : 9$
$x = \frac{9}{9}$
$x = 1$
$y = 1\frac{1}{3} - \frac{1}{2}x$
$y = 1\frac{1}{3} - \frac{1}{2} \cdot 1$
$y = \frac{5}{6}$
$L = \{1/\frac{5}{6}\}$

Aufgabe (26)

$I \qquad 3x + 6y = 8$
$II \qquad 9x + 0y = 9$
I nach x auflösen
$3x + 6y = 8$
$3x + 6y = 8 \qquad / - 6y$
$3x = 8 - 6y \qquad / : 3$
$x = 2\frac{2}{3} - 2y$
I in II
$9(2\frac{2}{3} - 2y) + 0y = 9$
$24 - 18y + 0y = 9 \qquad / - 24$
$-18y + 0y = 9 - 24$
$-18y = -15 \qquad / : (-18)$
$y = \frac{-15}{-18}$
$y = \frac{5}{6}$
$x = 2\frac{2}{3} - 2y$
$x = 2\frac{2}{3} - 2 \cdot \frac{5}{6}$
$x = 1$
$L = \{1/\frac{5}{6}\}$

$I \qquad 3x + 6y = 8$
$II \qquad 9x + 0y = 9$
I nach y auflösen
$3x + 6y = 8$
$3x + 6y = 8 \qquad / - 3x$
$6y = 8 - 3x \qquad / : 6$
$y = 1\frac{1}{3} - \frac{1}{2}x$
I in II
$9x + 0(1\frac{1}{3} - \frac{1}{2}x) = 9$
$0 - 0x + 0x = 9 \qquad / - 0$
$-0x + 0x = 9 - 0$
$9x = 9 \qquad / : 9$
$x = \frac{9}{9}$
$x = 1$
$y = 1\frac{1}{3} - \frac{1}{2}x$
$y = 1\frac{1}{3} - \frac{1}{2} \cdot 1$
$y = \frac{5}{6}$
$L = \{1/\frac{5}{6}\}$

2 Gleichsetzungsverfahren (2)

$I \quad a1 \cdot x + b1 \cdot y = c1$
$II \quad a2 \cdot x + b2 \cdot y = c42$

- beide Gleichungen nach x oder y auflösen
- Terme gleichsetzen
- Gleichung nach der Unbekannten auflösen
- zweite Unbekannte berechnen

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach y auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 3x$
$5y = 19 - 3x \quad / : 5$
$y = 3\frac{4}{5} - \frac{3}{5}x$
II nach y auflösen
$7x + 5y = 31$
$7x + 5y = 31 \quad / - 7x$
$5y = 31 - 7x \quad / : 5$
$y = 6\frac{1}{5} - 1\frac{2}{5}x$
I = II
$3\frac{4}{5} - \frac{3}{5}x = 6\frac{1}{5} - 1\frac{2}{5}x \quad / + \frac{3}{5}x$
$3\frac{4}{5} = 6\frac{1}{5} - \frac{4}{5}x \quad / - 6\frac{1}{5}$
$-2\frac{2}{5} = -\frac{4}{5}x \quad / : \left(-\frac{4}{5}\right)$
$x = 3$
x in I
$y = 3\frac{4}{5} - \frac{3}{5}3$
$y = 2$
$L = \{3/2\}$

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach x auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 5y$
$3x = 19 - 5y \quad / : 3$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
II nach x auflösen
$7x + 5y = 31$
$7x + 5y = 31 \quad / - 5y$
$7x = 31 - 5y \quad / : 7$
$x = 4\frac{3}{7} - \frac{5}{7}y$
I = II
$6\frac{1}{3} - 1\frac{2}{3}y = 4\frac{3}{7} - \frac{5}{7}y \quad / + 1\frac{2}{3}y$
$6\frac{1}{3} = 4\frac{3}{7} + \frac{20}{21}y \quad / - 4\frac{3}{7}$
$1\frac{19}{21} = \frac{20}{21}y \quad / : \frac{20}{21}$
$y = 2$
y in I
$x = 6\frac{1}{3} - 1\frac{2}{3}2$
$x = 3$
$L = \{3/2\}$

2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
$a1 \cdot x + b1 \cdot y = c1$
$a2 \cdot x + b2 \cdot y = c2$

Gesucht:
x und y

(1) $3x + 5y = 19$, $7x + 5y = 31$

(2) $1x + 1y = 10$, $1x - 1y = 4$

(3) $9x - 2y = 5$, $5x - 2y = 1$

(4) $9x - 2y = 1$, $-3x - 3y = -7$

(5) $1x + 3y = 9$, $3x - 2y = -6$

(6) $7x + 1\frac{1}{2}y = 1$, $2x - 5y = 22$

(7) $4x - 5y = 43$, $-3x - 7y = 0$

(8) $8x - 3y = 10$, $12x - 5y = 24$

(9) $-\frac{1}{2}x + 1y = 2$, $\frac{1}{2}x - 3y = -3$

(10) $-1x + 1y = 3$, $\frac{1}{2}x - 4y = 5$

(11) $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$

(12) $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$1x + 1y = 10\frac{2}{3}$

(13) $1\frac{1}{2}x - 2y = 9$
$\frac{2}{5}x + \frac{1}{3}y = 5$

(14) $2x + 3y = 4$
$\frac{1}{3}x - \frac{1}{5}y = 12$

(15) $1x + 2y = -4$
$-1x + 2y = 5$

(16) $2x + 4y = 4$
$-\frac{1}{2}x + 3y = 3$

(17) $-\frac{1}{2}x + 4y = 6$
$-2x - 8y = 2$

(18) $1x + 1y = 1$
$2x + 2y = 2$

(19) $1x + 1y = 2$
$3x + 3y = 3$

(20) $-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-\frac{2}{3}x + \frac{1}{9}y = 9$

(21) $2x - 7y = -8$
$7x - 1y = -9$

(22) $-7x + 9y = -3$
$5x - 6y = -4$

(23) $2x + 2y = 1\frac{7}{10}$
$3x + 6y = 3$

(24) $34x + 5y = 7$
$7x + 9y = 8$

(25) $34x + 5y = 7$
$7x + 9y = 8$

(26) $34x + 5y = 7$
$7x + 9y = 8$

(27) $2x + 5y = 7$
$-4x + 2y = -6$

(28) $2x + 5y = 7$
$-4x + 2y = -6$

2.2 Lösungen

Aufgabe (1)

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach y auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 3x$
$5y = 19 - 3x \quad / : 5$
$y = 3\frac{4}{5} - \frac{3}{5}x$
II nach y auflösen
$7x + 5y = 31$
$7x + 5y = 31 \quad / - 7x$
$5y = 31 - 7x \quad / : 5$
$y = 6\frac{1}{5} - 1\frac{2}{5}x$
I = II
$3\frac{4}{5} - \frac{3}{5}x = 6\frac{1}{5} - 1\frac{2}{5}x \quad / + \frac{3}{5}x / - 6\frac{1}{5}$
$3\frac{4}{5} - 6\frac{1}{5} = -1\frac{2}{5}x + \frac{3}{5}x$
$-2\frac{2}{5} = -\frac{4}{5}x \quad / : \left(-\frac{4}{5}\right)$
$x = 3$
x in I
$y = 3\frac{4}{5} - \frac{3}{5} \cdot 3$
$y = 2$
$L = \{3/2\}$

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
I nach x auflösen
$3x + 5y = 19$
$3x + 5y = 19 \quad / - 5y$
$3x = 19 - 5y \quad / : 3$
$x = 6\frac{1}{3} - 1\frac{2}{3}y$
II nach x auflösen
$7x + 5y = 31$
$7x + 5y = 31 \quad / - 5y$
$7x = 31 - 5y \quad / : 7$
$x = 4\frac{3}{7} - \frac{5}{7}y$
I = II
$6\frac{1}{3} - 1\frac{2}{3}y = 4\frac{3}{7} - \frac{5}{7}y \quad / + 1\frac{2}{3}y / - 4\frac{3}{7}$
$6\frac{1}{3} - 4\frac{3}{7} = -\frac{5}{7}y + 1\frac{2}{3}y$
$1\frac{19}{21} = \frac{20}{21}y \quad / : \frac{20}{21}$
$y = 2$
y in I
$x = 6\frac{1}{3} - 1\frac{2}{3} \cdot 2$
$x = 3$
$L = \{3/2\}$

Aufgabe (2)

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
I nach y auflösen
$1x + 1y = 10$
$1x + 1y = 10 \quad / - 1x$
$1y = 10 - 1x \quad / : 1$
$y = 10 - 1x$
II nach y auflösen
$1x - 1y = 4$
$1x - 1y = 4 \quad / - 1x$
$-1y = 4 - 1x \quad / : (-1)$
$y = -4 + 1x$
I = II
$10 - 1x = -4 + 1x \quad / + 1x / + 4$
$10 + 4 = 1x + 1x$
$14 = 2x \quad / : 2$
$x = 7$
x in I
$y = 10 - 1 \cdot 7$
$y = 3$
$L = \{7/3\}$

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
I nach x auflösen
$1x + 1y = 10$
$1x + 1y = 10 \quad / - 1y$
$1x = 10 - 1y \quad / : 1$
$x = 10 - 1y$
II nach x auflösen
$1x - 1y = 4$
$1x - 1y = 4 \quad / + 1y$
$1x = 4 + 1y \quad / : 1$
$x = 4 + 1y$
I = II
$10 - 1y = 4 + 1y \quad / + 1y / - 4$
$10 - 4 = 1y + 1y$
$6 = 2y \quad / : 2$
$y = 3$
y in I
$x = 10 - 1 \cdot 3$
$x = 7$
$L = \{7/3\}$

Aufgabe (3)

$I \qquad 9x - 2y = 5$
$II \qquad 5x - 2y = 1$
I nach y auflösen
$9x - 2y = 5$
$9x - 2y = 5 \qquad / - 9x$
$-2y = 5 - 9x \qquad / : (-2)$
$y = -2\frac{1}{2} + 4\frac{1}{2}x$
II nach y auflösen
$5x - 2y = 1$
$5x - 2y = 1 \qquad / - 5x$
$-2y = 1 - 5x \qquad / : (-2)$
$y = -\frac{1}{2} + 2\frac{1}{2}x$
I = II
$-2\frac{1}{2} + 4\frac{1}{2}x = -\frac{1}{2} + 2\frac{1}{2}x \qquad / - 4\frac{1}{2}x / + \frac{1}{2}$
$-2\frac{1}{2} + \frac{1}{2} = 2\frac{1}{2}x - 4\frac{1}{2}x$
$-2 = -2x \qquad / : (-2)$
$x = 1$
x in I
$y = -2\frac{1}{2} + 4\frac{1}{2} \cdot 1$
$y = 2$
$L = \{1/2\}$

$I \qquad 9x - 2y = 5$
$II \qquad 5x - 2y = 1$
I nach x auflösen
$9x - 2y = 5$
$9x - 2y = 5 \qquad / + 2y$
$9x = 5 + 2y \qquad / : 9$
$x = \frac{5}{9} + \frac{2}{9}y$
II nach x auflösen
$5x - 2y = 1$
$5x - 2y = 1 \qquad / + 2y$
$5x = 1 + 2y \qquad / : 5$
$x = \frac{1}{5} + \frac{2}{5}y$
I = II
$\frac{5}{9} + \frac{2}{9}y = \frac{1}{5} + \frac{2}{5}y \qquad / - \frac{2}{9}y / - \frac{1}{5}$
$\frac{5}{9} - \frac{1}{5} = \frac{2}{5}y - \frac{2}{9}y$
$\frac{16}{45} = \frac{8}{45}y \qquad / : \frac{8}{45}$
$y = 2$
y in I
$x = \frac{5}{9} + \frac{2}{9} \cdot 2$
$x = 1$
$L = \{1/2\}$

Aufgabe (4)

$I \qquad 9x - 2y = 1$
$II \qquad -3x - 3y = -7$
I nach y auflösen
$9x - 2y = 1$
$9x - 2y = 1 \qquad / - 9x$
$-2y = 1 - 9x \qquad / : (-2)$
$y = -\frac{1}{2} + 4\frac{1}{2}x$
II nach y auflösen
$-3x - 3y = -7$
$-3x - 3y = -7 \qquad / + 3x$
$-3y = -7 + 3x \qquad / : (-3)$
$y = 2\frac{1}{3} - 1x$
I = II
$-\frac{1}{2} + 4\frac{1}{2}x = 2\frac{1}{3} - 1x \qquad / - 4\frac{1}{2}x / - 2\frac{1}{3}$
$-\frac{1}{2} - 2\frac{1}{3} = -1x - 4\frac{1}{2}x$
$-2\frac{5}{6} = -5\frac{1}{2}x \qquad / : \left(-5\frac{1}{2}\right)$
$x = \frac{17}{33}$
x in I
$y = -\frac{1}{2} + 4\frac{1}{2} \cdot \frac{17}{33}$
$y = 1\frac{9}{11}$
$L = \{\frac{17}{33} / 1\frac{9}{11}\}$

$I \qquad 9x - 2y = 1$
$II \qquad -3x - 3y = -7$
I nach x auflösen
$9x - 2y = 1$
$9x - 2y = 1 \qquad / + 2y$
$9x = 1 + 2y \qquad / : 9$
$x = \frac{1}{9} + \frac{2}{9}y$
II nach x auflösen
$-3x - 3y = -7$
$-3x - 3y = -7 \qquad / + 3y$
$-3x = -7 + 3y \qquad / : (-3)$
$x = 2\frac{1}{3} - 1y$
I = II
$\frac{1}{9} + \frac{2}{9}y = 2\frac{1}{3} - 1y \qquad / - \frac{2}{9}y / - 2\frac{1}{3}$
$\frac{1}{9} - 2\frac{1}{3} = -1y - \frac{2}{9}y$
$-2\frac{2}{9} = -1\frac{2}{9}y \qquad / : \left(-1\frac{2}{9}\right)$
$y = 1\frac{9}{11}$
y in I
$x = \frac{1}{9} + \frac{2}{9} \cdot 1\frac{9}{11}$
$x = \frac{17}{33}$
$L = \{\frac{17}{33} / 1\frac{9}{11}\}$

Aufgabe (5)

$I \quad 1x + 3y = 9$
$II \quad 3x - 2y = -6$
I nach y auflösen
$1x + 3y = 9$
$1x + 3y = 9 \quad / - 1x$
$3y = 9 - 1x \quad / : 3$
$y = 3 - \frac{1}{3}x$
II nach y auflösen
$3x - 2y = -6$
$3x - 2y = -6 \quad / - 3x$
$-2y = -6 - 3x \quad / : (-2)$
$y = 3 + 1\frac{1}{2}x$
I = II
$3 - \frac{1}{3}x = 3 + 1\frac{1}{2}x \quad / + \frac{1}{3}x / - 3$
$3 - 3 = 1\frac{1}{2}x + \frac{1}{3}x$
$0 = 1\frac{5}{6}x \quad / : 1\frac{5}{6}$
$x = 0$
x in I
$y = 3 - \frac{1}{3} \cdot 0$
$y = 3$
$L = \{0/3\}$

$I \quad 1x + 3y = 9$
$II \quad 3x - 2y = -6$
I nach x auflösen
$1x + 3y = 9$
$1x + 3y = 9 \quad / - 3y$
$1x = 9 - 3y \quad / : 1$
$x = 9 - 3y$
II nach x auflösen
$3x - 2y = -6$
$3x - 2y = -6 \quad / + 2y$
$3x = -6 + 2y \quad / : 3$
$x = -2 + \frac{2}{3}y$
I = II
$9 - 3y = -2 + \frac{2}{3}y \quad / + 3y / + 2$
$9 + 2 = \frac{2}{3}y + 3y$
$11 = 3\frac{2}{3}y \quad / : 3\frac{2}{3}$
$y = 3$
y in I
$x = 9 - 3 \cdot 3$
$x = 0$
$L = \{0/3\}$

Aufgabe (6)

$I \quad 7x + 1\frac{1}{2}y = 1$
$II \quad 2x - 5y = 22$
I nach y auflösen
$7x + 1\frac{1}{2}y = 1$
$7x + 1\frac{1}{2}y = 1 \quad / - 7x$
$1\frac{1}{2}y = 1 - 7x \quad / : 1\frac{1}{2}$
$y = \frac{2}{3} - 4\frac{2}{3}x$
II nach y auflösen
$2x - 5y = 22$
$2x - 5y = 22 \quad / - 2x$
$-5y = 22 - 2x \quad / : (-5)$
$y = -4\frac{2}{5} + \frac{2}{5}x$
I = II
$\frac{2}{3} - 4\frac{2}{3}x = -4\frac{2}{5} + \frac{2}{5}x \quad / + 4\frac{2}{3}x / + 4\frac{2}{5}$
$\frac{2}{3} + 4\frac{2}{5} = \frac{2}{5}x + 4\frac{2}{3}x$
$5\frac{1}{15} = 5\frac{1}{15}x \quad / : 5\frac{1}{15}$
$x = 1$
x in I
$y = \frac{2}{3} - 4\frac{2}{3} \cdot 1$
$y = -4$
$L = \{1/-4\}$

$I \quad 7x + 1\frac{1}{2}y = 1$
$II \quad 2x - 5y = 22$
I nach x auflösen
$7x + 1\frac{1}{2}y = 1$
$7x + 1\frac{1}{2}y = 1 \quad / - 1\frac{1}{2}y$
$7x = 1 - 1\frac{1}{2}y \quad / : 7$
$x = \frac{1}{7} - \frac{3}{14}y$
II nach x auflösen
$2x - 5y = 22$
$2x - 5y = 22 \quad / + 5y$
$2x = 22 + 5y \quad / : 2$
$x = 11 + 2\frac{1}{2}y$
I = II
$\frac{1}{7} - \frac{3}{14}y = 11 + 2\frac{1}{2}y \quad / + \frac{3}{14}y / - 11$
$\frac{1}{7} - 11 = 2\frac{1}{2}y + \frac{3}{14}y$
$-10\frac{6}{7} = 2\frac{5}{7}y \quad / : 2\frac{5}{7}$
$y = -4$
y in I
$x = \frac{1}{7} - \frac{3}{14} \cdot -4$
$x = 1$
$L = \{1/-4\}$

Aufgabe (7)

$I \qquad 4x - 5y = 43$
$II \qquad -3x - 7y = 0$
I nach y auflösen
$4x - 5y = 43$
$4x - 5y = 43 \qquad / -4x$
$-5y = 43 - 4x \qquad / : (-5)$
$y = -8\frac{3}{5} + \frac{4}{5}x$
II nach y auflösen
$-3x - 7y = 0$
$-3x - 7y = 0 \qquad / +3x$
$-7y = 0 + 3x \qquad / : (-7)$
$y = 0 - \frac{3}{7}x$
I = II
$-8\frac{3}{5} + \frac{4}{5}x = 0 - \frac{3}{7}x \qquad / -\frac{4}{5}x / -0$
$-8\frac{3}{5} - 0 = -\frac{3}{7}x - \frac{4}{5}x$
$-8\frac{3}{5} = -1\frac{8}{35}x \qquad / : \left(-1\frac{8}{35}\right)$
$x = 7$
x in I
$y = -8\frac{3}{5} + \frac{4}{5} \cdot 7$
$y = -3$
$L = \{7/-3\}$

$I \qquad 4x - 5y = 43$
$II \qquad -3x - 7y = 0$
I nach x auflösen
$4x - 5y = 43$
$4x - 5y = 43 \qquad / +5y$
$4x = 43 + 5y \qquad / : 4$
$x = 10\frac{3}{4} + 1\frac{1}{4}y$
II nach x auflösen
$-3x - 7y = 0$
$-3x - 7y = 0 \qquad / +7y$
$-3x = 0 + 7y \qquad / : (-3)$
$x = 0 - 2\frac{1}{3}y$
I = II
$10\frac{3}{4} + 1\frac{1}{4}y = 0 - 2\frac{1}{3}y \qquad / -1\frac{1}{4}y / -0$
$10\frac{3}{4} - 0 = -2\frac{1}{3}y - 1\frac{1}{4}y$
$10\frac{3}{4} = -3\frac{7}{12}y \qquad / : \left(-3\frac{7}{12}\right)$
$y = -3$
y in I
$x = 10\frac{3}{4} + 1\frac{1}{4} \cdot -3$
$x = 7$
$L = \{7/-3\}$

Aufgabe (8)

$I \qquad 8x - 3y = 10$
$II \qquad 12x - 5y = 24$
I nach y auflösen
$8x - 3y = 10$
$8x - 3y = 10 \qquad / -8x$
$-3y = 10 - 8x \qquad / : (-3)$
$y = -3\frac{1}{3} + 2\frac{2}{3}x$
II nach y auflösen
$12x - 5y = 24$
$12x - 5y = 24 \qquad / -12x$
$-5y = 24 - 12x \qquad / : (-5)$
$y = -4\frac{4}{5} + 2\frac{2}{5}x$
I = II
$-3\frac{1}{3} + 2\frac{2}{3}x = -4\frac{4}{5} + 2\frac{2}{5}x \qquad / -2\frac{2}{3}x / +4\frac{4}{5}$
$-3\frac{1}{3} + 4\frac{4}{5} = 2\frac{2}{5}x - 2\frac{2}{3}x$
$1\frac{7}{15} = -\frac{4}{15}x \qquad / : \left(-\frac{4}{15}\right)$
$x = -5\frac{1}{2}$
x in I
$y = -3\frac{1}{3} + 2\frac{2}{3} \cdot -5\frac{1}{2}$
$y = -18$
$L = \{-5\frac{1}{2}/-18\}$

$I \qquad 8x - 3y = 10$
$II \qquad 12x - 5y = 24$
I nach x auflösen
$8x - 3y = 10$
$8x - 3y = 10 \qquad / +3y$
$8x = 10 + 3y \qquad / : 8$
$x = 1\frac{1}{4} + \frac{3}{8}y$
II nach x auflösen
$12x - 5y = 24$
$12x - 5y = 24 \qquad / +5y$
$12x = 24 + 5y \qquad / : 12$
$x = 2 + \frac{5}{12}y$
I = II
$1\frac{1}{4} + \frac{3}{8}y = 2 + \frac{5}{12}y \qquad / -\frac{3}{8}y / -2$
$1\frac{1}{4} - 2 = \frac{5}{12}y - \frac{3}{8}y$
$-\frac{3}{4} = \frac{1}{24}y \qquad / : \frac{1}{24}$
$y = -18$
y in I
$x = 1\frac{1}{4} + \frac{3}{8} \cdot -18$
$x = -5\frac{1}{2}$
$L = \{-5\frac{1}{2}/-18\}$

Aufgabe (9)

$I \qquad -\frac{1}{2}x + 1y = 2$
$II \qquad \frac{1}{2}x - 3y = -3$
I nach y auflösen
$-\frac{1}{2}x + 1y = 2$
$-\frac{1}{2}x + 1y = 2 \qquad / + \frac{1}{2}x$
$1y = 2 + \frac{1}{2}x \qquad / : 1$
$y = 2 + \frac{1}{2}x$
II nach y auflösen
$\frac{1}{2}x - 3y = -3$
$\frac{1}{2}x - 3y = -3 \qquad / - \frac{1}{2}x$
$-3y = -3 - \frac{1}{2}x \qquad / : (-3)$
$y = 1 + \frac{1}{6}x$
I = II
$2 + \frac{1}{2}x = 1 + \frac{1}{6}x \qquad / - \frac{1}{2}x / - 1$
$2 - 1 = \frac{1}{6}x - \frac{1}{2}x$
$1 = -\frac{1}{3}x \qquad / : \left(-\frac{1}{3}\right)$
$x = -3$
x in I
$y = 2 + \frac{1}{2} \cdot -3$
$y = \frac{1}{2}$
$L = \{-3/\frac{1}{2}\}$

$I \qquad -\frac{1}{2}x + 1y = 2$
$II \qquad \frac{1}{2}x - 3y = -3$
I nach x auflösen
$-\frac{1}{2}x + 1y = 2$
$-\frac{1}{2}x + 1y = 2 \qquad / - 1y$
$-\frac{1}{2}x = 2 - 1y \qquad / : \left(-\frac{1}{2}\right)$
$x = -4 + 2y$
II nach x auflösen
$\frac{1}{2}x - 3y = -3$
$\frac{1}{2}x - 3y = -3 \qquad / + 3y$
$\frac{1}{2}x = -3 + 3y \qquad / : \frac{1}{2}$
$x = -6 + 6y$
I = II
$-4 + 2y = -6 + 6y \qquad / - 2y / + 6$
$-4 + 6 = 6y - 2y$
$2 = 4y \qquad / : 4$
$y = \frac{1}{2}$
y in I
$x = -4 + 2 \cdot \frac{1}{2}$
$x = -3$
$L = \{-3/\frac{1}{2}\}$

Aufgabe (10)

$I \qquad -1x + 1y = 3$
$II \qquad \frac{1}{2}x - 4y = 5$
I nach y auflösen
$-1x + 1y = 3$
$-1x + 1y = 3 \qquad / + 1x$
$1y = 3 + 1x \qquad / : 1$
$y = 3 + 1x$
II nach y auflösen
$\frac{1}{2}x - 4y = 5$
$\frac{1}{2}x - 4y = 5 \qquad / - \frac{1}{2}x$
$-4y = 5 - \frac{1}{2}x \qquad / : (-4)$
$y = -1\frac{1}{4} + \frac{1}{8}x$
I = II
$3 + 1x = -1\frac{1}{4} + \frac{1}{8}x \qquad / - 1x / + 1\frac{1}{4}$
$3 + 1\frac{1}{4} = \frac{1}{8}x - 1x$
$4\frac{1}{4} = -\frac{7}{8}x \qquad / : \left(-\frac{7}{8}\right)$
$x = -4\frac{6}{7}$
x in I
$y = 3 + 1 \cdot -4\frac{6}{7}$
$y = -1\frac{6}{7}$
$L = \{-4\frac{6}{7} / -1\frac{6}{7}\}$

$I \qquad -1x + 1y = 3$
$II \qquad \frac{1}{2}x - 4y = 5$
I nach x auflösen
$-1x + 1y = 3$
$-1x + 1y = 3 \qquad / - 1y$
$-1x = 3 - 1y \qquad / : (-1)$
$x = -3 + 1y$
II nach x auflösen
$\frac{1}{2}x - 4y = 5$
$\frac{1}{2}x - 4y = 5 \qquad / + 4y$
$\frac{1}{2}x = 5 + 4y \qquad / : \frac{1}{2}$
$x = 10 + 8y$
I = II
$-3 + 1y = 10 + 8y \qquad / - 1y / - 10$
$-3 - 10 = 8y - 1y$
$-13 = 7y \qquad / : 7$
$y = -1\frac{6}{7}$
y in I
$x = -3 + 1 \cdot -1\frac{6}{7}$
$x = -4\frac{6}{7}$
$L = \{-4\frac{6}{7} / -1\frac{6}{7}\}$

Aufgabe (11)

$I \qquad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \qquad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
I nach y auflösen
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \qquad / - 1\frac{1}{5}x$
$-1\frac{1}{3}y = 5\frac{1}{3} - 1\frac{1}{5}x \qquad / : \left(-1\frac{1}{3}\right)$
$y = -4 + \frac{9}{10}x$
II nach y auflösen
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \qquad / - 2\frac{1}{2}x$
$-\frac{1}{4}y = 12\frac{3}{8} - 2\frac{1}{2}x \qquad / : \left(-\frac{1}{4}\right)$
$y = -49\frac{1}{2} + 10x$
I = II
$-4 + \frac{9}{10}x = -49\frac{1}{2} + 10x \qquad / - \frac{9}{10}x / + 49\frac{1}{2}$
$-4 + 49\frac{1}{2} = 10x - \frac{9}{10}x$
$45\frac{1}{2} = 9\frac{1}{10}x \qquad / : 9\frac{1}{10}$
$x = 5$
x in I
$y = -4 + \frac{9}{10} \cdot 5$
$y = \frac{1}{2}$
$L = \{5/\frac{1}{2}\}$

$I \qquad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \qquad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
I nach x auflösen
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \qquad / + 1\frac{1}{3}y$
$1\frac{1}{5}x = 5\frac{1}{3} + 1\frac{1}{3}y \qquad / : 1\frac{1}{5}$
$x = 4\frac{4}{9} + 1\frac{1}{9}y$
II nach x auflösen
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \qquad / + \frac{1}{4}y$
$2\frac{1}{2}x = 12\frac{3}{8} + \frac{1}{4}y \qquad / : 2\frac{1}{2}$
$x = 4\frac{19}{20} + \frac{1}{10}y$
I = II
$4\frac{4}{9} + 1\frac{1}{9}y = 4\frac{19}{20} + \frac{1}{10}y \qquad / - 1\frac{1}{9}y / - 4\frac{19}{20}$
$4\frac{4}{9} - 4\frac{19}{20} = \frac{1}{10}y - 1\frac{1}{9}y$
$-0,506 = -1\frac{1}{90}y \qquad / : \left(-1\frac{1}{90}\right)$
$y = \frac{1}{2}$
y in I
$x = 4\frac{4}{9} + 1\frac{1}{9} \cdot \frac{1}{2}$
$x = 5$
$L = \{5/\frac{1}{2}\}$

Aufgabe (12)

$I \qquad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \qquad 1x + 1y = 10\frac{2}{3}$
I nach y auflösen
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \qquad / - \frac{2}{3}x$
$-\frac{5}{7}y = \frac{2}{3} - \frac{2}{3}x \qquad / : \left(-\frac{5}{7}\right)$
$y = -\frac{14}{15} + \frac{14}{15}x$
II nach y auflösen
$1x + 1y = 10\frac{2}{3}$
$1x + 1y = 10\frac{2}{3} \qquad / - 1x$
$1y = 10\frac{2}{3} - 1x \qquad / : 1$
$y = 10\frac{2}{3} - 1x$
I = II
$-\frac{14}{15} + \frac{14}{15}x = 10\frac{2}{3} - 1x \qquad / - \frac{14}{15}x / - 10\frac{2}{3}$
$-\frac{14}{15} - 10\frac{2}{3} = -1x - \frac{14}{15}x$
$-11\frac{3}{5} = -1\frac{14}{15}x \qquad / : \left(-1\frac{14}{15}\right)$
$x = 6$
x in I
$y = -\frac{14}{15} + \frac{14}{15} \cdot 6$
$y = 4\frac{2}{3}$
$L = \{6/4\frac{2}{3}\}$

$I \qquad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \qquad 1x + 1y = 10\frac{2}{3}$
I nach x auflösen
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \qquad / + \frac{5}{7}y$
$\frac{2}{3}x = \frac{2}{3} + \frac{5}{7}y \qquad / : \frac{2}{3}$
$x = 1 + 1\frac{1}{14}y$
II nach x auflösen
$1x + 1y = 10\frac{2}{3}$
$1x + 1y = 10\frac{2}{3} \qquad / - 1y$
$1x = 10\frac{2}{3} - 1y \qquad / : 1$
$x = 10\frac{2}{3} - 1y$
I = II
$1 + 1\frac{1}{14}y = 10\frac{2}{3} - 1y \qquad / - 1\frac{1}{14}y / - 10\frac{2}{3}$
$1 - 10\frac{2}{3} = -1y - 1\frac{1}{14}y$
$-9\frac{2}{3} = -2\frac{1}{14}y \qquad / : \left(-2\frac{1}{14}\right)$
$y = 4\frac{2}{3}$
y in I
$x = 1 + 1\frac{1}{14} \cdot 4\frac{2}{3}$
$x = 6$
$L = \{6/4\frac{2}{3}\}$

Aufgabe (13)

$I \qquad 1\frac{1}{2}x - 2y = 9$
$II \qquad \frac{2}{5}x + \frac{1}{3}y = 5$
I nach y auflösen
$1\frac{1}{2}x - 2y = 9$
$1\frac{1}{2}x - 2y = 9 \qquad / - 1\frac{1}{2}x$
$-2y = 9 - 1\frac{1}{2}x \qquad / : (-2)$
$y = -4\frac{1}{2} + \frac{3}{4}x$
II nach y auflösen
$\frac{2}{5}x + \frac{1}{3}y = 5$
$\frac{2}{5}x + \frac{1}{3}y = 5 \qquad / - \frac{2}{5}x$
$\frac{1}{3}y = 5 - \frac{2}{5}x \qquad / : \frac{1}{3}$
$y = 15 - 1\frac{1}{5}x$
I = II
$-4\frac{1}{2} + \frac{3}{4}x = 15 - 1\frac{1}{5}x \qquad / - \frac{3}{4}x / - 15$
$-4\frac{1}{2} - 15 = -1\frac{1}{5}x - \frac{3}{4}x$
$-19\frac{1}{2} = -1\frac{19}{20}x \qquad / : \left(-1\frac{19}{20}\right)$
$x = 10$
x in I
$y = -4\frac{1}{2} + \frac{3}{4} \cdot 10$
$y = 3$
$L = \{10/3\}$

$I \qquad 1\frac{1}{2}x - 2y = 9$
$II \qquad \frac{2}{5}x + \frac{1}{3}y = 5$
I nach x auflösen
$1\frac{1}{2}x - 2y = 9$
$1\frac{1}{2}x - 2y = 9 \qquad / + 2y$
$1\frac{1}{2}x = 9 + 2y \qquad / : 1\frac{1}{2}$
$x = 6 + 1\frac{1}{3}y$
II nach x auflösen
$\frac{2}{5}x + \frac{1}{3}y = 5$
$\frac{2}{5}x + \frac{1}{3}y = 5 \qquad / - \frac{1}{3}y$
$\frac{2}{5}x = 5 - \frac{1}{3}y \qquad / : \frac{2}{5}$
$x = 12\frac{1}{2} - \frac{5}{6}y$
I = II
$6 + 1\frac{1}{3}y = 12\frac{1}{2} - \frac{5}{6}y \qquad / - 1\frac{1}{3}y / - 12\frac{1}{2}$
$6 - 12\frac{1}{2} = -\frac{5}{6}y - 1\frac{1}{3}y$
$-6\frac{1}{2} = -2\frac{1}{6}y \qquad / : \left(-2\frac{1}{6}\right)$
$y = 3$
y in I
$x = 6 + 1\frac{1}{3} \cdot 3$
$x = 10$
$L = \{10/3\}$

Aufgabe (14)

$I \qquad 2x + 3y = 4$
$II \qquad \frac{1}{3}x - \frac{1}{5}y = 12$
I nach y auflösen
$2x + 3y = 4$
$2x + 3y = 4 \qquad / - 2x$
$3y = 4 - 2x \qquad / : 3$
$y = 1\frac{1}{3} - \frac{2}{3}x$
II nach y auflösen
$\frac{1}{3}x - \frac{1}{5}y = 12$
$\frac{1}{3}x - \frac{1}{5}y = 12 \qquad / - \frac{1}{3}x$
$-\frac{1}{5}y = 12 - \frac{1}{3}x \qquad / : \left(-\frac{1}{5}\right)$
$y = -60 + 1\frac{2}{3}x$
I = II
$1\frac{1}{3} - \frac{2}{3}x = -60 + 1\frac{2}{3}x \qquad / + \frac{2}{3}x / + 60$
$1\frac{1}{3} + 60 = 1\frac{2}{3}x + \frac{2}{3}x$
$61\frac{1}{3} = 2\frac{1}{3}x \qquad / : 2\frac{1}{3}$
$x = 26\frac{2}{7}$
x in I
$y = 1\frac{1}{3} - \frac{2}{3} \cdot 26\frac{2}{7}$
$y = -16\frac{4}{21}$
$L = \{26\frac{2}{7} / - 16\frac{4}{21}\}$

$I \qquad 2x + 3y = 4$
$II \qquad \frac{1}{3}x - \frac{1}{5}y = 12$
I nach x auflösen
$2x + 3y = 4$
$2x + 3y = 4 \qquad / - 3y$
$2x = 4 - 3y \qquad / : 2$
$x = 2 - 1\frac{1}{2}y$
II nach x auflösen
$\frac{1}{3}x - \frac{1}{5}y = 12$
$\frac{1}{3}x - \frac{1}{5}y = 12 \qquad / + \frac{1}{5}y$
$\frac{1}{3}x = 12 + \frac{1}{5}y \qquad / : \frac{1}{3}$
$x = 36 + \frac{3}{5}y$
I = II
$2 - 1\frac{1}{2}y = 36 + \frac{3}{5}y \qquad / + 1\frac{1}{2}y / - 36$
$2 - 36 = \frac{3}{5}y + 1\frac{1}{2}y$
$-34 = 2\frac{1}{10}y \qquad / : 2\frac{1}{10}$
$y = -16\frac{4}{21}$
y in I
$x = 2 - 1\frac{1}{2} \cdot -16\frac{4}{21}$
$x = 26\frac{2}{7}$
$L = \{26\frac{2}{7} / - 16\frac{4}{21}\}$

Aufgabe (15)

$I \quad 1x + 2y = -4$
$II \quad -1x + 2y = 5$
I nach y auflösen
$1x + 2y = -4$
$1x + 2y = -4 \quad / -1x$
$2y = -4 - 1x \quad / : 2$
$y = -2 - \frac{1}{2}x$
II nach y auflösen
$-1x + 2y = 5$
$-1x + 2y = 5 \quad / +1x$
$2y = 5 + 1x \quad / : 2$
$y = 2\frac{1}{2} + \frac{1}{2}x$
I = II
$-2 - \frac{1}{2}x = 2\frac{1}{2} + \frac{1}{2}x \quad / + \frac{1}{2}x / -2\frac{1}{2}$
$-2 - 2\frac{1}{2} = \frac{1}{2}x + \frac{1}{2}x$
$-4\frac{1}{2} = 1x \quad / : 1$
$x = -4\frac{1}{2}$
x in I
$y = -2 - \frac{1}{2} \cdot -4\frac{1}{2}$
$y = \frac{1}{4}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

$I \quad 1x + 2y = -4$
$II \quad -1x + 2y = 5$
I nach x auflösen
$1x + 2y = -4$
$1x + 2y = -4 \quad / -2y$
$1x = -4 - 2y \quad / : 1$
$x = -4 - 2y$
II nach x auflösen
$-1x + 2y = 5$
$-1x + 2y = 5 \quad / -2y$
$-1x = 5 - 2y \quad / : (-1)$
$x = -5 + 2y$
I = II
$-4 - 2y = -5 + 2y \quad / +2y / +5$
$-4 + 5 = 2y + 2y$
$1 = 4y \quad / : 4$
$y = \frac{1}{4}$
y in I
$x = -4 - 2 \cdot \frac{1}{4}$
$x = -4\frac{1}{2}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

Aufgabe (16)

$I \quad 2x + 4y = 4$
$II \quad -\frac{1}{2}x + 3y = 3$
I nach y auflösen
$2x + 4y = 4$
$2x + 4y = 4 \quad / -2x$
$4y = 4 - 2x \quad / : 4$
$y = 1 - \frac{1}{2}x$
II nach y auflösen
$-\frac{1}{2}x + 3y = 3$
$-\frac{1}{2}x + 3y = 3 \quad / +\frac{1}{2}x$
$3y = 3 + \frac{1}{2}x \quad / : 3$
$y = 1 + \frac{1}{6}x$
I = II
$1 - \frac{1}{2}x = 1 + \frac{1}{6}x \quad / +\frac{1}{2}x / -1$
$1 - 1 = \frac{1}{6}x + \frac{1}{2}x$
$0 = \frac{2}{3}x \quad / : \frac{2}{3}$
$x = 0$
x in I
$y = 1 - \frac{1}{2} \cdot 0$
$y = 1$
$L = \{0/1\}$

$I \quad 2x + 4y = 4$
$II \quad -\frac{1}{2}x + 3y = 3$
I nach x auflösen
$2x + 4y = 4$
$2x + 4y = 4 \quad / -4y$
$2x = 4 - 4y \quad / : 2$
$x = 2 - 2y$
II nach x auflösen
$-\frac{1}{2}x + 3y = 3$
$-\frac{1}{2}x + 3y = 3 \quad / -3y$
$-\frac{1}{2}x = 3 - 3y \quad / : \left(-\frac{1}{2}\right)$
$x = -6 + 6y$
I = II
$2 - 2y = -6 + 6y \quad / +2y / +6$
$2 + 6 = 6y + 2y$
$8 = 8y \quad / : 8$
$y = 1$
y in I
$x = 2 - 2 \cdot 1$
$x = 0$
$L = \{0/1\}$

Aufgabe (17)

$I \qquad -\frac{1}{2}x + 4y = 6$
$II \qquad -2x - 8y = 2$
I nach y auflösen
$-\frac{1}{2}x + 4y = 6$
$-\frac{1}{2}x + 4y = 6 \qquad / + \frac{1}{2}x$
$4y = 6 + \frac{1}{2}x \qquad / : 4$
$y = 1\frac{1}{2} + \frac{1}{8}x$
II nach y auflösen
$-2x - 8y = 2$
$-2x - 8y = 2 \qquad / + 2x$
$-8y = 2 + 2x \qquad / : (-8)$
$y = -\frac{1}{4} - \frac{1}{4}x$
I = II
$1\frac{1}{2} + \frac{1}{8}x = -\frac{1}{4} - \frac{1}{4}x \qquad / - \frac{1}{8}x / + \frac{1}{4}$
$1\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}x - \frac{1}{8}x$
$1\frac{3}{4} = -\frac{3}{8}x \qquad / : \left(-\frac{3}{8}\right)$
$x = -4\frac{2}{3}$
x in I
$y = 1\frac{1}{2} + \frac{1}{8} \cdot -4\frac{2}{3}$
$y = \frac{11}{12}$
$L = \{-4\frac{2}{3} / \frac{11}{12}\}$

$I \qquad -\frac{1}{2}x + 4y = 6$
$II \qquad -2x - 8y = 2$
I nach x auflösen
$-\frac{1}{2}x + 4y = 6$
$-\frac{1}{2}x + 4y = 6 \qquad / - 4y$
$-\frac{1}{2}x = 6 - 4y \qquad / : \left(-\frac{1}{2}\right)$
$x = -12 + 8y$
II nach x auflösen
$-2x - 8y = 2$
$-2x - 8y = 2 \qquad / + 8y$
$-2x = 2 + 8y \qquad / : (-2)$
$x = -1 - 4y$
I = II
$-12 + 8y = -1 - 4y \qquad / - 8y / + 1$
$-12 + 1 = -4y - 8y$
$-11 = -12y \qquad / : (-12)$
$y = \frac{11}{12}$
y in I
$x = -12 + 8 \cdot \frac{11}{12}$
$x = -4\frac{2}{3}$
$L = \{-4\frac{2}{3} / \frac{11}{12}\}$

Aufgabe (18)

$I \qquad 1x + 1y = 1$
$II \qquad 2x + 2y = 2$
I nach y auflösen
$1x + 1y = 1$
$1x + 1y = 1 \qquad / - 1x$
$1y = 1 - 1x \qquad / : 1$
$y = 1 - 1x$
II nach y auflösen
$2x + 2y = 2$
$2x + 2y = 2 \qquad / - 2x$
$2y = 2 - 2x \qquad / : 2$
$y = 1 - 1x$
I = II
$1 - 1x = 1 - 1x \qquad / + 1x / - 1$
$1 - 1 = -1x + 1x$

$L = unendlich$

$I \qquad 1x + 1y = 1$
$II \qquad 2x + 2y = 2$
I nach x auflösen
$1x + 1y = 1$
$1x + 1y = 1 \qquad / - 1y$
$1x = 1 - 1y \qquad / : 1$
$x = 1 - 1y$
II nach x auflösen
$2x + 2y = 2$
$2x + 2y = 2 \qquad / - 2y$
$2x = 2 - 2y \qquad / : 2$
$x = 1 - 1y$
I = II
$1 - 1y = 1 - 1y \qquad / + 1y / - 1$
$1 - 1 = -1y + 1y$

$L = unendlich$

Aufgabe (19)

$I \quad 1x + 1y = 2$
$II \quad 3x + 3y = 3$
I nach y auflösen
$1x + 1y = 2$
$1x + 1y = 2 \quad / - 1x$
$1y = 2 - 1x \quad / : 1$
$y = 2 - 1x$
II nach y auflösen
$3x + 3y = 3$
$3x + 3y = 3 \quad / - 3x$
$3y = 3 - 3x \quad / : 3$
$y = 1 - 1x$
I = II
$2 - 1x = 1 - 1x \quad / + 1x / - 1$
$2 - 1 = -1x + 1x$

$L = \{\}$

$I \quad 1x + 1y = 2$
$II \quad 3x + 3y = 3$
I nach x auflösen
$1x + 1y = 2$
$1x + 1y = 2 \quad / - 1y$
$1x = 2 - 1y \quad / : 1$
$x = 2 - 1y$
II nach x auflösen
$3x + 3y = 3$
$3x + 3y = 3 \quad / - 3y$
$3x = 3 - 3y \quad / : 3$
$x = 1 - 1y$
I = II
$2 - 1y = 1 - 1y \quad / + 1y / - 1$
$2 - 1 = -1y + 1y$

$L = \{\}$

Aufgabe (20)

$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9$
I nach y auflösen
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / + 1\frac{4}{5}x$
$1\frac{1}{3}y = -1 + 1\frac{4}{5}x \quad / : 1\frac{1}{3}$
$y = -\frac{3}{4} + 1\frac{7}{20}x$
II nach y auflösen
$-\frac{2}{3}x + \frac{1}{9}y = 9$
$-\frac{2}{3}x + \frac{1}{9}y = 9 \quad / + \frac{2}{3}x$
$\frac{1}{9}y = 9 + \frac{2}{3}x \quad / : \frac{1}{9}$
$y = 81 + 6x$
I = II
$-\frac{3}{4} + 1\frac{7}{20}x = 81 + 6x \quad / - 1\frac{7}{20}x / - 81$
$-\frac{3}{4} - 81 = 6x - 1\frac{7}{20}x$
$-81\frac{3}{4} = 4\frac{13}{20}x \quad / : 4\frac{13}{20}$
$x = -17\frac{18}{31}$
x in I
$y = -\frac{3}{4} + 1\frac{7}{20} \cdot -17\frac{18}{31}$
$y = -24\frac{15}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9$
I nach x auflösen
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / - 1\frac{1}{3}y$
$-1\frac{4}{5}x = -1 - 1\frac{1}{3}y \quad / : \left(-1\frac{4}{5}\right)$
$x = \frac{5}{9} + \frac{20}{27}y$
II nach x auflösen
$-\frac{2}{3}x + \frac{1}{9}y = 9$
$-\frac{2}{3}x + \frac{1}{9}y = 9 \quad / - \frac{1}{9}y$
$-\frac{2}{3}x = 9 - \frac{1}{9}y \quad / : \left(-\frac{2}{3}\right)$
$x = -13\frac{1}{2} + \frac{1}{6}y$
I = II
$\frac{5}{9} + \frac{20}{27}y = -13\frac{1}{2} + \frac{1}{6}y \quad / - \frac{20}{27}y / + 13\frac{1}{2}$
$\frac{5}{9} + 13\frac{1}{2} = \frac{1}{6}y - \frac{20}{27}y$
$14\frac{1}{18} = -\frac{31}{54}y \quad / : \left(-\frac{31}{54}\right)$
$y = -24\frac{15}{31}$
y in I
$x = \frac{5}{9} + \frac{20}{27} \cdot -24\frac{15}{31}$
$x = -17\frac{18}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

Aufgabe (21)

$I \qquad 2x - 7y = -8$
$II \qquad 7x - 1y = -9$
I nach y auflösen
$2x - 7y = -8$
$2x - 7y = -8 \qquad / - 2x$
$-7y = -8 - 2x \qquad / : (-7)$
$y = 1\frac{1}{7} + \frac{2}{7}x$
II nach y auflösen
$7x - 1y = -9$
$7x - 1y = -9 \qquad / - 7x$
$-1y = -9 - 7x \qquad / : (-1)$
$y = 9 + 7x$
I = II
$1\frac{1}{7} + \frac{2}{7}x = 9 + 7x \qquad / - \frac{2}{7}x / - 9$
$1\frac{1}{7} - 9 = 7x - \frac{2}{7}x$
$-7\frac{6}{7} = 6\frac{5}{7}x \qquad / : 6\frac{5}{7}$
$x = -1\frac{8}{47}$
x in I
$y = 1\frac{1}{7} + \frac{2}{7} \cdot -1\frac{8}{47}$
$y = \frac{38}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

$I \qquad 2x - 7y = -8$
$II \qquad 7x - 1y = -9$
I nach x auflösen
$2x - 7y = -8$
$2x - 7y = -8 \qquad / + 7y$
$2x = -8 + 7y \qquad / : 2$
$x = -4 + 3\frac{1}{2}y$
II nach x auflösen
$7x - 1y = -9$
$7x - 1y = -9 \qquad / + 1y$
$7x = -9 + 1y \qquad / : 7$
$x = -1\frac{2}{7} + \frac{1}{7}y$
I = II
$-4 + 3\frac{1}{2}y = -1\frac{2}{7} + \frac{1}{7}y \qquad / - 3\frac{1}{2}y / + 1\frac{2}{7}$
$-4 + 1\frac{2}{7} = \frac{1}{7}y - 3\frac{1}{2}y$
$-2\frac{5}{7} = -3\frac{5}{14}y \qquad / : \left(-3\frac{5}{14}\right)$
$y = \frac{38}{47}$
y in I
$x = -4 + 3\frac{1}{2} \cdot \frac{38}{47}$
$x = -1\frac{8}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

Aufgabe (22)

$I \qquad -7x + 9y = -3$
$II \qquad 5x - 6y = -4$
I nach y auflösen
$-7x + 9y = -3$
$-7x + 9y = -3 \qquad / + 7x$
$9y = -3 + 7x \qquad / : 9$
$y = -\frac{1}{3} + \frac{7}{9}x$
II nach y auflösen
$5x - 6y = -4$
$5x - 6y = -4 \qquad / - 5x$
$-6y = -4 - 5x \qquad / : (-6)$
$y = \frac{2}{3} + \frac{5}{6}x$
I = II
$-\frac{1}{3} + \frac{7}{9}x = \frac{2}{3} + \frac{5}{6}x \qquad / - \frac{7}{9}x / - \frac{2}{3}$
$-\frac{1}{3} - \frac{2}{3} = \frac{5}{6}x - \frac{7}{9}x$
$-1 = \frac{1}{18}x \qquad / : \frac{1}{18}$
$x = -18$
x in I
$y = -\frac{1}{3} + \frac{7}{9} \cdot -18$
$y = -14\frac{1}{3}$
$L = \{-18 / -14\frac{1}{3}\}$

$I \qquad -7x + 9y = -3$
$II \qquad 5x - 6y = -4$
I nach x auflösen
$-7x + 9y = -3$
$-7x + 9y = -3 \qquad / - 9y$
$-7x = -3 - 9y \qquad / : (-7)$
$x = \frac{3}{7} + 1\frac{2}{7}y$
II nach x auflösen
$5x - 6y = -4$
$5x - 6y = -4 \qquad / + 6y$
$5x = -4 + 6y \qquad / : 5$
$x = -\frac{4}{5} + 1\frac{1}{5}y$
I = II
$\frac{3}{7} + 1\frac{2}{7}y = -\frac{4}{5} + 1\frac{1}{5}y \qquad / - 1\frac{2}{7}y / + \frac{4}{5}$
$\frac{3}{7} + \frac{4}{5} = 1\frac{1}{5}y - 1\frac{2}{7}y$
$1\frac{8}{35} = -\frac{3}{35}y \qquad / : \left(-\frac{3}{35}\right)$
$y = -14\frac{1}{3}$
y in I
$x = \frac{3}{7} + 1\frac{2}{7} \cdot -14\frac{1}{3}$
$x = -18$
$L = \{-18 / -14\frac{1}{3}\}$

Aufgabe (23)

$I \qquad 2x + 2y = 1\frac{7}{10}$
$II \qquad 3x + 6y = 3$
I nach y auflösen
$2x + 2y = 1\frac{7}{10}$
$2x + 2y = 1\frac{7}{10} \qquad / - 2x$
$2y = 1\frac{7}{10} - 2x \qquad / : 2$
$y = \frac{17}{20} - 1x$
II nach y auflösen
$3x + 6y = 3$
$3x + 6y = 3 \qquad / - 3x$
$6y = 3 - 3x \qquad / : 6$
$y = \frac{1}{2} - \frac{1}{2}x$
I = II
$\frac{17}{20} - 1x = \frac{1}{2} - \frac{1}{2}x \qquad / + 1x / - \frac{1}{2}$
$\frac{17}{20} - \frac{1}{2} = -\frac{1}{2}x + 1x$
$\frac{7}{20} = \frac{1}{2}x \qquad / : \frac{1}{2}$
$x = \frac{7}{10}$
x in I
$y = \frac{17}{20} - 1 \cdot \frac{7}{10}$
$y = \frac{3}{20}$
$L = \{\frac{7}{10} / \frac{3}{20}\}$

$I \qquad 2x + 2y = 1\frac{7}{10}$
$II \qquad 3x + 6y = 3$
I nach x auflösen
$2x + 2y = 1\frac{7}{10}$
$2x + 2y = 1\frac{7}{10} \qquad / - 2y$
$2x = 1\frac{7}{10} - 2y \qquad / : 2$
$x = \frac{17}{20} - 1y$
II nach x auflösen
$3x + 6y = 3$
$3x + 6y = 3 \qquad / - 6y$
$3x = 3 - 6y \qquad / : 3$
$x = 1 - 2y$
I = II
$\frac{17}{20} - 1y = 1 - 2y \qquad / + 1y / - 1$
$\frac{17}{20} - 1 = -2y + 1y$
$-\frac{3}{20} = -1y \qquad / : (-1)$
$y = \frac{3}{20}$
y in I
$x = \frac{17}{20} - 1 \cdot \frac{3}{20}$
$x = \frac{7}{10}$
$L = \{\frac{7}{10} / \frac{3}{20}\}$

Aufgabe (24)

$I \qquad 34x + 5y = 7$
$II \qquad 7x + 9y = 8$
I nach y auflösen
$34x + 5y = 7$
$34x + 5y = 7 \qquad / - 34x$
$5y = 7 - 34x \qquad / : 5$
$y = 1\frac{2}{5} - 6\frac{4}{5}x$
II nach y auflösen
$7x + 9y = 8$
$7x + 9y = 8 \qquad / - 7x$
$9y = 8 - 7x \qquad / : 9$
$y = \frac{8}{9} - \frac{7}{9}x$
I = II
$1\frac{2}{5} - 6\frac{4}{5}x = \frac{8}{9} - \frac{7}{9}x \qquad / + 6\frac{4}{5}x / - \frac{8}{9}$
$1\frac{2}{5} - \frac{8}{9} = -\frac{7}{9}x + 6\frac{4}{5}x$
$\frac{23}{45} = 6\frac{1}{45}x \qquad / : 6\frac{1}{45}$
$x = 0,0849$
x in I
$y = 1\frac{2}{5} - 6\frac{4}{5} \cdot 0,0849$
$y = 0,823$
$L = \{0,0849 / 0,823\}$

$I \qquad 34x + 5y = 7$
$II \qquad 7x + 9y = 8$
I nach x auflösen
$34x + 5y = 7$
$34x + 5y = 7 \qquad / - 5y$
$34x = 7 - 5y \qquad / : 34$
$x = \frac{7}{34} - \frac{5}{34}y$
II nach x auflösen
$7x + 9y = 8$
$7x + 9y = 8 \qquad / - 9y$
$7x = 8 - 9y \qquad / : 7$
$x = 1\frac{1}{7} - 1\frac{2}{7}y$
I = II
$\frac{7}{34} - \frac{5}{34}y = 1\frac{1}{7} - 1\frac{2}{7}y \qquad / + \frac{5}{34}y / - 1\frac{1}{7}$
$\frac{7}{34} - 1\frac{1}{7} = -1\frac{2}{7}y + \frac{5}{34}y$
$-0,937 = -1,14y \qquad / : (-1,14)$
$y = 0,823$
y in I
$x = \frac{7}{34} - \frac{5}{34} \cdot 0,823$
$x = 0,0849$
$L = \{0,0849 / 0,823\}$

Aufgabe (25)

$I \quad 34x + 5y = 7$
$II \quad 7x + 9y = 8$
I nach y auflösen
$34x + 5y = 7$
$34x + 5y = 7 \quad / - 34x$
$5y = 7 - 34x \quad / : 5$
$y = 1\frac{2}{5} - 6\frac{4}{5}x$
II nach y auflösen
$7x + 9y = 8$
$7x + 9y = 8 \quad / - 7x$
$9y = 8 - 7x \quad / : 9$
$y = \frac{8}{9} - \frac{7}{9}x$
I = II
$1\frac{2}{5} - 6\frac{4}{5}x = \frac{8}{9} - \frac{7}{9}x \quad / + 6\frac{4}{5}x / - \frac{8}{9}$
$1\frac{2}{5} - \frac{8}{9} = -\frac{7}{9}x + 6\frac{4}{5}x$
$\frac{23}{45} = 6\frac{1}{45}x \quad / : 6\frac{1}{45}$
$x = 0,0849$
x in I
$y = 1\frac{2}{5} - 6\frac{4}{5} \cdot 0,0849$
$y = 0,823$
$L = \{0,0849/0,823\}$

$I \quad 34x + 5y = 7$
$II \quad 7x + 9y = 8$
I nach x auflösen
$34x + 5y = 7$
$34x + 5y = 7 \quad / - 5y$
$34x = 7 - 5y \quad / : 34$
$x = \frac{7}{34} - \frac{5}{34}y$
II nach x auflösen
$7x + 9y = 8$
$7x + 9y = 8 \quad / - 9y$
$7x = 8 - 9y \quad / : 7$
$x = 1\frac{1}{7} - 1\frac{2}{7}y$
I = II
$\frac{7}{34} - \frac{5}{34}y = 1\frac{1}{7} - 1\frac{2}{7}y \quad / + \frac{5}{34}y / - 1\frac{1}{7}$
$\frac{7}{34} - 1\frac{1}{7} = -1\frac{2}{7}y + \frac{5}{34}y$
$-0,937 = -1,14y \quad / : (-1,14)$
$y = 0,823$
y in I
$x = \frac{7}{34} - \frac{5}{34} \cdot 0,823$
$x = 0,0849$
$L = \{0,0849/0,823\}$

Aufgabe (26)

$I \quad 34x + 5y = 7$
$II \quad 7x + 9y = 8$
I nach y auflösen
$34x + 5y = 7$
$34x + 5y = 7 \quad / - 34x$
$5y = 7 - 34x \quad / : 5$
$y = 1\frac{2}{5} - 6\frac{4}{5}x$
II nach y auflösen
$7x + 9y = 8$
$7x + 9y = 8 \quad / - 7x$
$9y = 8 - 7x \quad / : 9$
$y = \frac{8}{9} - \frac{7}{9}x$
I = II
$1\frac{2}{5} - 6\frac{4}{5}x = \frac{8}{9} - \frac{7}{9}x \quad / + 6\frac{4}{5}x / - \frac{8}{9}$
$1\frac{2}{5} - \frac{8}{9} = -\frac{7}{9}x + 6\frac{4}{5}x$
$\frac{23}{45} = 6\frac{1}{45}x \quad / : 6\frac{1}{45}$
$x = 0,0849$
x in I
$y = 1\frac{2}{5} - 6\frac{4}{5} \cdot 0,0849$
$y = 0,823$
$L = \{0,0849/0,823\}$

$I \quad 34x + 5y = 7$
$II \quad 7x + 9y = 8$
I nach x auflösen
$34x + 5y = 7$
$34x + 5y = 7 \quad / - 5y$
$34x = 7 - 5y \quad / : 34$
$x = \frac{7}{34} - \frac{5}{34}y$
II nach x auflösen
$7x + 9y = 8$
$7x + 9y = 8 \quad / - 9y$
$7x = 8 - 9y \quad / : 7$
$x = 1\frac{1}{7} - 1\frac{2}{7}y$
I = II
$\frac{7}{34} - \frac{5}{34}y = 1\frac{1}{7} - 1\frac{2}{7}y \quad / + \frac{5}{34}y / - 1\frac{1}{7}$
$\frac{7}{34} - 1\frac{1}{7} = -1\frac{2}{7}y + \frac{5}{34}y$
$-0,937 = -1,14y \quad / : (-1,14)$
$y = 0,823$
y in I
$x = \frac{7}{34} - \frac{5}{34} \cdot 0,823$
$x = 0,0849$
$L = \{0,0849/0,823\}$

Aufgabe (27)

$I \quad 2x + 5y = 7$
$II \quad -4x + 2y = -6$
I nach y auflösen
$2x + 5y = 7$
$2x + 5y = 7 \quad / -2x$
$5y = 7 - 2x \quad / : 5$
$y = 1\frac{2}{5} - \frac{2}{5}x$
II nach y auflösen
$-4x + 2y = -6$
$-4x + 2y = -6 \quad / + 4x$
$2y = -6 + 4x \quad / : 2$
$y = -3 + 2x$
I = II
$1\frac{2}{5} - \frac{2}{5}x = -3 + 2x \quad / + \frac{2}{5}x / + 3$
$1\frac{2}{5} + 3 = 2x + \frac{2}{5}x$
$4\frac{2}{5} = 2\frac{2}{5}x \quad / : 2\frac{2}{5}$
$x = 1\frac{5}{6}$
x in I
$y = 1\frac{2}{5} - \frac{2}{5} \cdot 1\frac{5}{6}$
$y = \frac{2}{3}$
$L = \{1\frac{5}{6} / \frac{2}{3}\}$

$I \quad 2x + 5y = 7$
$II \quad -4x + 2y = -6$
I nach x auflösen
$2x + 5y = 7$
$2x + 5y = 7 \quad / -5y$
$2x = 7 - 5y \quad / : 2$
$x = 3\frac{1}{2} - 2\frac{1}{2}y$
II nach x auflösen
$-4x + 2y = -6$
$-4x + 2y = -6 \quad / -2y$
$-4x = -6 - 2y \quad / : (-4)$
$x = 1\frac{1}{2} + \frac{1}{2}y$
I = II
$3\frac{1}{2} - 2\frac{1}{2}y = 1\frac{1}{2} + \frac{1}{2}y \quad / + 2\frac{1}{2}y / - 1\frac{1}{2}$
$3\frac{1}{2} - 1\frac{1}{2} = \frac{1}{2}y + 2\frac{1}{2}y$
$2 = 3y \quad / : 3$
$y = \frac{2}{3}$
y in I
$x = 3\frac{1}{2} - 2\frac{1}{2} \cdot \frac{2}{3}$
$x = 1\frac{5}{6}$
$L = \{1\frac{5}{6} / \frac{2}{3}\}$

Aufgabe (28)

$I \quad 2x + 5y = 7$
$II \quad -4x + 2y = -6$
I nach y auflösen
$2x + 5y = 7$
$2x + 5y = 7 \quad / -2x$
$5y = 7 - 2x \quad / : 5$
$y = 1\frac{2}{5} - \frac{2}{5}x$
II nach y auflösen
$-4x + 2y = -6$
$-4x + 2y = -6 \quad / + 4x$
$2y = -6 + 4x \quad / : 2$
$y = -3 + 2x$
I = II
$1\frac{2}{5} - \frac{2}{5}x = -3 + 2x \quad / + \frac{2}{5}x / + 3$
$1\frac{2}{5} + 3 = 2x + \frac{2}{5}x$
$4\frac{2}{5} = 2\frac{2}{5}x \quad / : 2\frac{2}{5}$
$x = 1\frac{5}{6}$
x in I
$y = 1\frac{2}{5} - \frac{2}{5} \cdot 1\frac{5}{6}$
$y = \frac{2}{3}$
$L = \{1\frac{5}{6} / \frac{2}{3}\}$

$I \quad 2x + 5y = 7$
$II \quad -4x + 2y = -6$
I nach x auflösen
$2x + 5y = 7$
$2x + 5y = 7 \quad / -5y$
$2x = 7 - 5y \quad / : 2$
$x = 3\frac{1}{2} - 2\frac{1}{2}y$
II nach x auflösen
$-4x + 2y = -6$
$-4x + 2y = -6 \quad / -2y$
$-4x = -6 - 2y \quad / : (-4)$
$x = 1\frac{1}{2} + \frac{1}{2}y$
I = II
$3\frac{1}{2} - 2\frac{1}{2}y = 1\frac{1}{2} + \frac{1}{2}y \quad / + 2\frac{1}{2}y / - 1\frac{1}{2}$
$3\frac{1}{2} - 1\frac{1}{2} = \frac{1}{2}y + 2\frac{1}{2}y$
$2 = 3y \quad / : 3$
$y = \frac{2}{3}$
y in I
$x = 3\frac{1}{2} - 2\frac{1}{2} \cdot \frac{2}{3}$
$x = 1\frac{5}{6}$
$L = \{1\frac{5}{6} / \frac{2}{3}\}$

3 Additionsverfahren (2)

$I \quad a1 \cdot x + b1 \cdot y = c1$
$II \quad a2 \cdot x + b2 \cdot y = c42$

- Terme mit x und y müssen untereinander stehen
- Gleichungen multiplizieren, so dass die Variablen beim spaltenweisen addieren herausfallen
- Gleichung nach der Unbekannten auflösen
- zweite Unbekannte berechnen

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
$I \quad 3x + 5y = 19 \quad / \cdot 7$
$II \quad 7x + 5y = 31 \quad / \cdot (-3)$
$I \quad 21x + 35y = 133$
$II \quad -21x - 15y = -93$
I + II
$21x - 21x + 35y - 15y = 133 - 93$
$20y = 40 \quad / : 20$
$y = \frac{40}{20}$
$y = 2$
y in I
$I \quad 3x + 5 \cdot 2 = 19$
$3x + 10 = 19 \quad / - 10$
$3x = 19 - 10$
$3x = 9 \quad / : 3$
$x = \frac{9}{3}$
$x = 3$
$L = \{3/2\}$

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
$I \quad 3x + 5y = 19 \quad / \cdot 1$
$II \quad 7x + 5y = 31 \quad / \cdot (-1)$
$I \quad 3x + 5y = 19$
$II \quad -7x - 5y = -31$
I + II
$3x - 7x + 5y - 5y = 19 - 31$
$-4x = -12 \quad / : (-4)$
$x = \frac{-12}{-4}$
$x = 3$
x in I
$I \quad 3 \cdot 3 + 5y = 19$
$5y + 9 = 19 \quad / - 9$
$5y = 19 - 9$
$5y = 10 \quad / : 5$
$y = \frac{10}{5}$
$y = 2$
$L = \{3/2\}$

3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
$a1 \cdot x + b1 \cdot y = c1$
$a2 \cdot x + b2 \cdot y = c2$

Gesucht:
x und y

(1) $3x + 5y = 19$, $7x + 5y = 31$

(2) $1x + 1y = 10$, $1x - 1y = 4$

(3) $9x - 2y = 5$, $5x - 2y = 1$

(4) $9x - 2y = 1$, $-3x - 3y = -7$

(5) $1x + 3y = 9$, $3x - 2y = -6$

(6) $7x + 1\frac{1}{2}y = 1$, $2x - 5y = 22$

(7) $4x - 5y = 43$, $-3x - 7y = 0$

(8) $8x - 3y = 10$, $12x - 5y = 24$

(9) $-\frac{1}{2}x + 1y = 2$, $\frac{1}{2}x - 3y = -3$

(10) $-1x + 1y = 3$, $\frac{1}{2}x - 4y = 5$

(11) $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$

(12) $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$1x + 1y = 10\frac{2}{3}$

(13) $1\frac{1}{2}x - 2y = 9$
$\frac{2}{5}x + \frac{1}{3}y = 5$

(14) $2x + 3y = 4$
$\frac{1}{3}x - \frac{1}{5}y = 12$

(15) $1x + 2y = -4$
$-1x + 2y = 5$

(16) $2x + 4y = 4$
$-\frac{1}{2}x + 3y = 3$

(17) $-\frac{1}{2}x + 4y = 6$
$-2x - 8y = 2$

(18) $1x + 1y = 1$
$2x + 2y = 2$

(19) $1x + 1y = 2$
$3x + 3y = 3$

(20) $-1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$-\frac{2}{3}x + \frac{1}{9}y = 9$

(21) $2x - 7y = -8$
$7x - 1y = -9$

(22) $-7x + 9y = -3$
$5x - 6y = -4$

(23) $2x + 2y = 1\frac{7}{10}$
$3x + 6y = 3$

(24) $5x + 6y = 7$
$4x + 6y = 4$

(25) $12x + 6y = 7$
$16x + 6y = 4$

3.2 Lösungen

Aufgabe (1)

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
$I \quad 3x + 5y = 19 \quad / \cdot 7$
$II \quad 7x + 5y = 31 \quad / \cdot (-3)$
$I \quad 21x + 35y = 133$
$II \quad -21x - 15y = -93$
I + II
$I \quad 21x - 21x + 35y - 15y = 133 - 93$
$20y = 40 \quad / : 20$
$y = \frac{40}{20}$
$y = 2$
y in I
$I \quad 3x + 5 \cdot 2 = 19$
$3x + 10 = 19 \quad / - 10$
$3x = 19 - 10$
$3x = 9 \quad / : 3$
$x = \frac{9}{3}$
$x = 3$
$L = \{3/2\}$

$I \quad 3x + 5y = 19$
$II \quad 7x + 5y = 31$
$I \quad 3x + 5y = 19 \quad / \cdot 1$
$II \quad 7x + 5y = 31 \quad / \cdot (-1)$
$I \quad 3x + 5y = 19$
$II \quad -7x - 5y = -31$
I + II
$I \quad 3x - 7x + 5y - 5y = 19 - 31$
$-4x = -12 \quad / : (-4)$
$x = \frac{-12}{-4}$
$x = 3$
x in I
$I \quad 3 \cdot 3 + 5y = 19$
$5y + 9 = 19 \quad / - 9$
$5y = 19 - 9$
$5y = 10 \quad / : 5$
$y = \frac{10}{5}$
$y = 2$
$L = \{3/2\}$

Aufgabe (2)

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
$I \quad 1x + 1y = 10 \quad / \cdot 1$
$II \quad 1x - 1y = 4 \quad / \cdot (-1)$
$I \quad 1x + 1y = 10$
$II \quad -1x + 1y = -4$
I + II
$I \quad 1x - 1x + 1y + 1y = 10 - 4$
$2y = 6 \quad / : 2$
$y = \frac{6}{2}$
$y = 3$
y in I
$I \quad 1x + 1 \cdot 3 = 10$
$1x + 3 = 10 \quad / - 3$
$1x = 10 - 3$
$1x = 7 \quad / : 1$
$x = \frac{7}{1}$
$x = 7$
$L = \{7/3\}$

$I \quad 1x + 1y = 10$
$II \quad 1x - 1y = 4$
$I \quad 1x + 1y = 10 \quad / \cdot (-1)$
$II \quad 1x - 1y = 4 \quad / \cdot (-1)$
$I \quad -1x - 1y = -10$
$II \quad -1x + 1y = -4$
I + II
$I \quad -1x - 1x - 1y + 1y = -10 - 4$
$-2x = -14 \quad / : (-2)$
$x = \frac{-14}{-2}$
$x = 7$
x in I
$I \quad 1 \cdot 7 + 1y = 10$
$1y + 7 = 10 \quad / - 7$
$1y = 10 - 7$
$1y = 3 \quad / : 1$
$y = \frac{3}{1}$
$y = 3$
$L = \{7/3\}$

Aufgabe (3)

$I \quad 9x - 2y = 5$
$II \quad 5x - 2y = 1$
$I \quad 9x - 2y = 5 \quad / \cdot 5$
$II \quad 5x - 2y = 1 \quad / \cdot (-9)$
$I \quad 45x - 10y = 25$
$II \quad -45x + 18y = -9$
I + II
$I \quad 45x - 45x - 10y + 18y = 25 - 9$
$8y = 16 \quad / : 8$
$y = \frac{16}{8}$
$y = 2$
y in I
$I \quad 9x - 2 \cdot 2 = 5$
$9x - 4 = 5 \quad / + 4$
$9x = 5 + 4$
$9x = 9 \quad / : 9$
$x = \frac{9}{9}$
$x = 1$
$L = \{1/2\}$

$I \quad 9x - 2y = 5$
$II \quad 5x - 2y = 1$
$I \quad 9x - 2y = 5 \quad / \cdot (-1)$
$II \quad 5x - 2y = 1 \quad / \cdot 1$
$I \quad -9x + 2y = -5$
$II \quad 5x - 2y = 1$
I + II
$I \quad -9x + 5x + 2y - 2y = -5 + 1$
$-4x = -4 \quad / : (-4)$
$x = \frac{-4}{-4}$
$x = 1$
x in I
$I \quad 9 \cdot 1 - 2y = 5$
$-2y + 9 = 5 \quad / - 9$
$-2y = 5 - 9$
$-2y = -4 \quad / : (-2)$
$y = \frac{-4}{-2}$
$y = 2$
$L = \{1/2\}$

Aufgabe (4)

$I \quad 9x - 2y = 1$
$II \quad -3x - 3y = -7$
$I \quad 9x - 2y = 1 \quad / \cdot (-1)$
$II \quad -3x - 3y = -7 \quad / \cdot (-3)$
$I \quad -9x + 2y = -1$
$II \quad 9x + 9y = 21$
I + II
$I \quad -9x + 9x + 2y + 9y = -1 + 21$
$11y = 20 \quad / : 11$
$y = \frac{20}{11}$
$y = 1\frac{9}{11}$
y in I
$I \quad 9x - 2 \cdot 1\frac{9}{11} = 1$
$9x - 3\frac{7}{11} = 1 \quad / + 3\frac{7}{11}$
$9x = 1 + 3\frac{7}{11}$
$9x = 4\frac{7}{11} \quad / : 9$
$x = \frac{4\frac{7}{11}}{9}$
$x = \frac{17}{33}$
$L = \{\frac{17}{33}/1\frac{9}{11}\}$

$I \quad 9x - 2y = 1$
$II \quad -3x - 3y = -7$
$I \quad 9x - 2y = 1 \quad / \cdot (-3)$
$II \quad -3x - 3y = -7 \quad / \cdot 2$
$I \quad -27x + 6y = -3$
$II \quad -6x - 6y = -14$
I + II
$I \quad -27x - 6x + 6y - 6y = -3 - 14$
$-33x = -17 \quad / : (-33)$
$x = \frac{-17}{-33}$
$x = \frac{17}{33}$
x in I
$I \quad 9 \cdot \frac{17}{33} - 2y = 1$
$-2y + 4\frac{7}{11} = 1 \quad / - 4\frac{7}{11}$
$-2y = 1 - 4\frac{7}{11}$
$-2y = -3\frac{7}{11} \quad / : (-2)$
$y = \frac{-3\frac{7}{11}}{-2}$
$y = 1\frac{9}{11}$
$L = \{\frac{17}{33}/1\frac{9}{11}\}$

Aufgabe (5)

$I \quad 1x + 3y = 9$
$II \quad 3x - 2y = -6$
$I \quad 1x + 3y = 9 \quad / \cdot 3$
$II \quad 3x - 2y = -6 \quad / \cdot (-1)$
$I \quad 3x + 9y = 27$
$II \quad -3x + 2y = 6$
I + II
$I \quad 3x - 3x + 9y + 2y = 27 + 6$
$11y = 33 \quad / : 11$
$y = \frac{33}{11}$
$y = 3$
y in I
$I \quad 1x + 3 \cdot 3 = 9$
$1x + 9 = 9 \quad / - 9$
$1x = 9 - 9$
$1x = 0 \quad / : 1$
$x = \frac{0}{1}$
$x = 0$
$L = \{0/3\}$

$I \quad 1x + 3y = 9$
$II \quad 3x - 2y = -6$
$I \quad 1x + 3y = 9 \quad / \cdot (-2)$
$II \quad 3x - 2y = -6 \quad / \cdot (-3)$
$I \quad -2x - 6y = -18$
$II \quad -9x + 6y = 18$
I + II
$I \quad -2x - 9x - 6y + 6y = -18 + 18$
$-11x = 0 \quad / : (-11)$
$x = \frac{0}{-11}$
$x = 0$
x in I
$I \quad 1 \cdot 0 + 3y = 9$
$3y + 0 = 9 \quad / - 0$
$3y = 9 - 0$
$3y = 9 \quad / : 3$
$y = \frac{9}{3}$
$y = 3$
$L = \{0/3\}$

Aufgabe (6)

$I \quad 7x + 1\frac{1}{2}y = 1$
$II \quad 2x - 5y = 22$
$I \quad 7x + 1\frac{1}{2}y = 1 \quad / \cdot 2$
$II \quad 2x - 5y = 22 \quad / \cdot (-7)$
$I \quad 14x + 3y = 2$
$II \quad -14x + 35y = -154$
I + II
$I \quad 14x - 14x + 3y + 35y = 2 - 154$
$38y = -152 \quad / : 38$
$y = \frac{-152}{38}$
$y = -4$
y in I
$I \quad 7x + 1\frac{1}{2} \cdot (-4) = 1$
$7x - 6 = 1 \quad / + 6$
$7x = 1 + 6$
$7x = 7 \quad / : 7$
$x = \frac{7}{7}$
$x = 1$
$L = \{1/-4\}$

$I \quad 7x + 1\frac{1}{2}y = 1$
$II \quad 2x - 5y = 22$
$I \quad 7x + 1\frac{1}{2}y = 1 \quad / \cdot 5$
$II \quad 2x - 5y = 22 \quad / \cdot 1\frac{1}{2}$
$I \quad 35x + 7\frac{1}{2}y = 5$
$II \quad 3x - 7\frac{1}{2}y = 33$
I + II
$I \quad 35x + 3x + 7\frac{1}{2}y - 7\frac{1}{2}y = 5 + 33$
$38x = 38 \quad / : 38$
$x = \frac{38}{38}$
$x = 1$
x in I
$I \quad 7 \cdot 1 + 1\frac{1}{2}y = 1$
$1\frac{1}{2}y + 7 = 1 \quad / - 7$
$1\frac{1}{2}y = 1 - 7$
$1\frac{1}{2}y = -6 \quad / : 1\frac{1}{2}$
$y = \frac{-6}{1\frac{1}{2}}$
$y = -4$
$L = \{1/-4\}$

Aufgabe (7)

$I \quad 4x - 5y = 43$
$II \quad -3x - 7y = 0$
$I \quad 4x - 5y = 43 \quad / \cdot (-3)$
$II \quad -3x - 7y = 0 \quad / \cdot (-4)$
$I \quad -12x + 15y = -129$
$II \quad 12x + 28y = 0$
I + II
$I \quad -12x + 12x + 15y + 28y = -129 + 0$
$43y = -129 \quad / : 43$
$y = \frac{-129}{43}$
$y = -3$
y in I
$I \quad 4x - 5 \cdot (-3) = 43$
$4x + 15 = 43 \quad / -15$
$4x = 43 - 15$
$4x = 28 \quad / : 4$
$x = \frac{28}{4}$
$x = 7$
$L = \{7/-3\}$

$I \quad 4x - 5y = 43$
$II \quad -3x - 7y = 0$
$I \quad 4x - 5y = 43 \quad / \cdot (-7)$
$II \quad -3x - 7y = 0 \quad / \cdot 5$
$I \quad -28x + 35y = -301$
$II \quad -15x - 35y = 0$
I + II
$I \quad -28x - 15x + 35y - 35y = -301 + 0$
$-43x = -301 \quad / : (-43)$
$x = \frac{-301}{-43}$
$x = 7$
x in I
$I \quad 4 \cdot 7 - 5y = 43$
$-5y + 28 = 43 \quad / -28$
$-5y = 43 - 28$
$-5y = 15 \quad / : (-5)$
$y = \frac{15}{-5}$
$y = -3$
$L = \{7/-3\}$

Aufgabe (8)

$I \quad 8x - 3y = 10$
$II \quad 12x - 5y = 24$
$I \quad 8x - 3y = 10 \quad / \cdot 3$
$II \quad 12x - 5y = 24 \quad / \cdot (-2)$
$I \quad 24x - 9y = 30$
$II \quad -24x + 10y = -48$
I + II
$I \quad 24x - 24x - 9y + 10y = 30 - 48$
$1y = -18 \quad / : 1$
$y = \frac{-18}{1}$
$y = -18$
y in I
$I \quad 8x - 3 \cdot (-18) = 10$
$8x + 54 = 10 \quad / -54$
$8x = 10 - 54$
$8x = -44 \quad / : 8$
$x = \frac{-44}{8}$
$x = -5\frac{1}{2}$
$L = \{-5\frac{1}{2}/-18\}$

$I \quad 8x - 3y = 10$
$II \quad 12x - 5y = 24$
$I \quad 8x - 3y = 10 \quad / \cdot (-5)$
$II \quad 12x - 5y = 24 \quad / \cdot 3$
$I \quad -40x + 15y = -50$
$II \quad 36x - 15y = 72$
I + II
$I \quad -40x + 36x + 15y - 15y = -50 + 72$
$-4x = 22 \quad / : (-4)$
$x = \frac{22}{-4}$
$x = -5\frac{1}{2}$
x in I
$I \quad 8 \cdot \left(-5\frac{1}{2}\right) - 3y = 10$
$-3y - 44 = 10 \quad / +44$
$-3y = 10 + 44$
$-3y = 54 \quad / : (-3)$
$y = \frac{54}{-3}$
$y = -18$
$L = \{-5\frac{1}{2}/-18\}$

Aufgabe (9)

$I \quad -\frac{1}{2}x + 1y = 2$
$II \quad \frac{1}{2}x - 3y = -3$
$I \quad -\frac{1}{2}x + 1y = 2 \quad / \cdot \left(-\frac{1}{2}\right)$
$II \quad \frac{1}{2}x - 3y = -3 \quad / \cdot \left(-\frac{1}{2}\right)$
$I \quad \frac{1}{4}x - \frac{1}{2}y = -1$
$II \quad -\frac{1}{4}x + 1\frac{1}{2}y = 1\frac{1}{2}$
I + II
$I \quad \frac{1}{4}x - \frac{1}{4}x - \frac{1}{2}y + 1\frac{1}{2}y = -1 + 1\frac{1}{2}$
$1y = \frac{1}{2} \quad / : 1$
$y = \frac{\frac{1}{2}}{1}$
$y = \frac{1}{2}$
y in I
$I \quad -\frac{1}{2}x + 1 \cdot \frac{1}{2} = 2$
$-\frac{1}{2}x + \frac{1}{2} = 2 \quad / -\frac{1}{2}$
$-\frac{1}{2}x = 2 - \frac{1}{2}$
$-\frac{1}{2}x = 1\frac{1}{2} \quad / : \left(-\frac{1}{2}\right)$
$x = \frac{1\frac{1}{2}}{-\frac{1}{2}}$
$x = -3$
$L = \{-3/\frac{1}{2}\}$

$I \quad -\frac{1}{2}x + 1y = 2$
$II \quad \frac{1}{2}x - 3y = -3$
$I \quad -\frac{1}{2}x + 1y = 2 \quad / \cdot (-3)$
$II \quad \frac{1}{2}x - 3y = -3 \quad / \cdot (-1)$
$I \quad 1\frac{1}{2}x - 3y = -6$
$II \quad -\frac{1}{2}x + 3y = 3$
I + II
$I \quad 1\frac{1}{2}x - \frac{1}{2}x - 3y + 3y = -6 + 3$
$1x = -3 \quad / : 1$
$x = \frac{-3}{1}$
$x = -3$
x in I
$I \quad -\frac{1}{2} \cdot (-3) + 1y = 2$
$1y + 1\frac{1}{2} = 2 \quad / -1\frac{1}{2}$
$1y = 2 - 1\frac{1}{2}$
$1y = \frac{1}{2} \quad / : 1$
$y = \frac{\frac{1}{2}}{1}$
$y = \frac{1}{2}$
$L = \{-3/\frac{1}{2}\}$

Aufgabe (10)

$I \quad -1x + 1y = 3$
$II \quad \frac{1}{2}x - 4y = 5$
$I \quad -1x + 1y = 3 \quad / \cdot \left(-\frac{1}{2}\right)$
$II \quad \frac{1}{2}x - 4y = 5 \quad / \cdot (-1)$
$I \quad \frac{1}{2}x - \frac{1}{2}y = -1\frac{1}{2}$
$II \quad -\frac{1}{2}x + 4y = -5$
I + II
$I \quad \frac{1}{2}x - \frac{1}{2}x - \frac{1}{2}y + 4y = -1\frac{1}{2} - 5$
$3\frac{1}{2}y = -6\frac{1}{2} \quad / : 3\frac{1}{2}$
$y = \frac{-6\frac{1}{2}}{3\frac{1}{2}}$
$y = -1\frac{6}{7}$
y in I
$I \quad -1x + 1 \cdot \left(-1\frac{6}{7}\right) = 3$
$-1x - 1\frac{6}{7} = 3 \quad / + 1\frac{6}{7}$
$-1x = 3 + 1\frac{6}{7}$
$-1x = 4\frac{6}{7} \quad / : (-1)$
$x = \frac{4\frac{6}{7}}{-1}$
$x = -4\frac{6}{7}$
$L = \{-4\frac{6}{7}/-1\frac{6}{7}\}$

$I \quad -1x + 1y = 3$
$II \quad \frac{1}{2}x - 4y = 5$
$I \quad -1x + 1y = 3 \quad / \cdot (-4)$
$II \quad \frac{1}{2}x - 4y = 5 \quad / \cdot (-1)$
$I \quad 4x - 4y = -12$
$II \quad -\frac{1}{2}x + 4y = -5$
I + II
$I \quad 4x - \frac{1}{2}x - 4y + 4y = -12 - 5$
$3\frac{1}{2}x = -17 \quad / : 3\frac{1}{2}$
$x = \frac{-17}{3\frac{1}{2}}$
$x = -4\frac{6}{7}$
x in I
$I \quad -1 \cdot \left(-4\frac{6}{7}\right) + 1y = 3$
$1y + 4\frac{6}{7} = 3 \quad / -4\frac{6}{7}$
$1y = 3 - 4\frac{6}{7}$
$1y = -1\frac{6}{7} \quad / : 1$
$y = \frac{-1\frac{6}{7}}{1}$
$y = -1\frac{6}{7}$
$L = \{-4\frac{6}{7}/-1\frac{6}{7}\}$

Aufgabe (11)

$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / \cdot 2\frac{1}{2}$
$II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / \cdot \left(-1\frac{1}{5}\right)$
$I \quad 3x - 3\frac{1}{3}y = 13\frac{1}{3}$
$II \quad -3x + \frac{3}{10}y = -14\frac{17}{20}$
I + II
$I \quad 3x - 3x - 3\frac{1}{3}y + \frac{3}{10}y = 13\frac{1}{3} - 14\frac{17}{20}$
$-3\frac{1}{30}y = -1\frac{31}{60} \quad / : \left(-3\frac{1}{30}\right)$
$y = \frac{-1\frac{31}{60}}{-3\frac{1}{30}}$
$y = \frac{1}{2}$
y in I
$I \quad 1\frac{1}{5}x - 1\frac{1}{3} \cdot \frac{1}{2} = 5\frac{1}{3}$
$1\frac{1}{5}x - \frac{2}{3} = 5\frac{1}{3} \quad / + \frac{2}{3}$
$1\frac{1}{5}x = 5\frac{1}{3} + \frac{2}{3}$
$1\frac{1}{5}x = 6 \quad / : 1\frac{1}{5}$
$x = \frac{6}{1\frac{1}{5}}$
$x = 5$
$L = \{5/\frac{1}{2}\}$

$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$
$II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$
$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / \cdot \left(-\frac{1}{4}\right)$
$II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / \cdot 1\frac{1}{3}$
$I \quad -\frac{3}{10}x + \frac{1}{3}y = -1\frac{1}{3}$
$II \quad 3\frac{1}{3}x - \frac{1}{3}y = 16\frac{1}{2}$
I + II
$I \quad -\frac{3}{10}x + 3\frac{1}{3}x + \frac{1}{3}y - \frac{1}{3}y = -1\frac{1}{3} + 16\frac{1}{2}$
$3\frac{1}{30}x = 15\frac{1}{6} \quad / : 3\frac{1}{30}$
$x = \frac{15\frac{1}{6}}{3\frac{1}{30}}$
$x = 5$
x in I
$I \quad 1\frac{1}{5} \cdot 5 - 1\frac{1}{3}y = 5\frac{1}{3}$
$-1\frac{1}{3}y + 6 = 5\frac{1}{3} \quad / - 6$
$-1\frac{1}{3}y = 5\frac{1}{3} - 6$
$-1\frac{1}{3}y = -\frac{2}{3} \quad / : \left(-1\frac{1}{3}\right)$
$y = \frac{-\frac{2}{3}}{-1\frac{1}{3}}$
$y = \frac{1}{2}$
$L = \{5/\frac{1}{2}\}$

Aufgabe (12)

$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \quad 1x + 1y = 10\frac{2}{3}$
$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / \cdot 1$
$II \quad 1x + 1y = 10\frac{2}{3} \quad / \cdot \left(-\frac{2}{3}\right)$
$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \quad -\frac{2}{3}x - \frac{2}{3}y = -7\frac{1}{9}$
I + II
$I \quad \frac{2}{3}x - \frac{2}{3}x - \frac{5}{7}y - \frac{2}{3}y = \frac{2}{3} - 7\frac{1}{9}$
$-1\frac{8}{21}y = -6\frac{4}{9} \quad / : \left(-1\frac{8}{21}\right)$
$y = \frac{-6\frac{4}{9}}{-1\frac{8}{21}}$
$y = 4\frac{2}{3}$
y in I
$I \quad \frac{2}{3}x - \frac{5}{7} \cdot 4\frac{2}{3} = \frac{2}{3}$
$\frac{2}{3}x - 3\frac{1}{3} = \frac{2}{3} \quad / + 3\frac{1}{3}$
$\frac{2}{3}x = \frac{2}{3} + 3\frac{1}{3}$
$\frac{2}{3}x = 4 \quad / : \frac{2}{3}$
$x = \frac{4}{\frac{2}{3}}$
$x = 6$
$L = \{6/4\frac{2}{3}\}$

$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$
$II \quad 1x + 1y = 10\frac{2}{3}$
$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / \cdot (-1)$
$II \quad 1x + 1y = 10\frac{2}{3} \quad / \cdot \left(-\frac{5}{7}\right)$
$I \quad -\frac{2}{3}x + \frac{5}{7}y = -\frac{2}{3}$
$II \quad -\frac{5}{7}x - \frac{5}{7}y = -7\frac{13}{21}$
I + II
$I \quad -\frac{2}{3}x - \frac{5}{7}x + \frac{5}{7}y - \frac{5}{7}y = -\frac{2}{3} - 7\frac{13}{21}$
$-1\frac{8}{21}x = -8\frac{2}{7} \quad / : \left(-1\frac{8}{21}\right)$
$x = \frac{-8\frac{2}{7}}{-1\frac{8}{21}}$
$x = 6$
x in I
$I \quad \frac{2}{3} \cdot 6 - \frac{5}{7}y = \frac{2}{3}$
$-\frac{5}{7}y + 4 = \frac{2}{3} \quad / - 4$
$-\frac{5}{7}y = \frac{2}{3} - 4$
$-\frac{5}{7}y = -3\frac{1}{3} \quad / : \left(-\frac{5}{7}\right)$
$y = \frac{-3\frac{1}{3}}{-\frac{5}{7}}$
$y = 4\frac{2}{3}$
$L = \{6/4\frac{2}{3}\}$

Aufgabe (13)

$I \quad 1\frac{1}{2}x - 2y = 9$
$II \quad \frac{2}{5}x + \frac{1}{3}y = 5$
$I \quad 1\frac{1}{2}x - 2y = 9 \quad / \cdot \frac{2}{5}$
$II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \quad / \cdot \left(-1\frac{1}{2}\right)$
$I \quad \frac{3}{5}x - \frac{4}{5}y = 3\frac{3}{5}$
$II \quad -\frac{3}{5}x - \frac{1}{2}y = -7\frac{1}{2}$
I + II
$I \quad \frac{3}{5}x - \frac{3}{5}x - \frac{4}{5}y - \frac{1}{2}y = 3\frac{3}{5} - 7\frac{1}{2}$
$-1\frac{3}{10}y = -3\frac{9}{10} \quad / : \left(-1\frac{3}{10}\right)$
$y = \frac{-3\frac{9}{10}}{-1\frac{3}{10}}$
$y = 3$
y in I
$I \quad 1\frac{1}{2}x - 2 \cdot 3 = 9$
$1\frac{1}{2}x - 6 = 9 \quad / + 6$
$1\frac{1}{2}x = 9 + 6$
$1\frac{1}{2}x = 15 \quad / : 1\frac{1}{2}$
$x = \frac{15}{1\frac{1}{2}}$
$x = 10$
$L = \{10/3\}$

$I \quad 1\frac{1}{2}x - 2y = 9$
$II \quad \frac{2}{5}x + \frac{1}{3}y = 5$
$I \quad 1\frac{1}{2}x - 2y = 9 \quad / \cdot \left(-\frac{1}{3}\right)$
$II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \quad / \cdot (-2)$
$I \quad -\frac{1}{2}x + \frac{2}{3}y = -3$
$II \quad -\frac{4}{5}x - \frac{2}{3}y = -10$
I + II
$I \quad -\frac{1}{2}x - \frac{4}{5}x + \frac{2}{3}y - \frac{2}{3}y = -3 - 10$
$-1\frac{3}{10}x = -13 \quad / : \left(-1\frac{3}{10}\right)$
$x = \frac{-13}{-1\frac{3}{10}}$
$x = 10$
x in I
$I \quad 1\frac{1}{2} \cdot 10 - 2y = 9$
$-2y + 15 = 9 \quad / - 15$
$-2y = 9 - 15$
$-2y = -6 \quad / : (-2)$
$y = \frac{-6}{-2}$
$y = 3$
$L = \{10/3\}$

Aufgabe (14)

$I \quad 2x + 3y = 4$
$II \quad \frac{1}{3}x - \frac{1}{5}y = 12$
$I \quad 2x + 3y = 4 \quad / \cdot \frac{1}{3}$
$II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \quad / \cdot (-2)$
$I \quad \frac{2}{3}x + 1y = 1\frac{1}{3}$
$II \quad -\frac{2}{3}x + \frac{2}{5}y = -24$
I + II
$I \quad \frac{2}{3}x - \frac{2}{3}x + 1y + \frac{2}{5}y = 1\frac{1}{3} - 24$
$1\frac{2}{5}y = -22\frac{2}{3} \quad / : 1\frac{2}{5}$
$y = \frac{-22\frac{2}{3}}{1\frac{2}{5}}$
$y = -16\frac{4}{21}$
y in I
$I \quad 2x + 3 \cdot \left(-16\frac{4}{21}\right) = 4$
$2x - 48\frac{4}{7} = 4 \quad / + 48\frac{4}{7}$
$2x = 4 + 48\frac{4}{7}$
$2x = 52\frac{4}{7} \quad / : 2$
$x = \frac{52\frac{4}{7}}{2}$
$x = 26\frac{2}{7}$
$L = \{26\frac{2}{7} / -16\frac{4}{21}\}$

$I \quad 2x + 3y = 4$
$II \quad \frac{1}{3}x - \frac{1}{5}y = 12$
$I \quad 2x + 3y = 4 \quad / \cdot \frac{1}{5}$
$II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \quad / \cdot 3$
$I \quad \frac{2}{5}x + \frac{3}{5}y = \frac{4}{5}$
$II \quad 1x - \frac{3}{5}y = 36$
I + II
$I \quad \frac{2}{5}x + 1x + \frac{3}{5}y - \frac{3}{5}y = \frac{4}{5} + 36$
$1\frac{2}{5}x = 36\frac{4}{5} \quad / : 1\frac{2}{5}$
$x = \frac{36\frac{4}{5}}{1\frac{2}{5}}$
$x = 26\frac{2}{7}$
x in I
$I \quad 2 \cdot 26\frac{2}{7} + 3y = 4$
$3y + 52\frac{4}{7} = 4 \quad / - 52\frac{4}{7}$
$3y = 4 - 52\frac{4}{7}$
$3y = -48\frac{4}{7} \quad / : 3$
$y = \frac{-48\frac{4}{7}}{3}$
$y = -16\frac{4}{21}$
$L = \{26\frac{2}{7} / -16\frac{4}{21}\}$

Aufgabe (15)

$I \quad 1x + 2y = -4$
$II \quad -1x + 2y = 5$
$I \quad 1x + 2y = -4 \quad / \cdot (-1)$
$II \quad -1x + 2y = 5 \quad / \cdot (-1)$
$I \quad -1x - 2y = 4$
$II \quad 1x - 2y = -5$
I + II
$I \quad -1x + 1x - 2y - 2y = 4 - 5$
$-4y = -1 \quad / : (-4)$
$y = \frac{-1}{-4}$
$y = \frac{1}{4}$
y in I
$I \quad 1x + 2 \cdot \frac{1}{4} = -4$
$1x + \frac{1}{2} = -4 \quad / - \frac{1}{2}$
$1x = -4 - \frac{1}{2}$
$1x = -4\frac{1}{2} \quad / : 1$
$x = \frac{-4\frac{1}{2}}{1}$
$x = -4\frac{1}{2}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

$I \quad 1x + 2y = -4$
$II \quad -1x + 2y = 5$
$I \quad 1x + 2y = -4 \quad / \cdot 1$
$II \quad -1x + 2y = 5 \quad / \cdot (-1)$
$I \quad 1x + 2y = -4$
$II \quad 1x - 2y = -5$
I + II
$I \quad 1x + 1x + 2y - 2y = -4 - 5$
$2x = -9 \quad / : 2$
$x = \frac{-9}{2}$
$x = -4\frac{1}{2}$
x in I
$I \quad 1 \cdot \left(-4\frac{1}{2}\right) + 2y = -4$
$2y - 4\frac{1}{2} = -4 \quad / + 4\frac{1}{2}$
$2y = -4 + 4\frac{1}{2}$
$2y = \frac{1}{2} \quad / : 2$
$y = \frac{\frac{1}{2}}{2}$
$y = \frac{1}{4}$
$L = \{-4\frac{1}{2}/\frac{1}{4}\}$

Aufgabe (16)

$I \quad 2x + 4y = 4$
$II \quad -\frac{1}{2}x + 3y = 3$
$I \quad 2x + 4y = 4 \quad / \cdot \frac{1}{2}$
$II \quad -\frac{1}{2}x + 3y = 3 \quad / \cdot 2$
$I \quad 1x + 2y = 2$
$II \quad -1x + 6y = 6$
I + II
$I \quad 1x - 1x + 2y + 6y = 2 + 6$
$8y = 8 \quad / : 8$
$y = \frac{8}{8}$
$y = 1$
y in I
$I \quad 2x + 4 \cdot 1 = 4$
$2x + 4 = 4 \quad / - 4$
$2x = 4 - 4$
$2x = 0 \quad / : 2$
$x = \frac{0}{2}$
$x = 0$
$L = \{0/1\}$

$I \quad 2x + 4y = 4$
$II \quad -\frac{1}{2}x + 3y = 3$
$I \quad 2x + 4y = 4 \quad / \cdot 3$
$II \quad -\frac{1}{2}x + 3y = 3 \quad / \cdot (-4)$
$I \quad 6x + 12y = 12$
$II \quad 2x - 12y = -12$
I + II
$I \quad 6x + 2x + 12y - 12y = 12 - 12$
$8x = 0 \quad / : 8$
$x = \frac{0}{8}$
$x = 0$
x in I
$I \quad 2 \cdot 0 + 4y = 4$
$4y + 0 = 4 \quad / - 0$
$4y = 4 - 0$
$4y = 4 \quad / : 4$
$y = \frac{4}{4}$
$y = 1$
$L = \{0/1\}$

Aufgabe (17)

$I \quad -\frac{1}{2}x + 4y = 6$
$II \quad -2x - 8y = 2$
$I \quad -\frac{1}{2}x + 4y = 6 \quad / \cdot (-2)$
$II \quad -2x - 8y = 2 \quad / \cdot \frac{1}{2}$
$I \quad 1x - 8y = -12$
$II \quad -1x - 4y = 1$
I + II
$I \quad 1x - 1x - 8y - 4y = -12 + 1$
$-12y = -11 \quad / : (-12)$
$y = \frac{-11}{-12}$
$y = \frac{11}{12}$
y in I
$I \quad -\frac{1}{2}x + 4 \cdot \frac{11}{12} = 6$
$-\frac{1}{2}x + 3\frac{2}{3} = 6 \quad / -3\frac{2}{3}$
$-\frac{1}{2}x = 6 - 3\frac{2}{3}$
$-\frac{1}{2}x = 2\frac{1}{3} \quad / : \left(-\frac{1}{2}\right)$
$x = \frac{2\frac{1}{3}}{-\frac{1}{2}}$
$x = -4\frac{2}{3}$
$L = \{-4\frac{2}{3} / \frac{11}{12}\}$

$I \quad -\frac{1}{2}x + 4y = 6$
$II \quad -2x - 8y = 2$
$I \quad -\frac{1}{2}x + 4y = 6 \quad / \cdot (-2)$
$II \quad -2x - 8y = 2 \quad / \cdot (-1)$
$I \quad 1x - 8y = -12$
$II \quad 2x + 8y = -2$
I + II
$I \quad 1x + 2x - 8y + 8y = -12 - 2$
$3x = -14 \quad / : 3$
$x = \frac{-14}{3}$
$x = -4\frac{2}{3}$
x in I
$I \quad -\frac{1}{2} \cdot \left(-4\frac{2}{3}\right) + 4y = 6$
$4y + 2\frac{1}{3} = 6 \quad / -2\frac{1}{3}$
$4y = 6 - 2\frac{1}{3}$
$4y = 3\frac{2}{3} \quad / : 4$
$y = \frac{3\frac{2}{3}}{4}$
$y = \frac{11}{12}$
$L = \{-4\frac{2}{3} / \frac{11}{12}\}$

Aufgabe (18)

$I \quad 1x + 1y = 1$
$II \quad 2x + 2y = 2$
$I \quad 1x + 1y = 1 \quad / \cdot 2$
$II \quad 2x + 2y = 2 \quad / \cdot (-1)$
$I \quad 2x + 2y = 2$
$II \quad -2x - 2y = -2$
I + II
$I \quad 2x - 2x + 2y - 2y = 2 - 2$

$L = unendlich$

$I \quad 1x + 1y = 1$
$II \quad 2x + 2y = 2$
$I \quad 1x + 1y = 1 \quad / \cdot 2$
$II \quad 2x + 2y = 2 \quad / \cdot (-1)$
$I \quad 2x + 2y = 2$
$II \quad -2x - 2y = -2$
I + II
$I \quad 2x - 2x + 2y - 2y = 2 - 2$

$L = unendlich$

Aufgabe (19)

$I \quad 1x + 1y = 2$
$II \quad 3x + 3y = 3$
$I \quad 1x + 1y = 2 \quad / \cdot 3$
$II \quad 3x + 3y = 3 \quad / \cdot (-1)$
$I \quad 3x + 3y = 6$
$II \quad -3x - 3y = -3$
I + II
$I \quad 3x - 3x + 3y - 3y = 6 - 3$

$L = \{\}$

$I \quad 1x + 1y = 2$
$II \quad 3x + 3y = 3$
$I \quad 1x + 1y = 2 \quad / \cdot 3$
$II \quad 3x + 3y = 3 \quad / \cdot (-1)$
$I \quad 3x + 3y = 6$
$II \quad -3x - 3y = -3$
I + II
$I \quad 3x - 3x + 3y - 3y = 6 - 3$

$L = \{\}$

Aufgabe (20)

$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9$
$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / \cdot \left(-\frac{2}{3}\right)$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \quad / \cdot 1\frac{4}{5}$
$I \quad 1\frac{1}{5}x - \frac{8}{9}y = \frac{2}{3}$
$II \quad -1\frac{1}{5}x + \frac{1}{5}y = 16\frac{1}{5}$
I + II
$I \quad 1\frac{1}{5}x - 1\frac{1}{5}x - \frac{8}{9}y + \frac{1}{5}y = \frac{2}{3} + 16\frac{1}{5}$
$-\frac{31}{45}y = 16\frac{13}{15} \quad / : \left(-\frac{31}{45}\right)$
$y = \frac{16\frac{13}{15}}{-\frac{31}{45}}$
$y = -24\frac{15}{31}$
y in I
$I \quad -1\frac{4}{5}x + 1\frac{1}{3} \cdot \left(-24\frac{15}{31}\right) = -1$
$-1\frac{4}{5}x - 32\frac{20}{31} = -1 \quad / + 32\frac{20}{31}$
$-1\frac{4}{5}x = -1 + 32\frac{20}{31}$
$-1\frac{4}{5}x = 31\frac{20}{31} \quad / : \left(-1\frac{4}{5}\right)$
$x = \frac{31\frac{20}{31}}{-1\frac{4}{5}}$
$x = -17\frac{18}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9$
$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / \cdot \frac{1}{9}$
$II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \quad / \cdot \left(-1\frac{1}{3}\right)$
$I \quad -\frac{1}{5}x + \frac{4}{27}y = -\frac{1}{9}$
$II \quad \frac{8}{9}x - \frac{4}{27}y = -12$
I + II
$I \quad -\frac{1}{5}x + \frac{8}{9}x + \frac{4}{27}y - \frac{4}{27}y = -\frac{1}{9} - 12$
$\frac{31}{45}x = -12\frac{1}{9} \quad / : \frac{31}{45}$
$x = \frac{-12\frac{1}{9}}{\frac{31}{45}}$
$x = -17\frac{18}{31}$
x in I
$I \quad -1\frac{4}{5} \cdot \left(-17\frac{18}{31}\right) + 1\frac{1}{3}y = -1$
$1\frac{1}{3}y + 31\frac{20}{31} = -1 \quad / - 31\frac{20}{31}$
$1\frac{1}{3}y = -1 - 31\frac{20}{31}$
$1\frac{1}{3}y = -32\frac{20}{31} \quad / : 1\frac{1}{3}$
$y = \frac{-32\frac{20}{31}}{1\frac{1}{3}}$
$y = -24\frac{15}{31}$
$L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$

Aufgabe (21)

$I \quad 2x - 7y = -8$
$II \quad 7x - 1y = -9$
$I \quad 2x - 7y = -8 \quad / \cdot 7$
$II \quad 7x - 1y = -9 \quad / \cdot (-2)$
$I \quad 14x - 49y = -56$
$II \quad -14x + 2y = 18$
I + II
$I \quad 14x - 14x - 49y + 2y = -56 + 18$
$-47y = -38 \quad / : (-47)$
$y = \frac{-38}{-47}$
$y = \frac{38}{47}$
y in I
$I \quad 2x - 7 \cdot \frac{38}{47} = -8$
$2x - 5\frac{31}{47} = -8 \quad / + 5\frac{31}{47}$
$2x = -8 + 5\frac{31}{47}$
$2x = -2\frac{16}{47} \quad / : 2$
$x = \frac{-2\frac{16}{47}}{2}$
$x = -1\frac{8}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

$I \quad 2x - 7y = -8$
$II \quad 7x - 1y = -9$
$I \quad 2x - 7y = -8 \quad / \cdot (-1)$
$II \quad 7x - 1y = -9 \quad / \cdot 7$
$I \quad -2x + 7y = 8$
$II \quad 49x - 7y = -63$
I + II
$I \quad -2x + 49x + 7y - 7y = 8 - 63$
$47x = -55 \quad / : 47$
$x = \frac{-55}{47}$
$x = -1\frac{8}{47}$
x in I
$I \quad 2 \cdot \left(-1\frac{8}{47}\right) - 7y = -8$
$-7y - 2\frac{16}{47} = -8 \quad / + 2\frac{16}{47}$
$-7y = -8 + 2\frac{16}{47}$
$-7y = -5\frac{31}{47} \quad / : (-7)$
$y = \frac{-5\frac{31}{47}}{-7}$
$y = \frac{38}{47}$
$L = \{-1\frac{8}{47} / \frac{38}{47}\}$

Aufgabe (22)

$I \quad -7x+9y=-3$
$II \quad 5x-6y=-4$
$I \quad -7x+9y=-3 \quad /\cdot 5$
$II \quad 5x-6y=-4 \quad /\cdot 7$
$I \quad -35x+45y=-15$
$II \quad 35x-42y=-28$
I + II
$I \quad -35x+35x+45y-42y=-15-28$
$3y=-43 \quad /:3$
$y=\frac{-43}{3}$
$y=-14\frac{1}{3}$
y in I
$I \quad -7x+9\cdot\left(-14\frac{1}{3}\right)=-3$
$-7x-129=-3 \quad /+129$
$-7x=-3+129$
$-7x=126 \quad /:(-7)$
$x=\frac{126}{-7}$
$x=-18$
$L=\{-18/-14\frac{1}{3}\}$

$I \quad -7x+9y=-3$
$II \quad 5x-6y=-4$
$I \quad -7x+9y=-3 \quad /\cdot(-2)$
$II \quad 5x-6y=-4 \quad /\cdot(-3)$
$I \quad 14x-18y=6$
$II \quad -15x+18y=12$
I + II
$I \quad 14x-15x-18y+18y=6+12$
$-1x=18 \quad /:(-1)$
$x=\frac{18}{-1}$
$x=-18$
x in I
$I \quad -7\cdot(-18)+9y=-3$
$9y+126=-3 \quad /-126$
$9y=-3-126$
$9y=-129 \quad /:9$
$y=\frac{-129}{9}$
$y=-14\frac{1}{3}$
$L=\{-18/-14\frac{1}{3}\}$

Aufgabe (23)

$I \quad 2x+2y=1\frac{7}{10}$
$II \quad 3x+6y=3$
$I \quad 2x+2y=1\frac{7}{10} \quad /\cdot 3$
$II \quad 3x+6y=3 \quad /\cdot(-2)$
$I \quad 6x+6y=5\frac{1}{10}$
$II \quad -6x-12y=-6$
I + II
$I \quad 6x-6x+6y-12y=5\frac{1}{10}-6$
$-6y=-\frac{9}{10} \quad /:(-6)$
$y=\frac{-\frac{9}{10}}{-6}$
$y=\frac{3}{20}$
y in I
$I \quad 2x+2\cdot\frac{3}{20}=1\frac{7}{10}$
$2x+\frac{3}{10}=1\frac{7}{10} \quad /-\frac{3}{10}$
$2x=1\frac{7}{10}-\frac{3}{10}$
$2x=1\frac{2}{5} \quad /:2$
$x=\frac{1\frac{2}{5}}{2}$
$x=\frac{7}{10}$
$L=\{\frac{7}{10}/\frac{3}{20}\}$

$I \quad 2x+2y=1\frac{7}{10}$
$II \quad 3x+6y=3$
$I \quad 2x+2y=1\frac{7}{10} \quad /\cdot 3$
$II \quad 3x+6y=3 \quad /\cdot(-1)$
$I \quad 6x+6y=5\frac{1}{10}$
$II \quad -3x-6y=-3$
I + II
$I \quad 6x-3x+6y-6y=5\frac{1}{10}-3$
$3x=2\frac{1}{10} \quad /:3$
$x=\frac{2\frac{1}{10}}{3}$
$x=\frac{7}{10}$
x in I
$I \quad 2\cdot\frac{7}{10}+2y=1\frac{7}{10}$
$2y+1\frac{2}{5}=1\frac{7}{10} \quad /-1\frac{2}{5}$
$2y=1\frac{7}{10}-1\frac{2}{5}$
$2y=\frac{3}{10} \quad /:2$
$y=\frac{\frac{3}{10}}{2}$
$y=\frac{3}{20}$
$L=\{\frac{7}{10}/\frac{3}{20}\}$

Aufgabe (24)

$I \quad 5x + 6y = 7$
$II \quad 4x + 6y = 4$
$I \quad 5x + 6y = 7 \quad / \cdot 4$
$II \quad 4x + 6y = 4 \quad / \cdot (-5)$
$I \quad 20x + 24y = 28$
$II \quad -20x - 30y = -20$
I + II
$I \quad 20x - 20x + 24y - 30y = 28 - 20$
$-6y = 8 \quad / : (-6)$
$y = \frac{8}{-6}$
$y = -1\frac{1}{3}$
y in I
$I \quad 5x + 6 \cdot \left(-1\frac{1}{3}\right) = 7$
$5x - 8 = 7 \quad / + 8$
$5x = 7 + 8$
$5x = 15 \quad / : 5$
$x = \frac{15}{5}$
$x = 3$
$L = \{3/ - 1\frac{1}{3}\}$

$I \quad 5x + 6y = 7$
$II \quad 4x + 6y = 4$
$I \quad 5x + 6y = 7 \quad / \cdot 1$
$II \quad 4x + 6y = 4 \quad / \cdot (-1)$
$I \quad 5x + 6y = 7$
$II \quad -4x - 6y = -4$
I + II
$I \quad 5x - 4x + 6y - 6y = 7 - 4$
$1x = 3 \quad / : 1$
$x = \frac{3}{1}$
$x = 3$
x in I
$I \quad 5 \cdot 3 + 6y = 7$
$6y + 15 = 7 \quad / - 15$
$6y = 7 - 15$
$6y = -8 \quad / : 6$
$y = \frac{-8}{6}$
$y = -1\frac{1}{3}$
$L = \{3/ - 1\frac{1}{3}\}$

Aufgabe (25)

$I \quad 12x + 6y = 7$
$II \quad 16x + 6y = 4$
$I \quad 12x + 6y = 7 \quad / \cdot 4$
$II \quad 16x + 6y = 4 \quad / \cdot (-3)$
$I \quad 48x + 24y = 28$
$II \quad -48x - 18y = -12$
I + II
$I \quad 48x - 48x + 24y - 18y = 28 - 12$
$6y = 16 \quad / : 6$
$y = \frac{16}{6}$
$y = 2\frac{2}{3}$
y in I
$I \quad 12x + 6 \cdot 2\frac{2}{3} = 7$
$12x + 16 = 7 \quad / - 16$
$12x = 7 - 16$
$12x = -9 \quad / : 12$
$x = \frac{-9}{12}$
$x = -\frac{3}{4}$
$L = \{-\frac{3}{4}/2\frac{2}{3}\}$

$I \quad 12x + 6y = 7$
$II \quad 16x + 6y = 4$
$I \quad 12x + 6y = 7 \quad / \cdot 1$
$II \quad 16x + 6y = 4 \quad / \cdot (-1)$
$I \quad 12x + 6y = 7$
$II \quad -16x - 6y = -4$
I + II
$I \quad 12x - 16x + 6y - 6y = 7 - 4$
$-4x = 3 \quad / : (-4)$
$x = \frac{3}{-4}$
$x = -\frac{3}{4}$
x in I
$I \quad 12 \cdot \left(-\frac{3}{4}\right) + 6y = 7$
$6y - 9 = 7 \quad / + 9$
$6y = 7 + 9$
$6y = 16 \quad / : 6$
$y = \frac{16}{6}$
$y = 2\frac{2}{3}$
$L = \{-\frac{3}{4}/2\frac{2}{3}\}$

4 2-reihige Determinante

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c$$

$$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 4 = 23$$

4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung

Gegeben: $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Gesucht:
Wert der Determinante D

(1) $D = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

(2) $D = \begin{vmatrix} -\frac{1}{2} & 0 \\ 2 & 3 \end{vmatrix}$

(3) $D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 6 & 0 \end{vmatrix}$

(4) $D = \begin{vmatrix} -2 & -8 \\ 0 & -3 \end{vmatrix}$

(5) $D = \begin{vmatrix} \frac{1}{4} & 0 \\ -2 & -1 \end{vmatrix}$

(6) $D = \begin{vmatrix} \frac{1}{4} & -1 \\ -2 & 4 \end{vmatrix}$

(7) $D = \begin{vmatrix} \frac{1}{2} & \frac{1}{5} \\ 2 & 5 \end{vmatrix}$

(8) $D = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix}$

(9) $D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 5 & 4 \end{vmatrix}$

(10) $D = \begin{vmatrix} -2 & 3 \\ 4 & -\frac{1}{3} \end{vmatrix}$

(11) $D = \begin{vmatrix} \frac{1}{2} & 6 \\ -2 & \frac{4}{5} \end{vmatrix}$

(12) $D = \begin{vmatrix} -\frac{1}{3} & \frac{2}{5} \\ 5 & 0 \end{vmatrix}$

(13) $D = \begin{vmatrix} -3 & \frac{2}{3} \\ \frac{1}{2} & \frac{2}{3} \end{vmatrix}$

(14) $D = \begin{vmatrix} 1 & 6 \\ 9 & 3 \end{vmatrix}$

(15) $D = \begin{vmatrix} 7 & 5 \\ 1 & 1 \end{vmatrix}$

(16) $D = \begin{vmatrix} 8 & 5 \\ 2 & 4 \end{vmatrix}$

(17) $D = \begin{vmatrix} \frac{7}{12} & 4\frac{3}{4} \\ \frac{1}{9} & \frac{6}{7} \end{vmatrix}$

(18) $D = \begin{vmatrix} 5\frac{1}{3} & 1\frac{3}{5} \\ \frac{2}{3} & \frac{11}{17} \end{vmatrix}$

(19) $D = \begin{vmatrix} \frac{1}{5} & \frac{1}{2} \\ 1\frac{1}{14} & \frac{11}{17} \end{vmatrix}$

(20) $D = \begin{vmatrix} 3 & 5 \\ 6 & 7 \end{vmatrix}$

(21) $D = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$

(22) $D = \begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix}$

(23) $D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$

4.2 Lösungen

Aufgabe (1)

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = (-2)$$

Aufgabe (2)

$$D = \begin{vmatrix} -\frac{1}{2} & 0 \\ 2 & 3 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 3 - 0 \cdot 2 = \left(-1\frac{1}{2}\right)$$

Aufgabe (3)

$$D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 6 & 0 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 0 - 2 \cdot 6 = (-12)$$

Aufgabe (4)

$$D = \begin{vmatrix} -2 & -8 \\ 0 & -3 \end{vmatrix} = (-2) \cdot (-3) - (-8) \cdot 0 = 6$$

Aufgabe (5)

$$D = \begin{vmatrix} \frac{1}{4} & 0 \\ -2 & -1 \end{vmatrix} = \frac{1}{4} \cdot (-1) - 0 \cdot (-2) = \left(-\frac{1}{4}\right)$$

Aufgabe (6)

$$D = \begin{vmatrix} \frac{1}{4} & -1 \\ -2 & 4 \end{vmatrix} = \frac{1}{4} \cdot 4 - (-1) \cdot (-2) = (-1)$$

Aufgabe (7)

$$D = \begin{vmatrix} \frac{1}{2} & \frac{1}{5} \\ 2 & 5 \end{vmatrix} = \frac{1}{2} \cdot 5 - \frac{1}{5} \cdot 2 = 2\frac{1}{10}$$

Aufgabe (8)

$$D = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = 2 \cdot 1 - 4 \cdot 0 = 2$$

Aufgabe (9)

$$D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 5 & 4 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 4 - 2 \cdot 5 = (-12)$$

Aufgabe (10)

$$D = \begin{vmatrix} -2 & 3 \\ 4 & -\frac{1}{3} \end{vmatrix} = (-2) \cdot \left(-\frac{1}{3}\right) - 3 \cdot 4 = \left(-11\frac{1}{3}\right)$$

Aufgabe (11)

$$D = \begin{vmatrix} \frac{1}{2} & 6 \\ -2 & \frac{4}{5} \end{vmatrix} = \frac{1}{2} \cdot \frac{4}{5} - 6 \cdot (-2) = 12\frac{2}{5}$$

Aufgabe (12)

$$D = \begin{vmatrix} -\frac{1}{3} & \frac{2}{5} \\ 5 & 0 \end{vmatrix} = \left(-\frac{1}{3}\right) \cdot 0 - \frac{2}{5} \cdot 5 = (-2)$$

Aufgabe (13)

$$D = \begin{vmatrix} -3 & \frac{2}{3} \\ \frac{1}{2} & \frac{2}{3} \end{vmatrix} = (-3) \cdot \frac{2}{3} - \frac{2}{3} \cdot \frac{1}{2} = \left(-2\frac{1}{3}\right)$$

Aufgabe (14)

$$D = \begin{vmatrix} 1 & 6 \\ 9 & 3 \end{vmatrix} = 1 \cdot 3 - 6 \cdot 9 = (-51)$$

Aufgabe (15)

$$D = \begin{vmatrix} 7 & 5 \\ 1 & 1 \end{vmatrix} = 7 \cdot 1 - 5 \cdot 1 = 2$$

Aufgabe (16)

$$D = \begin{vmatrix} 8 & 5 \\ 2 & 4 \end{vmatrix} = 8 \cdot 4 - 5 \cdot 2 = 22$$

Aufgabe (17)

$$D = \begin{vmatrix} \frac{7}{12} & 4\frac{3}{4} \\ \frac{1}{9} & \frac{6}{7} \end{vmatrix} = \frac{7}{12} \cdot \frac{6}{7} - 4\frac{3}{4} \cdot \frac{1}{9} = \left(-\frac{1}{36}\right)$$

Aufgabe (18)

$$D = \begin{vmatrix} 5\frac{1}{3} & 1\frac{3}{5} \\ \frac{2}{3} & \frac{11}{17} \end{vmatrix} = 5\frac{1}{3} \cdot \frac{11}{17} - 1\frac{3}{5} \cdot \frac{2}{3} = 2,38$$

Aufgabe (19)

$$D = \begin{vmatrix} \frac{1}{5} & \frac{1}{2} \\ 1\frac{1}{14} & \frac{11}{17} \end{vmatrix} = \frac{1}{5} \cdot \frac{11}{17} - \frac{1}{2} \cdot 1\frac{1}{14} = (-0,406)$$

Aufgabe (20)

$$D = \begin{vmatrix} 3 & 5 \\ 6 & 7 \end{vmatrix} = 3 \cdot 7 - 5 \cdot 6 = (-9)$$

Aufgabe (21)

$$D = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = 3 \cdot 6 - 4 \cdot 5 = (-2)$$

Aufgabe (22)

$$D = \begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix} = 4 \cdot 8 - 6 \cdot 7 = (-10)$$

Aufgabe (23)

$$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 4 = 23$$

5 3-reihige Determinante

Regel von Sarrus

$$D=\left|\begin{array}{ccc|cc} \overset{+}{a_1} & \overset{+}{b_1} & \overset{+}{c_1} & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}\right.$$

$D = a1 \cdot b2 \cdot c3 + b1 \cdot c2 \cdot a3 + c1 \cdot a2 \cdot b3$

$-c1 \cdot b2 \cdot a3 - a1 \cdot c2 \cdot b3 - b1 \cdot a2 \cdot c3$

$$D = \left|\begin{array}{ccc|cc} 11 & 13 & 4 & 11 & 13 \\ 12 & 14 & 5 & 12 & 14 \\ 9 & 3 & 3 & 9 & 3 \end{array}\right.$$

$D_h = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3$

$-4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$

5.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung

Gegeben: $D = \begin{vmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{vmatrix}$

Gesucht:
Wert der Determinante D

(1) $D = \begin{vmatrix} 1 & -2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & -9 \end{vmatrix}$

(2) $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

(3) $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 3 \\ 2 & 6 & 0 \end{vmatrix}$

(4) $D = \begin{vmatrix} -2 & -8 & 0 \\ -3 & \frac{1}{4} & 0 \\ -2 & -1 & 8 \end{vmatrix}$

(5) $D = \begin{vmatrix} \frac{1}{4} & -1 & -2 \\ 4 & 7 & \frac{1}{2} \\ \frac{1}{5} & 2 & 5 \end{vmatrix}$

(6) $D = \begin{vmatrix} \frac{1}{4} & 2 & 4 \\ 0 & 4 & -\frac{1}{2} \\ 2 & 5 & 4 \end{vmatrix}$

(7) $D = \begin{vmatrix} -2 & 3 & 4 \\ -\frac{1}{3} & \frac{1}{2} & 6 \\ -2 & 4 & \frac{4}{5} \end{vmatrix}$

(8) $D = \begin{vmatrix} 6 & 5 & 6 \\ 6 & 3 & 3 \\ 6 & 6 & 2 \end{vmatrix}$

(9) $D = \begin{vmatrix} 1 & 9 & 4 \\ 4 & 8 & 2 \\ 6 & 3 & 1 \end{vmatrix}$

(10) $D = \begin{vmatrix} 1 & 2 & 6 \\ 8 & 3 & 9 \\ 4 & 8 & 1 \end{vmatrix}$

(11) $D = \begin{vmatrix} \frac{14}{15} & 2\frac{4}{5} & 1 \\ 1\frac{6}{13} & 1\frac{1}{2} & 19 \\ 1\frac{3}{8} & \frac{5}{16} & \frac{1}{11} \end{vmatrix}$

(12) $D = \begin{vmatrix} \frac{1}{17} & 14 & \frac{1}{4} \\ 1\frac{2}{17} & \frac{1}{3} & 6\frac{1}{2} \\ \frac{2}{3} & \frac{8}{11} & \frac{8}{17} \end{vmatrix}$

(13) $D = \begin{vmatrix} 1\frac{4}{5} & \frac{9}{13} & 3\frac{3}{4} \\ \frac{2}{5} & 1\frac{5}{14} & 5 \\ \frac{5}{8} & \frac{1}{2} & 1 \end{vmatrix}$

(14) $D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 0 & 5 \\ 0 & 4 & 5 \end{vmatrix}$

(15) $D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{vmatrix}$

(16) $D = \begin{vmatrix} 4 & 6 & 0 \\ 0 & 7 & 8 \\ 0 & 0 & 7 \end{vmatrix}$

5.2 Lösungen

Aufgabe (1)

$$D = \left|\begin{array}{ccc|cc} 1 & -2 & 3 & 1 & -2 \\ -4 & 5 & 6 & -4 & 5 \\ 7 & 8 & -9 & 7 & 8 \end{array}\right.$$
$$D = 1 \cdot 5 \cdot (-9) + (-2) \cdot 6 \cdot 7 + 3 \cdot (-4) \cdot 8$$
$$- 3 \cdot 5 \cdot 7 - 1 \cdot 6 \cdot 8 - (-2) \cdot (-4) \cdot (-9) = -306$$

Aufgabe (2)

$$D = \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array}\right.$$
$$D = 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8$$
$$- 3 \cdot 5 \cdot 7 - 1 \cdot 6 \cdot 8 - 2 \cdot 4 \cdot 9 = 0$$

Aufgabe (3)

$$D = \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 2 & 3 & 4 & 2 \\ 2 & 6 & 0 & 2 & 6 \end{array}\right.$$
$$D = 1 \cdot 2 \cdot 0 + 2 \cdot 3 \cdot 2 + 3 \cdot 4 \cdot 6$$
$$- 3 \cdot 2 \cdot 2 - 1 \cdot 3 \cdot 6 - 2 \cdot 4 \cdot 0 = 54$$

Aufgabe (4)

$$D = \left|\begin{array}{ccc|cc} -2 & -8 & 0 & -2 & -8 \\ -3 & \frac{1}{4} & 0 & -3 & \frac{1}{4} \\ -2 & -1 & 8 & -2 & -1 \end{array}\right.$$
$$D = (-2) \cdot \tfrac{1}{4} \cdot 8 + (-8) \cdot 0 \cdot (-2) + 0 \cdot (-3) \cdot (-1)$$
$$- 0 \cdot \tfrac{1}{4} \cdot (-2) - (-2) \cdot 0 \cdot (-1) - (-8) \cdot (-3) \cdot 8 = -196$$

Aufgabe (5)

$$D = \left|\begin{array}{ccc|cc} \frac{1}{4} & -1 & -2 & \frac{1}{4} & -1 \\ 4 & 7 & \frac{1}{2} & 4 & 7 \\ \frac{1}{5} & 2 & 5 & \frac{1}{5} & 2 \end{array}\right.$$
$$D = \tfrac{1}{4} \cdot 7 \cdot 5 + (-1) \cdot \tfrac{1}{2} \cdot \tfrac{1}{5} + (-2) \cdot 4 \cdot 2$$
$$- (-2) \cdot 7 \cdot \tfrac{1}{5} - \tfrac{1}{4} \cdot \tfrac{1}{2} \cdot 2 - (-1) \cdot 4 \cdot 5 = 15\tfrac{1}{5}$$

Aufgabe (6)

$$D = \left|\begin{array}{ccc|cc} \frac{1}{4} & 2 & 4 & \frac{1}{4} & 2 \\ 0 & 4 & -\frac{1}{2} & 0 & 4 \\ 2 & 5 & 4 & 2 & 5 \end{array}\right.$$

$D = \frac{1}{4} \cdot 4 \cdot 4 + 2 \cdot \left(-\frac{1}{2}\right) \cdot 2 + 4 \cdot 0 \cdot 5$
$- 4 \cdot 4 \cdot 2 - \frac{1}{4} \cdot \left(-\frac{1}{2}\right) \cdot 5 - 2 \cdot 0 \cdot 4 = -29\frac{3}{8}$

Aufgabe (7)

$$D = \left| \begin{array}{ccc|cc} -2 & 3 & 4 & -2 & 3 \\ -\frac{1}{3} & \frac{1}{2} & 6 & -\frac{1}{3} & \frac{1}{2} \\ -2 & 4 & \frac{4}{5} & -2 & 4 \end{array} \right.$$
$D = (-2) \cdot \frac{1}{2} \cdot \frac{4}{5} + 3 \cdot 6 \cdot (-2) + 4 \cdot \left(-\frac{1}{3}\right) \cdot 4$
$- 4 \cdot \frac{1}{2} \cdot (-2) - (-2) \cdot 6 \cdot 4 - 3 \cdot \left(-\frac{1}{3}\right) \cdot \frac{4}{5} = 10\frac{2}{3}$

Aufgabe (8)

$$D = \left| \begin{array}{ccc|cc} 6 & 5 & 6 & 6 & 5 \\ 6 & 3 & 3 & 6 & 3 \\ 6 & 6 & 2 & 6 & 6 \end{array} \right.$$
$D = 6 \cdot 3 \cdot 2 + 5 \cdot 3 \cdot 6 + 6 \cdot 6 \cdot 6$
$- 6 \cdot 3 \cdot 6 - 6 \cdot 3 \cdot 6 - 5 \cdot 6 \cdot 2 = 66$

Aufgabe (9)

$$D = \left| \begin{array}{ccc|cc} 1 & 9 & 4 & 1 & 9 \\ 4 & 8 & 2 & 4 & 8 \\ 6 & 3 & 1 & 6 & 3 \end{array} \right.$$
$D = 1 \cdot 8 \cdot 1 + 9 \cdot 2 \cdot 6 + 4 \cdot 4 \cdot 3$
$- 4 \cdot 8 \cdot 6 - 1 \cdot 2 \cdot 3 - 9 \cdot 4 \cdot 1 = -70$

Aufgabe (10)

$$D = \left| \begin{array}{ccc|cc} 1 & 2 & 6 & 1 & 2 \\ 8 & 3 & 9 & 8 & 3 \\ 4 & 8 & 1 & 4 & 8 \end{array} \right.$$
$D = 1 \cdot 3 \cdot 1 + 2 \cdot 9 \cdot 4 + 6 \cdot 8 \cdot 8$
$- 6 \cdot 3 \cdot 4 - 1 \cdot 9 \cdot 8 - 2 \cdot 8 \cdot 1 = 299$

Aufgabe (11)

$$D = \left| \begin{array}{ccc|cc} \frac{14}{15} & 2\frac{4}{5} & 1 & \frac{14}{15} & 2\frac{4}{5} \\ 1\frac{6}{13} & 1\frac{1}{2} & 19 & 1\frac{6}{13} & 1\frac{1}{2} \\ 1\frac{3}{8} & \frac{5}{16} & \frac{1}{11} & 1\frac{3}{8} & \frac{5}{16} \end{array} \right.$$
$D = \frac{14}{15} \cdot 1\frac{1}{2} \cdot \frac{1}{11} + 2\frac{4}{5} \cdot 19 \cdot 1\frac{3}{8} + 1 \cdot 1\frac{6}{13} \cdot \frac{5}{16}$
$- 1 \cdot 1\frac{1}{2} \cdot 1\frac{3}{8} - \frac{14}{15} \cdot 19 \cdot \frac{5}{16} - 2\frac{4}{5} \cdot 1\frac{6}{13} \cdot \frac{1}{11} = 65,8$

Aufgabe (12)

$$D = \left| \begin{array}{ccc} \frac{1}{17} & 14 & \frac{1}{4} \\ 1\frac{2}{17} & \frac{1}{3} & 6\frac{1}{2} \\ \frac{2}{3} & \frac{8}{11} & \frac{8}{17} \end{array} \right| \begin{array}{cc} \frac{1}{17} & 14 \\ 1\frac{2}{17} & \frac{1}{3} \\ \frac{2}{3} & \frac{8}{11} \end{array}$$

$$D = \frac{1}{17} \cdot \frac{1}{3} \cdot \frac{8}{17} + 14 \cdot 6\frac{1}{2} \cdot \frac{2}{3} + \frac{1}{4} \cdot 1\frac{2}{17} \cdot \frac{8}{11}$$
$$- \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{2}{3} - \frac{1}{17} \cdot 6\frac{1}{2} \cdot \frac{8}{11} - 14 \cdot 1\frac{2}{17} \cdot \frac{8}{17} = 53,2$$

Aufgabe (13)

$$D = \left| \begin{array}{ccc} 1\frac{4}{5} & \frac{9}{13} & 3\frac{3}{4} \\ \frac{2}{5} & 1\frac{5}{14} & 5 \\ \frac{5}{8} & \frac{1}{2} & 1 \end{array} \right| \begin{array}{cc} 1\frac{4}{5} & \frac{9}{13} \\ \frac{2}{5} & 1\frac{5}{14} \\ \frac{5}{8} & \frac{1}{2} \end{array}$$

$$D = 1\frac{4}{5} \cdot 1\frac{5}{14} \cdot 1 + \frac{9}{13} \cdot 5 \cdot \frac{5}{8} + 3\frac{3}{4} \cdot \frac{2}{5} \cdot \frac{1}{2}$$
$$- 3\frac{3}{4} \cdot 1\frac{5}{14} \cdot \frac{5}{8} - 1\frac{4}{5} \cdot 5 \cdot \frac{1}{2} - \frac{9}{13} \cdot \frac{2}{5} \cdot 1 = -2,6$$

Aufgabe (14)

$$D = \left| \begin{array}{ccc} 2 & 4 & 0 \\ 0 & 0 & 5 \\ 0 & 4 & 5 \end{array} \right| \begin{array}{cc} 2 & 4 \\ 0 & 0 \\ 0 & 4 \end{array}$$

$$D = 2 \cdot 0 \cdot 5 + 4 \cdot 5 \cdot 0 + 0 \cdot 0 \cdot 4$$
$$- 0 \cdot 0 \cdot 0 - 2 \cdot 5 \cdot 4 - 4 \cdot 0 \cdot 5 = -40$$

Aufgabe (15)

$$D = \left| \begin{array}{ccc} 2 & 4 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{array} \right| \begin{array}{cc} 2 & 4 \\ 0 & 5 \\ 0 & 0 \end{array}$$

$$D = 2 \cdot 5 \cdot 6 + 4 \cdot 0 \cdot 0 + 0 \cdot 0 \cdot 0$$
$$- 0 \cdot 5 \cdot 0 - 2 \cdot 0 \cdot 0 - 4 \cdot 0 \cdot 6 = 60$$

Aufgabe (16)

$$D = \left| \begin{array}{ccc} 4 & 6 & 0 \\ 0 & 7 & 8 \\ 0 & 0 & 7 \end{array} \right| \begin{array}{cc} 4 & 6 \\ 0 & 7 \\ 0 & 0 \end{array}$$

$$D = 4 \cdot 7 \cdot 7 + 6 \cdot 8 \cdot 0 + 0 \cdot 0 \cdot 0$$
$$- 0 \cdot 7 \cdot 0 - 4 \cdot 8 \cdot 0 - 6 \cdot 0 \cdot 7 = 196$$

6 Determinantenverfahren (2)

$I \quad a1 \cdot x + b1 \cdot y = c1$

$II \quad a2 \cdot x + b2 \cdot y = c42$

$D_h = \begin{vmatrix} a1 & b1 \\ a2 & b2 \end{vmatrix} = a1 \cdot b2 - b1 \cdot a2$

$D_x = \begin{vmatrix} c1 & b1 \\ c2 & b2 \end{vmatrix} = c1 \cdot b2 - b1 \cdot c2$

$D_y = \begin{vmatrix} a1 & c1 \\ a2 & c2 \end{vmatrix} = a1 \cdot c2 - c1 \cdot a2$

- Eindeutige Lösung $D_h \neq 0$
  $x = \frac{D_x}{D_h}$
  $y = \frac{D_y}{D_h}$
- Keine Lösung $D_h = 0$
  $D_x \neq 0$ oder $D_y \neq 0$
- Unendlich viele Lösungen
  $D_h = D_x = D_y = 0$

$I \quad 3x + 5y = 19$

$II \quad 7x + 5y = 31$

$D_h = \begin{vmatrix} 3 & 5 \\ 7 & 5 \end{vmatrix} = 3 \cdot 5 - 5 \cdot 7 = -20$

$D_x = \begin{vmatrix} 19 & 5 \\ 31 & 5 \end{vmatrix} = 19 \cdot 5 - 5 \cdot 31 = -60$

$D_y = \begin{vmatrix} 3 & 19 \\ 7 & 31 \end{vmatrix} = 3 \cdot 31 - 19 \cdot 7 = -40$

$x = \frac{-60}{-20}$

$x = 3$

$y = \frac{-40}{-20}$

$y = 2$

$L = \{3/2\}$

6.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung

$D_h = \begin{vmatrix} a1 & b1 \\ a2 & b2 \end{vmatrix} = a1 \cdot b2 - b1 \cdot a2$

$D_x = \begin{vmatrix} c1 & b1 \\ c2 & b2 \end{vmatrix} = c1 \cdot b2 - b1 \cdot c2$

$D_y = \begin{vmatrix} a1 & c1 \\ a2 & c2 \end{vmatrix} = a1 \cdot c2 - c1 \cdot a2$

$x = \frac{D_x}{D_h}$

$y = \frac{D_y}{D_h}$

(1) $3x + 5y = 19$
$7x + 5y = 31$

(2) $1x + 1y = 10$
$1x - 1y = 4$

(3) $9x - 2y = 5$
$5x - 2y = 1$

(4) $9x - 2y = 1$
$-3x - 3y = -7$

(5) $1x + 3y = 9$
$3x - 2y = -6$

(6) $7x + 1\frac{1}{2}y = 1$
$2x - 5y = 22$

(7) $\begin{array}{l} 4x - 5y = 43 \\ -3x - 7y = 0 \end{array}$

(8) $\begin{array}{l} 8x - 3y = 10 \\ 12x - 5y = 24 \end{array}$

(9) $\begin{array}{l} -\frac{1}{2}x + 1y = 2 \\ \frac{1}{2}x - 3y = -3 \end{array}$

(10) $\begin{array}{l} -1x + 1y = 3 \\ \frac{1}{2}x - 4y = 5 \end{array}$

(11) $\begin{array}{l} 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\ 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \end{array}$

(12) $\begin{array}{l} \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\ 1x + 1y = 10\frac{2}{3} \end{array}$

(13) $\begin{array}{l} 1\frac{1}{2}x - 2y = 9 \\ \frac{2}{5}x + \frac{1}{3}y = 5 \end{array}$

(14) $\begin{array}{l} 2x + 3y = 4 \\ \frac{1}{3}x - \frac{1}{5}y = 12 \end{array}$

(15) $\begin{array}{l} 1x + 2y = -4 \\ -1x + 2y = 5 \end{array}$

(16) $\begin{array}{l} 2x + 4y = 4 \\ -\frac{1}{2}x + 3y = 3 \end{array}$

(17) $\begin{array}{l} -\frac{1}{2}x + 4y = 6 \\ -2x - 8y = 2 \end{array}$

(18) $\begin{array}{l} 1x + 1y = 1 \\ 2x + 2y = 2 \end{array}$

(19) $\begin{array}{l} 1x + 1y = 2 \\ 3x + 3y = 3 \end{array}$

(20) $\begin{array}{l} -1\frac{4}{5}x + 1\frac{1}{3}y = -1 \\ -\frac{2}{3}x + \frac{1}{9}y = 9 \end{array}$

(21) $\begin{array}{l} 2x - 7y = -8 \\ 7x - 1y = -9 \end{array}$

(22) $\begin{array}{l} -7x + 9y = -3 \\ 5x - 6y = -4 \end{array}$

(23) $\begin{array}{l} 2x + 2y = 1\frac{7}{10} \\ 3x + 6y = 3 \end{array}$

(24) $\begin{array}{l} 4x + 6y = 7 \\ 5x + 6y = 5 \end{array}$

6.2 Lösungen

Aufgabe (1)

$D_h = \begin{vmatrix} 3 & 5 \\ 7 & 5 \end{vmatrix} = 3 \cdot 5 - 5 \cdot 7 = -20$
$D_x = \begin{vmatrix} 19 & 5 \\ 31 & 5 \end{vmatrix} = 19 \cdot 5 - 5 \cdot 31 = -60$
$D_y = \begin{vmatrix} 3 & 19 \\ 7 & 31 \end{vmatrix} = 3 \cdot 31 - 19 \cdot 7 = -40$
$x = \frac{-60}{-20}$
$x = 3$
$y = \frac{-40}{-20}$
$y = 2$
$L = \{3/2\}$

Aufgabe (2)

$D_h = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1 \cdot (-1) - 1 \cdot 1 = -2$
$D_x = \begin{vmatrix} 10 & 1 \\ 4 & -1 \end{vmatrix} = 10 \cdot (-1) - 1 \cdot 4 = -14$
$D_y = \begin{vmatrix} 1 & 10 \\ 1 & 4 \end{vmatrix} = 1 \cdot 4 - 10 \cdot 1 = -6$
$x = \frac{-14}{-2}$
$x = 7$
$y = \frac{-6}{-2}$
$y = 3$
$L = \{7/3\}$

Aufgabe (3)

$D_h = \begin{vmatrix} 9 & -2 \\ 5 & -2 \end{vmatrix} = 9 \cdot (-2) - (-2) \cdot 5 = -8$
$D_x = \begin{vmatrix} 5 & -2 \\ 1 & -2 \end{vmatrix} = 5 \cdot (-2) - (-2) \cdot 1 = -8$
$D_y = \begin{vmatrix} 9 & 5 \\ 5 & 1 \end{vmatrix} = 9 \cdot 1 - 5 \cdot 5 = -16$
$x = \frac{-8}{-8}$
$x = 1$
$y = \frac{-16}{-8}$
$y = 2$
$L = \{1/2\}$

Aufgabe (4)

$D_h = \begin{vmatrix} 9 & -2 \\ -3 & -3 \end{vmatrix} = 9 \cdot (-3) - (-2) \cdot (-3) = -33$

$D_x = \begin{vmatrix} 1 & -2 \\ -7 & -3 \end{vmatrix} = 1 \cdot (-3) - (-2) \cdot (-7) = -17$
$D_y = \begin{vmatrix} 9 & 1 \\ -3 & -7 \end{vmatrix} = 9 \cdot (-7) - 1 \cdot (-3) = -60$
$x = \frac{-17}{-33}$
$x = \frac{17}{33}$
$y = \frac{-60}{-33}$
$y = 1\frac{9}{11}$
$L = \{\frac{17}{33}/1\frac{9}{11}\}$

Aufgabe (5)

$D_h = \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = 1 \cdot (-2) - 3 \cdot 3 = -11$
$D_x = \begin{vmatrix} 9 & 3 \\ -6 & -2 \end{vmatrix} = 9 \cdot (-2) - 3 \cdot (-6) = 0$
$D_y = \begin{vmatrix} 1 & 9 \\ 3 & -6 \end{vmatrix} = 1 \cdot (-6) - 9 \cdot 3 = -33$
$x = \frac{0}{-11}$
$x = 0$
$y = \frac{-33}{-11}$
$y = 3$
$L = \{0/3\}$

Aufgabe (6)

$D_h = \begin{vmatrix} 7 & 1\frac{1}{2} \\ 2 & -5 \end{vmatrix} = 7 \cdot (-5) - 1\frac{1}{2} \cdot 2 = -38$
$D_x = \begin{vmatrix} 1 & 1\frac{1}{2} \\ 22 & -5 \end{vmatrix} = 1 \cdot (-5) - 1\frac{1}{2} \cdot 22 = -38$
$D_y = \begin{vmatrix} 7 & 1 \\ 2 & 22 \end{vmatrix} = 7 \cdot 22 - 1 \cdot 2 = 152$
$x = \frac{-38}{-38}$
$x = 1$
$y = \frac{152}{-38}$
$y = -4$
$L = \{1/-4\}$

Aufgabe (7)

$D_h = \begin{vmatrix} 4 & -5 \\ -3 & -7 \end{vmatrix} = 4 \cdot (-7) - (-5) \cdot (-3) = -43$
$D_x = \begin{vmatrix} 43 & -5 \\ 0 & -7 \end{vmatrix} = 43 \cdot (-7) - (-5) \cdot 0 = -301$
$D_y = \begin{vmatrix} 4 & 43 \\ -3 & 0 \end{vmatrix} = 4 \cdot 0 - 43 \cdot (-3) = 129$
$x = \frac{-301}{-43}$

$x = 7$
$y = \frac{129}{-43}$
$y = -3$
$L = \{7/-3\}$

Aufgabe (8)

$D_h = \begin{vmatrix} 8 & -3 \\ 12 & -5 \end{vmatrix} = 8 \cdot (-5) - (-3) \cdot 12 = -4$
$D_x = \begin{vmatrix} 10 & -3 \\ 24 & -5 \end{vmatrix} = 10 \cdot (-5) - (-3) \cdot 24 = 22$
$D_y = \begin{vmatrix} 8 & 10 \\ 12 & 24 \end{vmatrix} = 8 \cdot 24 - 10 \cdot 12 = 72$
$x = \frac{22}{-4}$
$x = -5\frac{1}{2}$
$y = \frac{72}{-4}$
$y = -18$
$L = \{-5\frac{1}{2}/-18\}$

Aufgabe (9)

$D_h = \begin{vmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2} \cdot (-3) - 1 \cdot \frac{1}{2} = 1$
$D_x = \begin{vmatrix} 2 & 1 \\ -3 & -3 \end{vmatrix} = 2 \cdot (-3) - 1 \cdot (-3) = -3$
$D_y = \begin{vmatrix} -\frac{1}{2} & 2 \\ \frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2} \cdot (-3) - 2 \cdot \frac{1}{2} = \frac{1}{2}$
$x = \frac{-3}{1}$
$x = -3$
$y = \frac{\frac{1}{2}}{1}$
$y = \frac{1}{2}$
$L = \{-3/\frac{1}{2}\}$

Aufgabe (10)

$D_h = \begin{vmatrix} -1 & 1 \\ \frac{1}{2} & -4 \end{vmatrix} = -1 \cdot (-4) - 1 \cdot \frac{1}{2} = 3\frac{1}{2}$
$D_x = \begin{vmatrix} 3 & 1 \\ 5 & -4 \end{vmatrix} = 3 \cdot (-4) - 1 \cdot 5 = -17$
$D_y = \begin{vmatrix} -1 & 3 \\ \frac{1}{2} & 5 \end{vmatrix} = -1 \cdot 5 - 3 \cdot \frac{1}{2} = -6\frac{1}{2}$
$x = \frac{-17}{3\frac{1}{2}}$
$x = -4\frac{6}{7}$
$y = \frac{-6\frac{1}{2}}{3\frac{1}{2}}$
$y = -1\frac{6}{7}$
$L = \{-4\frac{6}{7}/-1\frac{6}{7}\}$

Aufgabe (11)

$D_h = \begin{vmatrix} 1\frac{1}{5} & -1\frac{1}{3} \\ 2\frac{1}{2} & -\frac{1}{4} \end{vmatrix} = 1\frac{1}{5} \cdot \left(-\frac{1}{4}\right) - \left(-1\frac{1}{3}\right) \cdot 2\frac{1}{2} = 3\frac{1}{30}$

$D_x = \begin{vmatrix} 5\frac{1}{3} & -1\frac{1}{3} \\ 12\frac{3}{8} & -\frac{1}{4} \end{vmatrix} = 5\frac{1}{3} \cdot \left(-\frac{1}{4}\right) - \left(-1\frac{1}{3}\right) \cdot 12\frac{3}{8} = 15\frac{1}{6}$

$D_y = \begin{vmatrix} 1\frac{1}{5} & 5\frac{1}{3} \\ 2\frac{1}{2} & 12\frac{3}{8} \end{vmatrix} = 1\frac{1}{5} \cdot 12\frac{3}{8} - 5\frac{1}{3} \cdot 2\frac{1}{2} = 1\frac{31}{60}$

$x = \frac{15\frac{1}{6}}{3\frac{1}{30}}$

$x = 5$

$y = \frac{1\frac{31}{60}}{3\frac{1}{30}}$

$y = \frac{1}{2}$

$L = \{5/\frac{1}{2}\}$

Aufgabe (12)

$D_h = \begin{vmatrix} \frac{2}{3} & -\frac{5}{7} \\ 1 & 1 \end{vmatrix} = \frac{2}{3} \cdot 1 - \left(-\frac{5}{7}\right) \cdot 1 = 1\frac{8}{21}$

$D_x = \begin{vmatrix} \frac{2}{3} & -\frac{5}{7} \\ 10\frac{2}{3} & 1 \end{vmatrix} = \frac{2}{3} \cdot 1 - \left(-\frac{5}{7}\right) \cdot 10\frac{2}{3} = 8\frac{2}{7}$

$D_y = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} \\ 1 & 10\frac{2}{3} \end{vmatrix} = \frac{2}{3} \cdot 10\frac{2}{3} - \frac{2}{3} \cdot 1 = 6\frac{4}{9}$

$x = \frac{8\frac{2}{7}}{1\frac{8}{21}}$

$x = 6$

$y = \frac{6\frac{4}{9}}{1\frac{8}{21}}$

$y = 4\frac{2}{3}$

$L = \{6/4\frac{2}{3}\}$

Aufgabe (13)

$D_h = \begin{vmatrix} 1\frac{1}{2} & -2 \\ \frac{2}{5} & \frac{1}{3} \end{vmatrix} = 1\frac{1}{2} \cdot \frac{1}{3} - (-2) \cdot \frac{2}{5} = 1\frac{3}{10}$

$D_x = \begin{vmatrix} 9 & -2 \\ 5 & \frac{1}{3} \end{vmatrix} = 9 \cdot \frac{1}{3} - (-2) \cdot 5 = 13$

$D_y = \begin{vmatrix} 1\frac{1}{2} & 9 \\ \frac{2}{5} & 5 \end{vmatrix} = 1\frac{1}{2} \cdot 5 - 9 \cdot \frac{2}{5} = 3\frac{9}{10}$

$x = \frac{13}{1\frac{3}{10}}$

$x = 10$

$y = \frac{3\frac{9}{10}}{1\frac{3}{10}}$

$y = 3$

$L = \{10/3\}$

Aufgabe (14)

$D_h = \begin{vmatrix} 2 & 3 \\ \frac{1}{3} & -\frac{1}{5} \end{vmatrix} = 2 \cdot \left(-\frac{1}{5}\right) - 3 \cdot \frac{1}{3} = -1\frac{2}{5}$
$D_x = \begin{vmatrix} 4 & 3 \\ 12 & -\frac{1}{5} \end{vmatrix} = 4 \cdot \left(-\frac{1}{5}\right) - 3 \cdot 12 = -36\frac{4}{5}$
$D_y = \begin{vmatrix} 2 & 4 \\ \frac{1}{3} & 12 \end{vmatrix} = 2 \cdot 12 - 4 \cdot \frac{1}{3} = 22\frac{2}{3}$
$x = \frac{-36\frac{4}{5}}{-1\frac{2}{5}}$
$x = 26\frac{2}{7}$
$y = \frac{22\frac{2}{3}}{-1\frac{2}{5}}$
$y = -16\frac{4}{21}$
$L = \{26\frac{2}{7} / -16\frac{4}{21}\}$

Aufgabe (15)

$D_h = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 1 \cdot 2 - 2 \cdot (-1) = 4$
$D_x = \begin{vmatrix} -4 & 2 \\ 5 & 2 \end{vmatrix} = -4 \cdot 2 - 2 \cdot 5 = -18$
$D_y = \begin{vmatrix} 1 & -4 \\ -1 & 5 \end{vmatrix} = 1 \cdot 5 - (-4) \cdot (-1) = 1$
$x = \frac{-18}{4}$
$x = -4\frac{1}{2}$
$y = \frac{1}{4}$
$y = \frac{1}{4}$
$L = \{-4\frac{1}{2} / \frac{1}{4}\}$

Aufgabe (16)

$D_h = \begin{vmatrix} 2 & 4 \\ -\frac{1}{2} & 3 \end{vmatrix} = 2 \cdot 3 - 4 \cdot \left(-\frac{1}{2}\right) = 8$
$D_x = \begin{vmatrix} 4 & 4 \\ 3 & 3 \end{vmatrix} = 4 \cdot 3 - 4 \cdot 3 = 0$
$D_y = \begin{vmatrix} 2 & 4 \\ -\frac{1}{2} & 3 \end{vmatrix} = 2 \cdot 3 - 4 \cdot \left(-\frac{1}{2}\right) = 8$
$x = \frac{0}{8}$
$x = 0$
$y = \frac{8}{8}$
$y = 1$
$L = \{0/1\}$

Aufgabe (17)

$D_h = \begin{vmatrix} -\frac{1}{2} & 4 \\ -2 & -8 \end{vmatrix} = -\frac{1}{2} \cdot (-8) - 4 \cdot (-2) = 12$

$D_x = \begin{vmatrix} 6 & 4 \\ 2 & -8 \end{vmatrix} = 6 \cdot (-8) - 4 \cdot 2 = -56$
$D_y = \begin{vmatrix} -\frac{1}{2} & 6 \\ -2 & 2 \end{vmatrix} = -\frac{1}{2} \cdot 2 - 6 \cdot (-2) = 11$
$x = \frac{-56}{12}$
$x = -4\frac{2}{3}$
$y = \frac{11}{12}$
$y = \frac{11}{12}$
$L = \{-4\frac{2}{3}/\frac{11}{12}\}$

Aufgabe (18)

$D_h = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0$
$D_x = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0$
$D_y = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0$
$L = \{\text{unendlich}\}$

Aufgabe (19)

$D_h = \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 1 \cdot 3 - 1 \cdot 3 = 0$
$D_x = \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 3 = 3$
$D_y = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 1 \cdot 3 - 2 \cdot 3 = -3$
$L = \{\}$

Aufgabe (20)

$D_h = \begin{vmatrix} -1\frac{4}{5} & 1\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{vmatrix} = -1\frac{4}{5} \cdot \frac{1}{9} - 1\frac{1}{3} \cdot \left(-\frac{2}{3}\right) = \frac{31}{45}$
$D_x = \begin{vmatrix} -1 & 1\frac{1}{3} \\ 9 & \frac{1}{9} \end{vmatrix} = -1 \cdot \frac{1}{9} - 1\frac{1}{3} \cdot 9 = -12\frac{1}{9}$
$D_y = \begin{vmatrix} -1\frac{4}{5} & -1 \\ -\frac{2}{3} & 9 \end{vmatrix} = -1\frac{4}{5} \cdot 9 - (-1) \cdot \left(-\frac{2}{3}\right) = -16\frac{13}{15}$
$x = \frac{-12\frac{1}{9}}{\frac{31}{45}}$
$x = -17\frac{18}{31}$
$y = \frac{-16\frac{13}{15}}{\frac{31}{45}}$
$y = -24\frac{15}{31}$
$L = \{-17\frac{18}{31}/-24\frac{15}{31}\}$

Aufgabe (21)

$D_h = \begin{vmatrix} 2 & -7 \\ 7 & -1 \end{vmatrix} = 2 \cdot (-1) - (-7) \cdot 7 = 47$
$D_x = \begin{vmatrix} -8 & -7 \\ -9 & -1 \end{vmatrix} = -8 \cdot (-1) - (-7) \cdot (-9) = -55$
$D_y = \begin{vmatrix} 2 & -8 \\ 7 & -9 \end{vmatrix} = 2 \cdot (-9) - (-8) \cdot 7 = 38$
$x = \frac{-55}{47}$
$x = -1\frac{8}{47}$
$y = \frac{38}{47}$
$y = \frac{38}{47}$
$L = \{-1\frac{8}{47}/\frac{38}{47}\}$

Aufgabe (22)

$D_h = \begin{vmatrix} -7 & 9 \\ 5 & -6 \end{vmatrix} = -7 \cdot (-6) - 9 \cdot 5 = -3$
$D_x = \begin{vmatrix} -3 & 9 \\ -4 & -6 \end{vmatrix} = -3 \cdot (-6) - 9 \cdot (-4) = 54$
$D_y = \begin{vmatrix} -7 & -3 \\ 5 & -4 \end{vmatrix} = -7 \cdot (-4) - (-3) \cdot 5 = 43$
$x = \frac{54}{-3}$
$x = -18$
$y = \frac{43}{-3}$
$y = -14\frac{1}{3}$
$L = \{-18/-14\frac{1}{3}\}$

Aufgabe (23)

$D_h = \begin{vmatrix} 2 & 2 \\ 3 & 6 \end{vmatrix} = 2 \cdot 6 - 2 \cdot 3 = 6$
$D_x = \begin{vmatrix} 1\frac{7}{10} & 2 \\ 3 & 6 \end{vmatrix} = 1\frac{7}{10} \cdot 6 - 2 \cdot 3 = 4\frac{1}{5}$
$D_y = \begin{vmatrix} 2 & 1\frac{7}{10} \\ 3 & 3 \end{vmatrix} = 2 \cdot 3 - 1\frac{7}{10} \cdot 3 = \frac{9}{10}$
$x = \frac{4\frac{1}{5}}{6}$
$x = \frac{7}{10}$
$y = \frac{\frac{9}{10}}{6}$
$y = \frac{3}{20}$
$L = \{\frac{7}{10}/\frac{3}{20}\}$

Aufgabe (24)

$D_h = \begin{vmatrix} 4 & 6 \\ 5 & 6 \end{vmatrix} = 4 \cdot 6 - 6 \cdot 5 = -6$
$D_x = \begin{vmatrix} 7 & 6 \\ 5 & 6 \end{vmatrix} = 7 \cdot 6 - 6 \cdot 5 = 12$

$D_y = \begin{vmatrix} 4 & 7 \\ 5 & 5 \end{vmatrix} = 4 \cdot 5 - 7 \cdot 5 = -15$
$x = \frac{12}{-6}$
$x = -2$
$y = \frac{-15}{-6}$
$y = 2\frac{1}{2}$
$L = \{-2/2\frac{1}{2}\}$

7 Determinantenverfahren (3)

$a1x + b1y + c1z = d1$
$a2x + b2y + c2z = d2$
$a3x + b3y + c3z = d3$

$$D_h = \left|\begin{array}{ccc|cc} a1 & b1 & c1 & a1 & b1 \\ a2 & b2 & c2 & a2 & b2 \\ a3 & b3 & c3 & a3 & b3 \end{array}\right.$$

$D_h = a1 \cdot b2 \cdot c3 + b1 \cdot c2 \cdot a3 + c1 \cdot a2 \cdot b3 - c1 \cdot b2 \cdot a3 - a1 \cdot c2 \cdot b3 - b1 \cdot a2 \cdot c3$

$$D_x = \left|\begin{array}{ccc|cc} d1 & b1 & c1 & d1 & b1 \\ d2 & b2 & c2 & d2 & b2 \\ d3 & b3 & c3 & d3 & b3 \end{array}\right.$$

$D_x = d1 \cdot b2 \cdot c3 + b1 \cdot c2 \cdot d3 + c1 \cdot d2 \cdot b3 - c1 \cdot b2 \cdot d3 - d1 \cdot c2 \cdot b3 - b1 \cdot d2 \cdot c3$

$$D_y = \left|\begin{array}{ccc|cc} a1 & d1 & c1 & a1 & d1 \\ a2 & d2 & c2 & a2 & d2 \\ a3 & d3 & c3 & a3 & d3 \end{array}\right.$$

$D_y = a1 \cdot d2 \cdot c3 + d1 \cdot c2 \cdot a3 + c1 \cdot a2 \cdot d3 - c1 \cdot d2 \cdot a3 - a1 \cdot c2 \cdot d3 - d1 \cdot a2 \cdot c3$

$$D_z = \left|\begin{array}{ccc|cc} a1 & b1 & d1 & a1 & b1 \\ a2 & b2 & d2 & a2 & b2 \\ a3 & b3 & d3 & a3 & b3 \end{array}\right.$$

$D_z = a1 \cdot b2 \cdot d3 + b1 \cdot d2 \cdot a3 + d1 \cdot a2 \cdot b3 - d1 \cdot b2 \cdot a3 - a1 \cdot d2 \cdot b3 - b1 \cdot a2 \cdot d3 = 0$

- Eindeutige Lösung $D_h \neq 0$
  $x = \frac{D_x}{D_h}$
  $y = \frac{D_y}{D_h}$
  $z = \frac{D_z}{D_h}$
- Keine Lösung $D_h = 0$ $D_x \neq 0$ oder $D_y \neq 0$ oder $D_z \neq 0$
- Unendlich viele Lösungen
  $D_h = D_x = D_y = D_z = 0$

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

$$D_h = \left|\begin{array}{ccc|cc} 11 & 13 & 4 & 11 & 13 \\ 12 & 14 & 5 & 12 & 14 \\ 9 & 3 & 3 & 9 & 3 \end{array}\right.$$

$D_h = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3$
$-4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$

$$D_x = \left|\begin{array}{ccc|cc} 37 & 13 & 4 & 37 & 13 \\ 40 & 14 & 5 & 40 & 14 \\ 15 & 3 & 3 & 15 & 3 \end{array}\right.$$

$D_x = 37 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 15 + 4 \cdot 40 \cdot 3$
$-4 \cdot 14 \cdot 15 - 37 \cdot 5 \cdot 3 - 13 \cdot 40 \cdot 3 = 54$

$$D_y = \left|\begin{array}{ccc|cc} 11 & 37 & 4 & 11 & 37 \\ 12 & 40 & 5 & 12 & 40 \\ 9 & 15 & 3 & 9 & 15 \end{array}\right.$$

$D_y = 11 \cdot 40 \cdot 3 + 37 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 15$
$-4 \cdot 40 \cdot 9 - 11 \cdot 5 \cdot 15 - 37 \cdot 12 \cdot 3 = 108$

$$D_z = \left|\begin{array}{ccc|cc} 11 & 13 & 37 & 11 & 13 \\ 12 & 14 & 40 & 12 & 14 \\ 9 & 3 & 15 & 9 & 3 \end{array}\right.$$

$D_z = 11 \cdot 14 \cdot 15 + 13 \cdot 40 \cdot 9 + 37 \cdot 12 \cdot 3$
$-37 \cdot 14 \cdot 9 - 11 \cdot 40 \cdot 3 - 13 \cdot 12 \cdot 15 = 0$
$x = \frac{54}{54}$
$x = 1$
$y = \frac{108}{54}$
$y = 2$
$z = \frac{0}{54}$
$z = 0$
$L = \{1/2/0\}$

7.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$
$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$
$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$

Gesucht:
x,y,z

(1)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(2)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(3)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x + 2y + -3z &= 7 \end{aligned}$$

(4)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(5)
$$\begin{aligned} 1x + 2 + 3z &= 4 \\ 2x + 3y + 2z &= 6 \\ 0x + 2y + 6z &= 0 \end{aligned}$$

(6)
$$\begin{aligned} -2x - 8 + 0z &= 1 \\ 1x + 4y + 0z &= -\tfrac{1}{2} \\ 8x - 2y + -1z &= 8 \end{aligned}$$

(7)
$$\begin{aligned} -2x + 2 + 4z &= 0 \\ 4x - \tfrac{1}{2}y + 2z &= 5 \\ 4x - 2y + -1z &= 8 \end{aligned}$$

(8)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(9)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x - 2y + -3z &= 7 \end{aligned}$$

(10)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(11)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(12)
$$\begin{aligned} 2x + 3 + 4z &= 175 \\ 4x + 6y + 5z &= 287 \\ 3x + 2y + 8z &= 257 \end{aligned}$$

(13)
$$\begin{aligned} 6x + 4 + 9z &= 32 \\ 5x + 7y + 10z &= 17 \\ 4x + 8y + 5z &= 100 \end{aligned}$$

(14)
$$\begin{aligned} 1x + 1 + 0z &= 1 \\ 1x + 0y + 1z &= 6 \\ 0x + 1y + -1z &= 5 \end{aligned}$$

(15)
$$\begin{aligned} 1x - 2 + 3z &= 9 \\ 3x + 8y + 9z &= 5 \\ 2x + 3y + 6z &= 7 \end{aligned}$$

(16)
$$\begin{aligned} 6x + 4 + 5z &= 8 \\ 4x + 2y + 3z &= 7 \\ 5x + 3y + 4z &= 9 \end{aligned}$$

(17)
$$\begin{aligned} 1x + 3 + -2z &= 3 \\ 3x + 2y + 1z &= 2 \\ 0x + 1y + 3z &= 5 \end{aligned}$$

7.2 Lösungen

Aufgabe (1)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

$$D_h = \left|\begin{array}{ccc|cc} 11 & 13 & 4 & 11 & 13 \\ 12 & 14 & 5 & 12 & 14 \\ 9 & 3 & 3 & 9 & 3 \end{array}\right.$$

$D_h = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3 - 4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$

$$D_x = \left|\begin{array}{ccc|cc} 37 & 13 & 4 & 37 & 13 \\ 40 & 14 & 5 & 40 & 14 \\ 15 & 3 & 3 & 15 & 3 \end{array}\right.$$

$D_x = 37 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 15 + 4 \cdot 40 \cdot 3 - 4 \cdot 14 \cdot 15 - 37 \cdot 5 \cdot 3 - 13 \cdot 40 \cdot 3 = 54$

$$D_y = \left|\begin{array}{ccc|cc} 11 & 37 & 4 & 11 & 37 \\ 12 & 40 & 5 & 12 & 40 \\ 9 & 15 & 3 & 9 & 15 \end{array}\right.$$

$D_y = 11 \cdot 40 \cdot 3 + 37 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 15 - 4 \cdot 40 \cdot 9 - 11 \cdot 5 \cdot 15 - 37 \cdot 12 \cdot 3 = 108$

$$D_z = \left|\begin{array}{ccc|cc} 11 & 13 & 37 & 11 & 13 \\ 12 & 14 & 40 & 12 & 14 \\ 9 & 3 & 15 & 9 & 3 \end{array}\right.$$

$D_z = 11 \cdot 14 \cdot 15 + 13 \cdot 40 \cdot 9 + 37 \cdot 12 \cdot 3 - 37 \cdot 14 \cdot 9 - 11 \cdot 40 \cdot 3 - 13 \cdot 12 \cdot 15 = 0$
$x = \frac{54}{54}$
$x = 1$
$y = \frac{108}{54}$
$y = 2$
$z = \frac{0}{54}$
$z = 0$
$L = \{1/2/0\}$

Aufgabe (2)

$9x + 5y + 4z = 13$
$6x + 3y - 5z = 17$
$3x - 10y + 6z = 23$

$$D_h = \left|\begin{array}{ccc|cc} 9 & 5 & 4 & 9 & 5 \\ 6 & 3 & -5 & 6 & 3 \\ 3 & -10 & 6 & 3 & -10 \end{array}\right.$$

$D_h = 9 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot (-10) - 4 \cdot 3 \cdot 3 - 9 \cdot (-5) \cdot (-10) - 5 \cdot 6 \cdot 6 = -819$

$$D_x = \left|\begin{array}{ccc|cc} 13 & 5 & 4 & 13 & 5 \\ 17 & 3 & -5 & 17 & 3 \\ 23 & -10 & 6 & 23 & -10 \end{array}\right.$$

$D_x = 13 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 23 + 4 \cdot 17 \cdot (-10) - 4 \cdot 3 \cdot 23 - 13 \cdot (-5) \cdot (-10) - 5 \cdot 17 \cdot 6 = -2,46 \cdot 10^3$

$$D_y = \left|\begin{array}{ccc|cc} 9 & 13 & 4 & 9 & 13 \\ 6 & 17 & -5 & 6 & 17 \\ 3 & 23 & 6 & 3 & 23 \end{array}\right.$$

$D_y = 9 \cdot 17 \cdot 6 + 13 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot 23 - 4 \cdot 17 \cdot 3 - 9 \cdot (-5) \cdot 23 - 13 \cdot 6 \cdot 6 = 1,64 \cdot 10^3$

$$D_z = \left|\begin{array}{ccc|cc} 9 & 5 & 13 & 9 & 5 \\ 6 & 3 & 17 & 6 & 3 \\ 3 & -10 & 23 & 3 & -10 \end{array}\right.$$

$D_z = 9 \cdot 3 \cdot 23 + 5 \cdot 17 \cdot 3 + 13 \cdot 6 \cdot (-10) - 13 \cdot 3 \cdot 3 - 9 \cdot 17 \cdot (-10) - 5 \cdot 6 \cdot 23 = 819$
$x = \frac{-2,46 \cdot 10^3}{-819}$
$x = 3$

$y = \frac{1{,}64 \cdot 10^3}{-819}$
$y = -2$
$z = \frac{819}{-819}$
$z = -1$
$L = \{3/-2/-1\}$

Aufgabe (3)

$4x + -3y + 2z = 10$
$5x + 6y - 7z = 4$
$10x + 2y - 3z = 7$

$$D_h = \left|\begin{array}{ccc|cc} 4 & -3 & 2 & 4 & -3 \\ 5 & 6 & -7 & 5 & 6 \\ 10 & 2 & -3 & 10 & 2 \end{array}\right.$$

$D_h = 4 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot 2 - 2 \cdot 6 \cdot 10 - 4 \cdot (-7) \cdot 2 - (-3) \cdot 5 \cdot (-3) = 49$

$$D_x = \left|\begin{array}{ccc|cc} 10 & -3 & 2 & 10 & -3 \\ 4 & 6 & -7 & 4 & 6 \\ 7 & 2 & -3 & 7 & 2 \end{array}\right.$$

$D_x = 10 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 7 + 2 \cdot 4 \cdot 2 - 2 \cdot 6 \cdot 7 - 10 \cdot (-7) \cdot 2 - (-3) \cdot 4 \cdot (-3) = 3$

$$D_y = \left|\begin{array}{ccc|cc} 4 & 10 & 2 & 4 & 10 \\ 5 & 4 & -7 & 5 & 4 \\ 10 & 7 & -3 & 10 & 7 \end{array}\right.$$

$D_y = 4 \cdot 4 \cdot (-3) + 10 \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot 7 - 2 \cdot 4 \cdot 10 - 4 \cdot (-7) \cdot 7 - 10 \cdot 5 \cdot (-3) = -412$

$$D_z = \left|\begin{array}{ccc|cc} 4 & -3 & 10 & 4 & -3 \\ 5 & 6 & 4 & 5 & 6 \\ 10 & 2 & 7 & 10 & 2 \end{array}\right.$$

$D_z = 4 \cdot 6 \cdot 7 + (-3) \cdot 4 \cdot 10 + 10 \cdot 5 \cdot 2 - 10 \cdot 6 \cdot 10 - 4 \cdot 4 \cdot 2 - (-3) \cdot 5 \cdot 7 = -379$
$x = \frac{3}{49}$
$x = \frac{3}{49}$
$y = \frac{-412}{49}$
$y = -8\frac{20}{49}$
$z = \frac{-379}{49}$
$z = -7\frac{36}{49}$
$L = \{\frac{3}{49}/-8\frac{20}{49}/-7\frac{36}{49}\}$

Aufgabe (4)

$2x + 3y + -4z = 16$
$4x + 9y - 1z = 58$
$1x + 6y + 2z = 34$

$$D_h = \left|\begin{array}{ccc|cc} 2 & 3 & -4 & 2 & 3 \\ 4 & 9 & -1 & 4 & 9 \\ 1 & 6 & 2 & 1 & 6 \end{array}\right.$$

$D_h = 2 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 6 - (-4) \cdot 9 \cdot 1 - 2 \cdot (-1) \cdot 6 - 3 \cdot 4 \cdot 2 = -39$

$$D_x = \left|\begin{array}{ccc|cc} 16 & 3 & -4 & 16 & 3 \\ 58 & 9 & -1 & 58 & 9 \\ 34 & 6 & 2 & 34 & 6 \end{array}\right.$$

$D_x = 16 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 34 + (-4) \cdot 58 \cdot 6 - (-4) \cdot 9 \cdot 34 - 16 \cdot (-1) \cdot 6 - 3 \cdot 58 \cdot 2 = -234$

$$D_y = \left|\begin{array}{ccc|cc} 2 & 16 & -4 & 2 & 16 \\ 4 & 58 & -1 & 4 & 58 \\ 1 & 34 & 2 & 1 & 34 \end{array}\right.$$

$D_y = 2 \cdot 58 \cdot 2 + 16 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 34 - (-4) \cdot 58 \cdot 1 - 2 \cdot (-1) \cdot 34 - 16 \cdot 4 \cdot 2 = -156$

$$D_z = \left|\begin{array}{ccc|cc} 2 & 3 & 16 & 2 & 3 \\ 4 & 9 & 58 & 4 & 9 \\ 1 & 6 & 34 & 1 & 6 \end{array}\right.$$
$D_z = 2 \cdot 9 \cdot 34 + 3 \cdot 58 \cdot 1 + 16 \cdot 4 \cdot 6 - 16 \cdot 9 \cdot 1 - 2 \cdot 58 \cdot 6 - 3 \cdot 4 \cdot 34 = -78$
$x = \frac{-234}{-39}$
$x = 6$
$y = \frac{-156}{-39}$
$y = 4$
$z = \frac{-78}{-39}$
$z = 2$
$L = \{6/4/2\}$

Aufgabe (5)

$1x + 2y + 3z = 4$
$2x + 3y + 2z = 6$
$0x + 2y + 6z = 0$
$$D_h = \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 2 & 3 & 2 & 2 & 3 \\ 0 & 2 & 6 & 0 & 2 \end{array}\right.$$
$D_h = 1 \cdot 3 \cdot 6 + 2 \cdot 2 \cdot 0 + 3 \cdot 2 \cdot 2 - 3 \cdot 3 \cdot 0 - 1 \cdot 2 \cdot 2 - 2 \cdot 2 \cdot 6 = 2$
$$D_x = \left|\begin{array}{ccc|cc} 4 & 2 & 3 & 4 & 2 \\ 6 & 3 & 2 & 6 & 3 \\ 0 & 2 & 6 & 0 & 2 \end{array}\right.$$
$D_x = 4 \cdot 3 \cdot 6 + 2 \cdot 2 \cdot 0 + 3 \cdot 6 \cdot 2 - 3 \cdot 3 \cdot 0 - 4 \cdot 2 \cdot 2 - 2 \cdot 6 \cdot 6 = 20$
$$D_y = \left|\begin{array}{ccc|cc} 1 & 4 & 3 & 1 & 4 \\ 2 & 6 & 2 & 2 & 6 \\ 0 & 0 & 6 & 0 & 0 \end{array}\right.$$
$D_y = 1 \cdot 6 \cdot 6 + 4 \cdot 2 \cdot 0 + 3 \cdot 2 \cdot 0 - 3 \cdot 6 \cdot 0 - 1 \cdot 2 \cdot 0 - 4 \cdot 2 \cdot 6 = -12$
$$D_z = \left|\begin{array}{ccc|cc} 1 & 2 & 4 & 1 & 2 \\ 2 & 3 & 6 & 2 & 3 \\ 0 & 2 & 0 & 0 & 2 \end{array}\right.$$
$D_z = 1 \cdot 3 \cdot 0 + 2 \cdot 6 \cdot 0 + 4 \cdot 2 \cdot 2 - 4 \cdot 3 \cdot 0 - 1 \cdot 6 \cdot 2 - 2 \cdot 2 \cdot 0 = 4$
$x = \frac{20}{2}$
$x = 10$
$y = \frac{-12}{2}$
$y = -6$
$z = \frac{4}{2}$
$z = 2$
$L = \{10/-6/2\}$

Aufgabe (6)

$-2x + -8y + 0z = 1$
$1x + 4y + 0z = -\frac{1}{2}$
$8x - 2y - 1z = 8$
$$D_h = \left|\begin{array}{ccc|cc} -2 & -8 & 0 & -2 & -8 \\ 1 & 4 & 0 & 1 & 4 \\ 8 & -2 & -1 & 8 & -2 \end{array}\right.$$
$D_h = (-2) \cdot 4 \cdot (-1) + (-8) \cdot 0 \cdot 8 + 0 \cdot 1 \cdot (-2) - 0 \cdot 4 \cdot 8 - (-2) \cdot 0 \cdot (-2) - (-8) \cdot 1 \cdot (-1) = 0$
$$D_x = \left|\begin{array}{ccc|cc} 1 & -8 & 0 & 1 & -8 \\ -\frac{1}{2} & 4 & 0 & -\frac{1}{2} & 4 \\ 8 & -2 & -1 & 8 & -2 \end{array}\right.$$

$D_x = 1 \cdot 4 \cdot (-1) + (-8) \cdot 0 \cdot 8 + 0 \cdot \left(-\frac{1}{2}\right) \cdot (-2) - 0 \cdot 4 \cdot 8 - 1 \cdot 0 \cdot (-2) - (-8) \cdot \left(-\frac{1}{2}\right) \cdot (-1) = 0$

$D_y = \left| \begin{array}{ccc|cc} -2 & 1 & 0 & -2 & 1 \\ 1 & -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ 8 & 8 & -1 & 8 & 8 \end{array} \right.$

$D_y = (-2) \cdot \left(-\frac{1}{2}\right) \cdot (-1) + 1 \cdot 0 \cdot 8 + 0 \cdot 1 \cdot 8 - 0 \cdot \left(-\frac{1}{2}\right) \cdot 8 - (-2) \cdot 0 \cdot 8 - 1 \cdot 1 \cdot (-1) = 0$

$D_z = \left| \begin{array}{ccc|cc} -2 & -8 & 1 & -2 & -8 \\ 1 & 4 & -\frac{1}{2} & 1 & 4 \\ 8 & -2 & 8 & 8 & -2 \end{array} \right.$

$D_z = (-2) \cdot 4 \cdot 8 + (-8) \cdot \left(-\frac{1}{2}\right) \cdot 8 + 1 \cdot 1 \cdot (-2) - 1 \cdot 4 \cdot 8 - (-2) \cdot \left(-\frac{1}{2}\right) \cdot (-2) - (-8) \cdot 1 \cdot 8 = 0$

$L = unendlich$

Aufgabe (7)

$-2x + 2y + 4z = 0$
$4x - \frac{1}{2}y + 2z = 5$
$4x - 2y - 1z = 8$

$D_h = \left| \begin{array}{ccc|cc} -2 & 2 & 4 & -2 & 2 \\ 4 & -\frac{1}{2} & 2 & 4 & -\frac{1}{2} \\ 4 & -2 & -1 & 4 & -2 \end{array} \right.$

$D_h = (-2) \cdot \left(-\frac{1}{2}\right) \cdot (-1) + 2 \cdot 2 \cdot 4 + 4 \cdot 4 \cdot (-2) - 4 \cdot \left(-\frac{1}{2}\right) \cdot 4 - (-2) \cdot 2 \cdot (-2) - 2 \cdot 4 \cdot (-1) = -9$

$D_x = \left| \begin{array}{ccc|cc} 0 & 2 & 4 & 0 & 2 \\ 5 & -\frac{1}{2} & 2 & 5 & -\frac{1}{2} \\ 8 & -2 & -1 & 8 & -2 \end{array} \right.$

$D_x = 0 \cdot \left(-\frac{1}{2}\right) \cdot (-1) + 2 \cdot 2 \cdot 8 + 4 \cdot 5 \cdot (-2) - 4 \cdot \left(-\frac{1}{2}\right) \cdot 8 - 0 \cdot 2 \cdot (-2) - 2 \cdot 5 \cdot (-1) = 18$

$D_y = \left| \begin{array}{ccc|cc} -2 & 0 & 4 & -2 & 0 \\ 4 & 5 & 2 & 4 & 5 \\ 4 & 8 & -1 & 4 & 8 \end{array} \right.$

$D_y = (-2) \cdot 5 \cdot (-1) + 0 \cdot 2 \cdot 4 + 4 \cdot 4 \cdot 8 - 4 \cdot 5 \cdot 4 - (-2) \cdot 2 \cdot 8 - 0 \cdot 4 \cdot (-1) = 90$

$D_z = \left| \begin{array}{ccc|cc} -2 & 2 & 0 & -2 & 2 \\ 4 & -\frac{1}{2} & 5 & 4 & -\frac{1}{2} \\ 4 & -2 & 8 & 4 & -2 \end{array} \right.$

$D_z = (-2) \cdot \left(-\frac{1}{2}\right) \cdot 8 + 2 \cdot 5 \cdot 4 + 0 \cdot 4 \cdot (-2) - 0 \cdot \left(-\frac{1}{2}\right) \cdot 4 - (-2) \cdot 5 \cdot (-2) - 2 \cdot 4 \cdot 8 = -36$
$x = \frac{18}{-9}$
$x = -2$
$y = \frac{90}{-9}$
$y = -10$
$z = \frac{-36}{-9}$
$z = 4$
$L = \{-2/-10/4\}$

Aufgabe (8)

$2x + 3y + -4z = 16$
$4x + 9y - 1z = 58$
$1x + 6y + 2z = 34$

$D_h = \left| \begin{array}{ccc|cc} 2 & 3 & -4 & 2 & 3 \\ 4 & 9 & -1 & 4 & 9 \\ 1 & 6 & 2 & 1 & 6 \end{array} \right.$

$D_h = 2 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 6 - (-4) \cdot 9 \cdot 1 - 2 \cdot (-1) \cdot 6 - 3 \cdot 4 \cdot 2 = -39$

$D_x = \left| \begin{array}{ccc|cc} 16 & 3 & -4 & 16 & 3 \\ 58 & 9 & -1 & 58 & 9 \\ 34 & 6 & 2 & 34 & 6 \end{array} \right.$

$D_x = 16 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 34 + (-4) \cdot 58 \cdot 6 - (-4) \cdot 9 \cdot 34 - 16 \cdot (-1) \cdot 6 - 3 \cdot 58 \cdot 2 = -234$

$$D_y = \left|\begin{array}{ccc|cc} 2 & 16 & -4 & 2 & 16 \\ 4 & 58 & -1 & 4 & 58 \\ 1 & 34 & 2 & 1 & 34 \end{array}\right.$$

$D_y = 2 \cdot 58 \cdot 2 + 16 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 34 - (-4) \cdot 58 \cdot 1 - 2 \cdot (-1) \cdot 34 - 16 \cdot 4 \cdot 2 = -156$

$$D_z = \left|\begin{array}{ccc|cc} 2 & 3 & 16 & 2 & 3 \\ 4 & 9 & 58 & 4 & 9 \\ 1 & 6 & 34 & 1 & 6 \end{array}\right.$$

$D_z = 2 \cdot 9 \cdot 34 + 3 \cdot 58 \cdot 1 + 16 \cdot 4 \cdot 6 - 16 \cdot 9 \cdot 1 - 2 \cdot 58 \cdot 6 - 3 \cdot 4 \cdot 34 = -78$

$x = \frac{-234}{-39}$

$x = 6$

$y = \frac{-156}{-39}$

$y = 4$

$z = \frac{-78}{-39}$

$z = 2$

$L = \{6/4/2\}$

Aufgabe (9)

$4x + -3y + 2z = 10$

$5x + 6y - 7z = 4$

$10x - 2y - 3z = 7$

$$D_h = \left|\begin{array}{ccc|cc} 4 & -3 & 2 & 4 & -3 \\ 5 & 6 & -7 & 5 & 6 \\ 10 & -2 & -3 & 10 & -2 \end{array}\right.$$

$D_h = 4 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot (-2) - 2 \cdot 6 \cdot 10 - 4 \cdot (-7) \cdot (-2) - (-3) \cdot 5 \cdot (-3) = -103$

$$D_x = \left|\begin{array}{ccc|cc} 10 & -3 & 2 & 10 & -3 \\ 4 & 6 & -7 & 4 & 6 \\ 7 & -2 & -3 & 7 & -2 \end{array}\right.$$

$D_x = 10 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 7 + 2 \cdot 4 \cdot (-2) - 2 \cdot 6 \cdot 7 - 10 \cdot (-7) \cdot (-2) - (-3) \cdot 4 \cdot (-3) = -309$

$$D_y = \left|\begin{array}{ccc|cc} 4 & 10 & 2 & 4 & 10 \\ 5 & 4 & -7 & 5 & 4 \\ 10 & 7 & -3 & 10 & 7 \end{array}\right.$$

$D_y = 4 \cdot 4 \cdot (-3) + 10 \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot 7 - 2 \cdot 4 \cdot 10 - 4 \cdot (-7) \cdot 7 - 10 \cdot 5 \cdot (-3) = -412$

$$D_z = \left|\begin{array}{ccc|cc} 4 & -3 & 10 & 4 & -3 \\ 5 & 6 & 4 & 5 & 6 \\ 10 & -2 & 7 & 10 & -2 \end{array}\right.$$

$D_z = 4 \cdot 6 \cdot 7 + (-3) \cdot 4 \cdot 10 + 10 \cdot 5 \cdot (-2) - 10 \cdot 6 \cdot 10 - 4 \cdot 4 \cdot (-2) - (-3) \cdot 5 \cdot 7 = -515$

$x = \frac{-309}{-103}$

$x = 3$

$y = \frac{-412}{-103}$

$y = 4$

$z = \frac{-515}{-103}$

$z = 5$

$L = \{3/4/5\}$

Aufgabe (10)

$9x + 5y + 4z = 13$

$6x + 3y - 5z = 17$

$3x - 10y + 6z = 23$

$$D_h = \begin{vmatrix} 9 & 5 & 4 \\ 6 & 3 & -5 \\ 3 & -10 & 6 \end{vmatrix} \begin{matrix} 9 & 5 \\ 6 & 3 \\ 3 & -10 \end{matrix}$$
$D_h = 9 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot (-10) - 4 \cdot 3 \cdot 3 - 9 \cdot (-5) \cdot (-10) - 5 \cdot 6 \cdot 6 = -819$
$$D_x = \begin{vmatrix} 13 & 5 & 4 \\ 17 & 3 & -5 \\ 23 & -10 & 6 \end{vmatrix} \begin{matrix} 13 & 5 \\ 17 & 3 \\ 23 & -10 \end{matrix}$$
$D_x = 13 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 23 + 4 \cdot 17 \cdot (-10) - 4 \cdot 3 \cdot 23 - 13 \cdot (-5) \cdot (-10) - 5 \cdot 17 \cdot 6 = -2,46 \cdot 10^3$
$$D_y = \begin{vmatrix} 9 & 13 & 4 \\ 6 & 17 & -5 \\ 3 & 23 & 6 \end{vmatrix} \begin{matrix} 9 & 13 \\ 6 & 17 \\ 3 & 23 \end{matrix}$$
$D_y = 9 \cdot 17 \cdot 6 + 13 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot 23 - 4 \cdot 17 \cdot 3 - 9 \cdot (-5) \cdot 23 - 13 \cdot 6 \cdot 6 = 1,64 \cdot 10^3$
$$D_z = \begin{vmatrix} 9 & 5 & 13 \\ 6 & 3 & 17 \\ 3 & -10 & 23 \end{vmatrix} \begin{matrix} 9 & 5 \\ 6 & 3 \\ 3 & -10 \end{matrix}$$
$D_z = 9 \cdot 3 \cdot 23 + 5 \cdot 17 \cdot 3 + 13 \cdot 6 \cdot (-10) - 13 \cdot 3 \cdot 3 - 9 \cdot 17 \cdot (-10) - 5 \cdot 6 \cdot 23 = 819$
$x = \frac{-2,46 \cdot 10^3}{-819}$
$x = 3$
$y = \frac{1,64 \cdot 10^3}{-819}$
$y = -2$
$z = \frac{819}{-819}$
$z = -1$
$L = \{3/-2/-1\}$

Aufgabe (11)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$
$$D_h = \begin{vmatrix} 11 & 13 & 4 \\ 12 & 14 & 5 \\ 9 & 3 & 3 \end{vmatrix} \begin{matrix} 11 & 13 \\ 12 & 14 \\ 9 & 3 \end{matrix}$$
$D_h = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3 - 4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$
$$D_x = \begin{vmatrix} 37 & 13 & 4 \\ 40 & 14 & 5 \\ 15 & 3 & 3 \end{vmatrix} \begin{matrix} 37 & 13 \\ 40 & 14 \\ 15 & 3 \end{matrix}$$
$D_x = 37 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 15 + 4 \cdot 40 \cdot 3 - 4 \cdot 14 \cdot 15 - 37 \cdot 5 \cdot 3 - 13 \cdot 40 \cdot 3 = 54$
$$D_y = \begin{vmatrix} 11 & 37 & 4 \\ 12 & 40 & 5 \\ 9 & 15 & 3 \end{vmatrix} \begin{matrix} 11 & 37 \\ 12 & 40 \\ 9 & 15 \end{matrix}$$
$D_y = 11 \cdot 40 \cdot 3 + 37 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 15 - 4 \cdot 40 \cdot 9 - 11 \cdot 5 \cdot 15 - 37 \cdot 12 \cdot 3 = 108$
$$D_z = \begin{vmatrix} 11 & 13 & 37 \\ 12 & 14 & 40 \\ 9 & 3 & 15 \end{vmatrix} \begin{matrix} 11 & 13 \\ 12 & 14 \\ 9 & 3 \end{matrix}$$
$D_z = 11 \cdot 14 \cdot 15 + 13 \cdot 40 \cdot 9 + 37 \cdot 12 \cdot 3 - 37 \cdot 14 \cdot 9 - 11 \cdot 40 \cdot 3 - 13 \cdot 12 \cdot 15 = 0$
$x = \frac{54}{54}$
$x = 1$
$y = \frac{108}{54}$
$y = 2$
$z = \frac{0}{54}$
$z = 0$
$L = \{1/2/0\}$

Aufgabe (12)

$2x + 3y + 4z = 175$
$4x + 6y + 5z = 287$
$3x + 2y + 8z = 257$

$$D_h = \left|\begin{array}{ccc|cc} 2 & 3 & 4 & 2 & 3 \\ 4 & 6 & 5 & 4 & 6 \\ 3 & 2 & 8 & 3 & 2 \end{array}\right.$$

$D_h = 2 \cdot 6 \cdot 8 + 3 \cdot 5 \cdot 3 + 4 \cdot 4 \cdot 2 - 4 \cdot 6 \cdot 3 - 2 \cdot 5 \cdot 2 - 3 \cdot 4 \cdot 8 = -15$

$$D_x = \left|\begin{array}{ccc|cc} 175 & 3 & 4 & 175 & 3 \\ 287 & 6 & 5 & 287 & 6 \\ 257 & 2 & 8 & 257 & 2 \end{array}\right.$$

$D_x = 175 \cdot 6 \cdot 8 + 3 \cdot 5 \cdot 257 + 4 \cdot 287 \cdot 2 - 4 \cdot 6 \cdot 257 - 175 \cdot 5 \cdot 2 - 3 \cdot 287 \cdot 8 = -255$

$$D_y = \left|\begin{array}{ccc|cc} 2 & 175 & 4 & 2 & 175 \\ 4 & 287 & 5 & 4 & 287 \\ 3 & 257 & 8 & 3 & 257 \end{array}\right.$$

$D_y = 2 \cdot 287 \cdot 8 + 175 \cdot 5 \cdot 3 + 4 \cdot 4 \cdot 257 - 4 \cdot 287 \cdot 3 - 2 \cdot 5 \cdot 257 - 175 \cdot 4 \cdot 8 = -285$

$$D_z = \left|\begin{array}{ccc|cc} 2 & 3 & 175 & 2 & 3 \\ 4 & 6 & 287 & 4 & 6 \\ 3 & 2 & 257 & 3 & 2 \end{array}\right.$$

$D_z = 2 \cdot 6 \cdot 257 + 3 \cdot 287 \cdot 3 + 175 \cdot 4 \cdot 2 - 175 \cdot 6 \cdot 3 - 2 \cdot 287 \cdot 2 - 3 \cdot 4 \cdot 257 = -315$
$x = \frac{-255}{-15}$
$x = 17$
$y = \frac{-285}{-15}$
$y = 19$
$z = \frac{-315}{-15}$
$z = 21$
$L = \{17/19/21\}$

Aufgabe (13)

$6x + 4y + 9z = 32$
$5x + 7y + 10z = 17$
$4x + 8y + 5z = 100$

$$D_h = \left|\begin{array}{ccc|cc} 6 & 4 & 9 & 6 & 4 \\ 5 & 7 & 10 & 5 & 7 \\ 4 & 8 & 5 & 4 & 8 \end{array}\right.$$

$D_h = 6 \cdot 7 \cdot 5 + 4 \cdot 10 \cdot 4 + 9 \cdot 5 \cdot 8 - 9 \cdot 7 \cdot 4 - 6 \cdot 10 \cdot 8 - 4 \cdot 5 \cdot 5 = -102$

$$D_x = \left|\begin{array}{ccc|cc} 32 & 4 & 9 & 32 & 4 \\ 17 & 7 & 10 & 17 & 7 \\ 100 & 8 & 5 & 100 & 8 \end{array}\right.$$

$D_x = 32 \cdot 7 \cdot 5 + 4 \cdot 10 \cdot 100 + 9 \cdot 17 \cdot 8 - 9 \cdot 7 \cdot 100 - 32 \cdot 10 \cdot 8 - 4 \cdot 17 \cdot 5 = -2,86 \cdot 10^3$

$$D_y = \left|\begin{array}{ccc|cc} 6 & 32 & 9 & 6 & 32 \\ 5 & 17 & 10 & 5 & 17 \\ 4 & 100 & 5 & 4 & 100 \end{array}\right.$$

$D_y = 6 \cdot 17 \cdot 5 + 32 \cdot 10 \cdot 4 + 9 \cdot 5 \cdot 100 - 9 \cdot 17 \cdot 4 - 6 \cdot 10 \cdot 100 - 32 \cdot 5 \cdot 5 = -1,12 \cdot 10^3$

$$D_z = \left|\begin{array}{ccc|cc} 6 & 4 & 32 & 6 & 4 \\ 5 & 7 & 17 & 5 & 7 \\ 4 & 8 & 100 & 4 & 8 \end{array}\right.$$

$D_z = 6 \cdot 7 \cdot 100 + 4 \cdot 17 \cdot 4 + 32 \cdot 5 \cdot 8 - 32 \cdot 7 \cdot 4 - 6 \cdot 17 \cdot 8 - 4 \cdot 5 \cdot 100 = 2,04 \cdot 10^3$
$x = \frac{-2,86 \cdot 10^3}{-102}$
$x = 28$
$y = \frac{-1,12 \cdot 10^3}{-102}$
$y = 11$
$z = \frac{2,04 \cdot 10^3}{-102}$
$z = -20$

$L = \{28/11/-20\}$

Aufgabe (14)

$1x + 1y + 0z = 1$
$1x + 0y + 1z = 6$
$0x + 1y - 1z = 5$

$$D_h = \left|\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{array}\right.$$

$D_h = 1 \cdot 0 \cdot (-1) + 1 \cdot 1 \cdot 0 + 0 \cdot 1 \cdot 1 - 0 \cdot 0 \cdot 0 - 1 \cdot 1 \cdot 1 - 1 \cdot 1 \cdot (-1) = 0$

$$D_x = \left|\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 6 & 0 & 1 & 6 & 0 \\ 5 & 1 & -1 & 5 & 1 \end{array}\right.$$

$D_x = 1 \cdot 0 \cdot (-1) + 1 \cdot 1 \cdot 5 + 0 \cdot 6 \cdot 1 - 0 \cdot 0 \cdot 5 - 1 \cdot 1 \cdot 1 - 1 \cdot 6 \cdot (-1) = 10$

$$D_y = \left|\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 1 & 6 & 1 & 1 & 6 \\ 0 & 5 & -1 & 0 & 5 \end{array}\right.$$

$D_y = 1 \cdot 6 \cdot (-1) + 1 \cdot 1 \cdot 0 + 0 \cdot 1 \cdot 5 - 0 \cdot 6 \cdot 0 - 1 \cdot 1 \cdot 5 - 1 \cdot 1 \cdot (-1) = -10$

$$D_z = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 6 & 1 & 0 \\ 0 & 1 & 5 & 0 & 1 \end{array}\right.$$

$D_z = 1 \cdot 0 \cdot 5 + 1 \cdot 6 \cdot 0 + 1 \cdot 1 \cdot 1 - 1 \cdot 0 \cdot 0 - 1 \cdot 6 \cdot 1 - 1 \cdot 1 \cdot 5 = -10$

$L = \{\}$

Aufgabe (15)

$1x + -2y + 3z = 9$
$3x + 8y + 9z = 5$
$2x + 3y + 6z = 7$

$$D_h = \left|\begin{array}{ccc|cc} 1 & -2 & 3 & 1 & -2 \\ 3 & 8 & 9 & 3 & 8 \\ 2 & 3 & 6 & 2 & 3 \end{array}\right.$$

$D_h = 1 \cdot 8 \cdot 6 + (-2) \cdot 9 \cdot 2 + 3 \cdot 3 \cdot 3 - 3 \cdot 8 \cdot 2 - 1 \cdot 9 \cdot 3 - (-2) \cdot 3 \cdot 6 = 0$

$$D_x = \left|\begin{array}{ccc|cc} 9 & -2 & 3 & 9 & -2 \\ 5 & 8 & 9 & 5 & 8 \\ 7 & 3 & 6 & 7 & 3 \end{array}\right.$$

$D_x = 9 \cdot 8 \cdot 6 + (-2) \cdot 9 \cdot 7 + 3 \cdot 5 \cdot 3 - 3 \cdot 8 \cdot 7 - 9 \cdot 9 \cdot 3 - (-2) \cdot 5 \cdot 6 = 0$

$$D_y = \left|\begin{array}{ccc|cc} 1 & 9 & 3 & 1 & 9 \\ 3 & 5 & 9 & 3 & 5 \\ 2 & 7 & 6 & 2 & 7 \end{array}\right.$$

$D_y = 1 \cdot 5 \cdot 6 + 9 \cdot 9 \cdot 2 + 3 \cdot 3 \cdot 7 - 3 \cdot 5 \cdot 2 - 1 \cdot 9 \cdot 7 - 9 \cdot 3 \cdot 6 = 0$

$$D_z = \left|\begin{array}{ccc|cc} 1 & -2 & 9 & 1 & -2 \\ 3 & 8 & 5 & 3 & 8 \\ 2 & 3 & 7 & 2 & 3 \end{array}\right.$$

$D_z = 1 \cdot 8 \cdot 7 + (-2) \cdot 5 \cdot 2 + 9 \cdot 3 \cdot 3 - 9 \cdot 8 \cdot 2 - 1 \cdot 5 \cdot 3 - (-2) \cdot 3 \cdot 7 = 0$

$L = unendlich$

Aufgabe (16)

$6x + 4y + 5z = 8$
$4x + 2y + 3z = 7$
$5x + 3y + 4z = 9$

$$D_h = \left|\begin{array}{ccc|cc} 6 & 4 & 5 & 6 & 4 \\ 4 & 2 & 3 & 4 & 2 \\ 5 & 3 & 4 & 5 & 3 \end{array}\right.$$

$D_h = 6 \cdot 2 \cdot 4 + 4 \cdot 3 \cdot 5 + 5 \cdot 4 \cdot 3 - 5 \cdot 2 \cdot 5 - 6 \cdot 3 \cdot 3 - 4 \cdot 4 \cdot 4 = 0$

$$D_x = \left|\begin{array}{ccc|cc} 8 & 4 & 5 & 8 & 4 \\ 7 & 2 & 3 & 7 & 2 \\ 9 & 3 & 4 & 9 & 3 \end{array}\right.$$

$D_x = 8 \cdot 2 \cdot 4 + 4 \cdot 3 \cdot 9 + 5 \cdot 7 \cdot 3 - 5 \cdot 2 \cdot 9 - 8 \cdot 3 \cdot 3 - 4 \cdot 7 \cdot 4 = 3$

$$D_y = \left|\begin{array}{ccc|cc} 6 & 8 & 5 & 6 & 8 \\ 4 & 7 & 3 & 4 & 7 \\ 5 & 9 & 4 & 5 & 9 \end{array}\right.$$

$D_y = 6 \cdot 7 \cdot 4 + 8 \cdot 3 \cdot 5 + 5 \cdot 4 \cdot 9 - 5 \cdot 7 \cdot 5 - 6 \cdot 3 \cdot 9 - 8 \cdot 4 \cdot 4 = 3$

$$D_z = \left|\begin{array}{ccc|cc} 6 & 4 & 8 & 6 & 4 \\ 4 & 2 & 7 & 4 & 2 \\ 5 & 3 & 9 & 5 & 3 \end{array}\right.$$

$D_z = 6 \cdot 2 \cdot 9 + 4 \cdot 7 \cdot 5 + 8 \cdot 4 \cdot 3 - 8 \cdot 2 \cdot 5 - 6 \cdot 7 \cdot 3 - 4 \cdot 4 \cdot 9 = -6$

$L = \{\}$

Aufgabe (17)

$1x + 3y + -2z = 3$
$3x + 2y + 1z = 2$
$0x + 1y + 3z = 5$

$$D_h = \left|\begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 3 & 2 & 1 & 3 & 2 \\ 0 & 1 & 3 & 0 & 1 \end{array}\right.$$

$D_h = 1 \cdot 2 \cdot 3 + 3 \cdot 1 \cdot 0 + (-2) \cdot 3 \cdot 1 - (-2) \cdot 2 \cdot 0 - 1 \cdot 1 \cdot 1 - 3 \cdot 3 \cdot 3 = -28$

$$D_x = \left|\begin{array}{ccc|cc} 3 & 3 & -2 & 3 & 3 \\ 2 & 2 & 1 & 2 & 2 \\ 5 & 1 & 3 & 5 & 1 \end{array}\right.$$

$D_x = 3 \cdot 2 \cdot 3 + 3 \cdot 1 \cdot 5 + (-2) \cdot 2 \cdot 1 - (-2) \cdot 2 \cdot 5 - 3 \cdot 1 \cdot 1 - 3 \cdot 2 \cdot 3 = 28$

$$D_y = \left|\begin{array}{ccc|cc} 1 & 3 & -2 & 1 & 3 \\ 3 & 2 & 1 & 3 & 2 \\ 0 & 5 & 3 & 0 & 5 \end{array}\right.$$

$D_y = 1 \cdot 2 \cdot 3 + 3 \cdot 1 \cdot 0 + (-2) \cdot 3 \cdot 5 - (-2) \cdot 2 \cdot 0 - 1 \cdot 1 \cdot 5 - 3 \cdot 3 \cdot 3 = -56$

$$D_z = \left|\begin{array}{ccc|cc} 1 & 3 & 3 & 1 & 3 \\ 3 & 2 & 2 & 3 & 2 \\ 0 & 1 & 5 & 0 & 1 \end{array}\right.$$

$D_z = 1 \cdot 2 \cdot 5 + 3 \cdot 2 \cdot 0 + 3 \cdot 3 \cdot 1 - 3 \cdot 2 \cdot 0 - 1 \cdot 2 \cdot 1 - 3 \cdot 3 \cdot 5 = -28$
$x = \frac{28}{-28}$
$x = -1$
$y = \frac{-56}{-28}$
$y = 2$
$z = \frac{-28}{-28}$
$z = 1$
$L = \{-1/2/1\}$

8 Gaußsches Eliminationsverfahren

$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$
$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$
$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$

Koeffizientenmatrix erstellen

x	y	z	
$a1$	$b1$	$c1$	$d1$
$a2$	$b2$	$c2$	$d2$
$a3$	$b3$	$c3$	$d3$

x	y	z	
$Zeile1Spalte1$	$z1s2$	$z1s3$	$z1s4$
$z2s1$	$z2s2$	$z2s3$	$z2s4$
$z3s1$	$z3s2$	$z3s3$	$z3s4$

Die Lösungsmenge ändert sich nicht durch:

- Multiplizieren oder Dividieren der Zeilen mit einer Zahl
- Addieren oder Subtrahieren der Zeilen
- Vertauschen der Zeilen

Umformen in die Stufenform

- Eindeutige Lösung

x	y	z	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
0	$z2s2$	$z2s3$	$z2s4$
0	0	$z3s3$	$z3s4$

  Rückwärtseinsetzen

  $z = \frac{z3s3}{z3s4}$

  z in die 2. Zeile einsetzen → y

  z und y in die 1. Zeile einsetzen → x

- Keine Lösung

x	y	z	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
0	$z2s2$	$z2s3$	$z2s4$
0	0	0	$z3s4$

- Unendlich viele Lösungen

x	y	z	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
0	$z2s2$	$z2s3$	$z2s4$
0	0	0	0

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

Zeile2 = Zeile2 · 11 − Zeile1 · 12
$z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0$
$z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2$
$z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7$
$z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4$

x	y	z	
11	13	4	37
0	−2	7	−4
9	3	3	15

Zeile3 = Zeile3 · 11 − Zeile1 · 9
$z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0$
$z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84$
$z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3$
$z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168$

x	y	z	
11	13	4	37
0	−2	7	−4
0	−84	−3	−168

Zeile3 = Zeile3 · (−2) − Zeile2 · (−84)
$z3s2 = (-84) \cdot -2 - (-2) \cdot (-84) = 0$
$z3s3 = (-3) \cdot -2 - 7 \cdot (-84) = 594$
$z3s4 = (-168) \cdot -2 - (-4) \cdot (-84) = 0$

x	y	z	
11	13	4	37
0	−2	7	−4
0	0	594	0

$z = \frac{0}{594} = 0$
$y \cdot (-2) + 7 \cdot 0 = (-4)$
$y = 2$
$x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$
$x = 1$
$L = \{1/2/0\}$

8.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:

$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$
$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$
$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$

Gesucht:
x,y,z

(1)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(2)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(3)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x + 2y + -3z &= 7 \end{aligned}$$

(4)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(5)
$$\begin{aligned} 1x + 2 + 3z &= 4 \\ 2x + 3y + 2z &= 6 \\ 0x + 2y + 6z &= 0 \end{aligned}$$

(6)
$$\begin{aligned} -2x - 8 + 0z &= 1 \\ 1x + 4y + 0z &= -\tfrac{1}{2} \\ 8x - 2y + -1z &= 8 \end{aligned}$$

(7)
$$\begin{aligned} -2x + 2 + 4z &= 0 \\ 4x - \tfrac{1}{2}y + 2z &= 5 \\ 4x - 2y + -1z &= 8 \end{aligned}$$

(8)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(9)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x - 2y + -3z &= 7 \end{aligned}$$

(10)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(11)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(12)
$$\begin{aligned} 2x + 3 + 4z &= 175 \\ 4x + 6y + 5z &= 287 \\ 3x + 2y + 8z &= 257 \end{aligned}$$

(13)
$$\begin{aligned} 6x + 4 + 9z &= 32 \\ 5x + 7y + 10z &= 17 \\ 4x + 8y + 5z &= 100 \end{aligned}$$

(14)
$$\begin{aligned} 1x + 1 + 0z &= 1 \\ 1x + 0y + 1z &= 6 \\ 0x + 1y + -1z &= 5 \end{aligned}$$

(15)
$$\begin{aligned} 1x - 2 + 3z &= 9 \\ 3x + 8y + 9z &= 5 \\ 2x + 3y + 6z &= 7 \end{aligned}$$

(16)
$$\begin{aligned} 6x + 4 + 5z &= 8 \\ 4x + 2y + 3z &= 7 \\ 5x + 3y + 4z &= 9 \end{aligned}$$

(17)
$$\begin{aligned} 1x + 3 + -2z &= 3 \\ 3x + 2y + 1z &= 2 \\ 0x + 1y + 3z &= 5 \end{aligned}$$

(18)
$$\begin{aligned} 4x + 6 + 8z &= 0 \\ 5x + 6y + 67z &= 8 \\ 8x + 87y + 6z &= 6 \end{aligned}$$

8.2 Lösungen

Aufgabe (1)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

Zeile2 = Zeile2 · 11-Zeile1 · 12
$z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0$
$z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2$
$z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7$
$z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4$

x	y	z	
11	13	4	37
0	-2	7	-4
9	3	3	15

Zeile3 = Zeile3 · 11-Zeile1 · 9
$z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0$
$z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84$
$z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3$
$z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168$

x	y	z	
11	13	4	37
0	-2	7	-4
0	-84	-3	-168

Zeile3 = Zeile3 · (−2)-Zeile2 · (−84)
$z3s2 = (-84) \cdot -2 - (-2) \cdot (-84) = 0$
$z3s3 = (-3) \cdot -2 - 7 \cdot (-84) = 594$
$z3s4 = (-168) \cdot -2 - (-4) \cdot (-84) = 0$

x	y	z	
11	13	4	37
0	-2	7	-4
0	0	594	0

$z = \frac{0}{594} = 0$
$y \cdot (-2) + 7 \cdot 0 = (-4)$
$y = 2$
$x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$
$x = 1$
$L = \{1/2/0\}$

Aufgabe (2)

$9x + 5y + 4z = 13$
$6x + 3y - 5z = 17$
$3x - 10y + 6z = 23$

x	y	z	
9	5	4	13
6	3	-5	17
3	-10	6	23

Zeile2 = Zeile2 · 9-Zeile1 · 6
$z2s1 = 6 \cdot 9 - 9 \cdot 6 = 0$
$z2s2 = 3 \cdot 9 - 5 \cdot 6 = -3$
$z2s3 = (-5) \cdot 9 - 4 \cdot 6 = -69$
$z2s4 = 17 \cdot 9 - 13 \cdot 6 = 75$

x	y	z	
9	5	4	13
0	-3	-69	75
3	-10	6	23

Zeile3 = Zeile3 · 9-Zeile1 · 3
$z3s1 = 3 \cdot 9 - 9 \cdot 3 = 0$
$z3s2 = (-10) \cdot 9 - 5 \cdot 3 = -105$
$z3s3 = 6 \cdot 9 - 4 \cdot 3 = 42$
$z3s4 = 23 \cdot 9 - 13 \cdot 3 = 168$

x	y	z	
9	5	4	13
0	-3	-69	75
0	-105	42	168

Zeile3 = Zeile3 · (−3)-Zeile2 · (−105)
$z3s2 = (-105) \cdot -3 - (-3) \cdot (-105) = 0$
$z3s3 = 42 \cdot -3 - (-69) \cdot (-105) = -7,37 \cdot 10^3$
$z3s4 = 168 \cdot -3 - 75 \cdot (-105) = 7,37 \cdot 10^3$

x	y	z	
9	5	4	13
0	-3	-69	75
0	0	$-7,37 \cdot 10^3$	$7,37 \cdot 10^3$

$z = \frac{7,37 \cdot 10^3}{-7,37 \cdot 10^3} = -1$
$y \cdot (-3) + (-69) \cdot -1 = 75$
$y = -2$
$x \cdot 9 + 5 \cdot -2 + 4 \cdot (-1) = 13$
$x = 3$
$L = \{3/-2/-1\}$

Aufgabe (3)

$4x - 3y + 2z = 10$
$5x + 6y - 7z = 4$
$10x + 2y - 3z = 7$

x	y	z	
4	−3	2	10
5	6	−7	4
10	2	−3	7

Zeile2 = Zeile2 · 4-Zeile1 · 5
$z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0$
$z2s2 = 6 \cdot 4 - (-3) \cdot 5 = 39$
$z2s3 = (-7) \cdot 4 - 2 \cdot 5 = -38$
$z2s4 = 4 \cdot 4 - 10 \cdot 5 = -34$

x	y	z	
4	−3	2	10
0	39	−38	−34
10	2	−3	7

Zeile3 = Zeile3 · 4-Zeile1 · 10
$z3s1 = 10 \cdot 4 - 4 \cdot 10 = 0$
$z3s2 = 2 \cdot 4 - (-3) \cdot 10 = 38$
$z3s3 = (-3) \cdot 4 - 2 \cdot 10 = -32$
$z3s4 = 7 \cdot 4 - 10 \cdot 10 = -72$

x	y	z	
4	−3	2	10
0	39	−38	−34
0	38	−32	−72

Zeile3 = Zeile3 · 39-Zeile2 · 38
$z3s2 = 38 \cdot 39 - 39 \cdot 38 = 0$
$z3s3 = (-32) \cdot 39 - (-38) \cdot 38 = 196$
$z3s4 = (-72) \cdot 39 - (-34) \cdot 38 = -1,52 \cdot 10^3$

x	y	z	
4	−3	2	10
0	39	−38	−34
0	0	196	$-1,52 \cdot 10^3$

$z = \frac{-1,52 \cdot 10^3}{196} = -7\frac{36}{49}$
$y \cdot 39 + (-38) \cdot -7\frac{36}{49} = (-34)$
$y = -8\frac{20}{49}$
$x \cdot 4 + (-3) \cdot -8\frac{20}{49} + 2 \cdot (-7\frac{36}{49}) = 10$
$x = \frac{3}{49}$
$L = \{\frac{3}{49} / -8\frac{20}{49} / -7\frac{36}{49}\}$

Aufgabe (4)

$2x + 3y - 4z = 16$
$4x + 9y - z = 58$
$x + 6y + 2z = 34$

x	y	z	
2	3	−4	16
4	9	−1	58
1	6	2	34

Zeile2 = Zeile2 · 2-Zeile1 · 4
$z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0$
$z2s2 = 9 \cdot 2 - 3 \cdot 4 = 6$
$z2s3 = (-1) \cdot 2 - (-4) \cdot 4 = 14$
$z2s4 = 58 \cdot 2 - 16 \cdot 4 = 52$

x	y	z	
2	3	−4	16
0	6	14	52
1	6	2	34

Zeile3 = Zeile3 · 2-Zeile1 · 1
$z3s1 = 1 \cdot 2 - 2 \cdot 1 = 0$
$z3s2 = 6 \cdot 2 - 3 \cdot 1 = 9$
$z3s3 = 2 \cdot 2 - (-4) \cdot 1 = 8$
$z3s4 = 34 \cdot 2 - 16 \cdot 1 = 52$

x	y	z	
2	3	−4	16
0	6	14	52
0	9	8	52

Zeile3 = Zeile3 · 6-Zeile2 · 9
$z3s2 = 9 \cdot 6 - 6 \cdot 9 = 0$
$z3s3 = 8 \cdot 6 - 14 \cdot 9 = -78$
$z3s4 = 52 \cdot 6 - 52 \cdot 9 = -156$

x	y	z	
2	3	−4	16
0	6	14	52
0	0	−78	−156

$z = \frac{-156}{-78} = 2$
$y \cdot 6 + 14 \cdot 2 = 52$
$y = 4$
$x \cdot 2 + 3 \cdot 4 + (-4) \cdot 2 = 16$
$x = 6$
$L = \{6/4/2\}$

Aufgabe (5)

$x + 2y + 3z = 4$
$2x + 3y + 2z = 6$
$2y + 6z = 0$

x	y	z	
1	2	3	4
2	3	2	6
0	2	6	0

Zeile2 = Zeile2 · 1-Zeile1 · 2
$z2s1 = 2 \cdot 1 - 1 \cdot 2 = 0$
$z2s2 = 3 \cdot 1 - 2 \cdot 2 = -1$
$z2s3 = 2 \cdot 1 - 3 \cdot 2 = -4$
$z2s4 = 6 \cdot 1 - 4 \cdot 2 = -2$

x	y	z	
1	2	3	4
0	-1	-4	-2
0	2	6	0

Zeile3 = Zeile3 · (-1)-Zeile2 · 2
$z3s2 = 2 \cdot -1 - (-1) \cdot 2 = 0$
$z3s3 = 6 \cdot -1 - (-4) \cdot 2 = 2$
$z3s4 = 0 \cdot -1 - (-2) \cdot 2 = 4$

x	y	z	
1	2	3	4
0	-1	-4	-2
0	0	2	4

$z = \frac{4}{2} = 2$
$y \cdot (-1) + (-4) \cdot 2 = (-2)$
$y = -6$
$x \cdot 1 + 2 \cdot -6 + 3 \cdot 2 = 4$
$x = 10$
$L = \{10/-6/2\}$

Aufgabe (6)

$-2x - 8y = 1$
$x + 4y = -\frac{1}{2}$
$8x - 2y - z = 8$

x	y	z	
-2	-8	0	1
1	4	0	$-\frac{1}{2}$
8	-2	-1	8

Zeile2 = Zeile2 · (-2)-Zeile1 · 1
$z2s1 = 1 \cdot -2 - (-2) \cdot 1 = 0$
$z2s2 = 4 \cdot -2 - (-8) \cdot 1 = 0$
$z2s3 = 0 \cdot -2 - 0 \cdot 1 = 0$
$z2s4 = (-\frac{1}{2}) \cdot -2 - 1 \cdot 1 = 0$

x	y	z	
-2	-8	0	1
0	0	0	0
8	-2	-1	8

Zeile3 = Zeile3 · (-2)-Zeile1 · 8
$z3s1 = 8 \cdot -2 - (-2) \cdot 8 = 0$
$z3s2 = (-2) \cdot -2 - (-8) \cdot 8 = 68$
$z3s3 = (-1) \cdot -2 - 0 \cdot 8 = 2$
$z3s4 = 8 \cdot -2 - 1 \cdot 8 = -24$

x	y	z	
-2	-8	0	1
0	0	0	0
0	68	2	-24

Zeilen vertauschen

x	y	z	
-2	-8	0	1
0	68	2	-24
0	0	0	0

$L = unendlich$

Aufgabe (7)

$-2x + 2y + 4z = 0$
$4x - \frac{1}{2}y + 2z = 5$
$4x - 2y - z = 8$

x	y	z	
-2	2	4	0
4	$-\frac{1}{2}$	2	5
4	-2	-1	8

Zeile2 = Zeile2 · (−2)-Zeile1 · 4
$z2s1 = 4 \cdot -2 - (-2) \cdot 4 = 0$
$z2s2 = (-\frac{1}{2}) \cdot -2 - 2 \cdot 4 = -7$
$z2s3 = 2 \cdot -2 - 4 \cdot 4 = -20$
$z2s4 = 5 \cdot -2 - 0 \cdot 4 = -10$

x	y	z	
−2	2	4	0
0	−7	−20	−10
4	−2	−1	8

Zeile3 = Zeile3 · (−2)-Zeile1 · 4
$z3s1 = 4 \cdot -2 - (-2) \cdot 4 = 0$
$z3s2 = (-2) \cdot -2 - 2 \cdot 4 = -4$
$z3s3 = (-1) \cdot -2 - 4 \cdot 4 = -14$
$z3s4 = 8 \cdot -2 - 0 \cdot 4 = -16$

x	y	z	
−2	2	4	0
0	−7	−20	−10
0	−4	−14	−16

Zeile3 = Zeile3 · (−7)-Zeile2 · (−4)
$z3s2 = (-4) \cdot -7 - (-7) \cdot (-4) = 0$
$z3s3 = (-14) \cdot -7 - (-20) \cdot (-4) = 18$
$z3s4 = (-16) \cdot -7 - (-10) \cdot (-4) = 72$

x	y	z	
−2	2	4	0
0	−7	−20	−10
0	0	18	72

$z = \frac{72}{18} = 4$
$y \cdot (-7) + (-20) \cdot 4 = (-10)$
$y = -10$
$x \cdot (-2) + 2 \cdot -10 + 4 \cdot 4 = 0$
$x = -2$
$L = \{-2/-10/4\}$

Aufgabe (8)

$2x + 3y - 4z = 16$
$4x + 9y - z = 58$
$x + 6y + 2z = 34$

x	y	z	
2	3	−4	16
4	9	−1	58
1	6	2	34

Zeile2 = Zeile2 · 2-Zeile1 · 4
$z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0$
$z2s2 = 9 \cdot 2 - 3 \cdot 4 = 6$
$z2s3 = (-1) \cdot 2 - (-4) \cdot 4 = 14$
$z2s4 = 58 \cdot 2 - 16 \cdot 4 = 52$

x	y	z	
2	3	−4	16
0	6	14	52
1	6	2	34

Zeile3 = Zeile3 · 2-Zeile1 · 1
$z3s1 = 1 \cdot 2 - 2 \cdot 1 = 0$
$z3s2 = 6 \cdot 2 - 3 \cdot 1 = 9$
$z3s3 = 2 \cdot 2 - (-4) \cdot 1 = 8$
$z3s4 = 34 \cdot 2 - 16 \cdot 1 = 52$

x	y	z	
2	3	−4	16
0	6	14	52
0	9	8	52

Zeile3 = Zeile3 · 6-Zeile2 · 9
$z3s2 = 9 \cdot 6 - 6 \cdot 9 = 0$
$z3s3 = 8 \cdot 6 - 14 \cdot 9 = -78$
$z3s4 = 52 \cdot 6 - 52 \cdot 9 = -156$

x	y	z	
2	3	−4	16
0	6	14	52
0	0	−78	−156

$z = \frac{-156}{-78} = 2$
$y \cdot 6 + 14 \cdot 2 = 52$
$y = 4$
$x \cdot 2 + 3 \cdot 4 + (-4) \cdot 2 = 16$
$x = 6$
$L = \{6/4/2\}$

Aufgabe (9)

$4x - 3y + 2z = 10$
$5x + 6y - 7z = 4$
$10x - 2y - 3z = 7$

x	y	z	
4	−3	2	10
5	6	−7	4
10	−2	−3	7

Zeile2 = Zeile2 · 4-Zeile1 · 5
$z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0$
$z2s2 = 6 \cdot 4 - (-3) \cdot 5 = 39$
$z2s3 = (-7) \cdot 4 - 2 \cdot 5 = -38$
$z2s4 = 4 \cdot 4 - 10 \cdot 5 = -34$

x	y	z	
4	−3	2	10
0	39	−38	−34
10	−2	−3	7

Zeile3 = Zeile3 · 4-Zeile1 · 10
$z3s1 = 10 \cdot 4 - 4 \cdot 10 = 0$
$z3s2 = (-2) \cdot 4 - (-3) \cdot 10 = 22$
$z3s3 = (-3) \cdot 4 - 2 \cdot 10 = -32$
$z3s4 = 7 \cdot 4 - 10 \cdot 10 = -72$

x	y	z	
4	−3	2	10
0	39	−38	−34
0	22	−32	−72

Zeile3 = Zeile3 · 39-Zeile2 · 22
$z3s2 = 22 \cdot 39 - 39 \cdot 22 = 0$
$z3s3 = (-32) \cdot 39 - (-38) \cdot 22 = -412$
$z3s4 = (-72) \cdot 39 - (-34) \cdot 22 = -2,06 \cdot 10^3$

x	y	z	
4	−3	2	10
0	39	−38	−34
0	0	−412	$-2,06 \cdot 10^3$

$z = \frac{-2,06 \cdot 10^3}{-412} = 5$
$y \cdot 39 + (-38) \cdot 5 = (-34)$
$y = 4$
$x \cdot 4 + (-3) \cdot 4 + 2 \cdot 5 = 10$
$x = 3$
$L = \{3/4/5\}$

Aufgabe (10)

$9x + 5y + 4z = 13$
$6x + 3y - 5z = 17$
$3x - 10y + 6z = 23$

x	y	z	
9	5	4	13
6	3	−5	17
3	−10	6	23

Zeile2 = Zeile2 · 9-Zeile1 · 6
$z2s1 = 6 \cdot 9 - 9 \cdot 6 = 0$
$z2s2 = 3 \cdot 9 - 5 \cdot 6 = -3$
$z2s3 = (-5) \cdot 9 - 4 \cdot 6 = -69$
$z2s4 = 17 \cdot 9 - 13 \cdot 6 = 75$

x	y	z	
9	5	4	13
0	−3	−69	75
3	−10	6	23

Zeile3 = Zeile3 · 9-Zeile1 · 3
$z3s1 = 3 \cdot 9 - 9 \cdot 3 = 0$
$z3s2 = (-10) \cdot 9 - 5 \cdot 3 = -105$
$z3s3 = 6 \cdot 9 - 4 \cdot 3 = 42$
$z3s4 = 23 \cdot 9 - 13 \cdot 3 = 168$

x	y	z	
9	5	4	13
0	−3	−69	75
0	−105	42	168

Zeile3 = Zeile3 · (−3)-Zeile2 · (−105)
$z3s2 = (-105) \cdot -3 - (-3) \cdot (-105) = 0$
$z3s3 = 42 \cdot -3 - (-69) \cdot (-105) = -7,37 \cdot 10^3$
$z3s4 = 168 \cdot -3 - 75 \cdot (-105) = 7,37 \cdot 10^3$

x	y	z	
9	5	4	13
0	−3	−69	75
0	0	$-7,37 \cdot 10^3$	$7,37 \cdot 10^3$

$z = \frac{7,37 \cdot 10^3}{-7,37 \cdot 10^3} = -1$
$y \cdot (-3) + (-69) \cdot -1 = 75$
$y = -2$
$x \cdot 9 + 5 \cdot -2 + 4 \cdot (-1) = 13$
$x = 3$
$L = \{3/-2/-1\}$

Aufgabe (11)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

Zeile2 = Zeile2 · 11-Zeile1 · 12
$z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0$
$z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2$
$z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7$
$z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4$

x	y	z	
11	13	4	37
0	-2	7	-4
9	3	3	15

Zeile3 = Zeile3 · 11-Zeile1 · 9
$z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0$
$z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84$
$z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3$
$z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168$

x	y	z	
11	13	4	37
0	-2	7	-4
0	-84	-3	-168

Zeile3 = Zeile3 · (−2)-Zeile2 · (−84)
$z3s2 = (-84) \cdot -2 - (-2) \cdot (-84) = 0$
$z3s3 = (-3) \cdot -2 - 7 \cdot (-84) = 594$
$z3s4 = (-168) \cdot -2 - (-4) \cdot (-84) = 0$

x	y	z	
11	13	4	37
0	-2	7	-4
0	0	594	0

$z = \frac{0}{594} = 0$
$y \cdot (-2) + 7 \cdot 0 = (-4)$
$y = 2$
$x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$
$x = 1$
$L = \{1/2/0\}$

Aufgabe (12)

$2x + 3y + 4z = 175$
$4x + 6y + 5z = 287$
$3x + 2y + 8z = 257$

x	y	z	
2	3	4	175
4	6	5	287
3	2	8	257

Zeile2 = Zeile2 · 2-Zeile1 · 4
$z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0$
$z2s2 = 6 \cdot 2 - 3 \cdot 4 = 0$
$z2s3 = 5 \cdot 2 - 4 \cdot 4 = -6$
$z2s4 = 287 \cdot 2 - 175 \cdot 4 = -126$

x	y	z	
2	3	4	175
0	0	-6	-126
3	2	8	257

Zeile3 = Zeile3 · 2-Zeile1 · 3
$z3s1 = 3 \cdot 2 - 2 \cdot 3 = 0$
$z3s2 = 2 \cdot 2 - 3 \cdot 3 = -5$
$z3s3 = 8 \cdot 2 - 4 \cdot 3 = 4$
$z3s4 = 257 \cdot 2 - 175 \cdot 3 = -11$

x	y	z	
2	3	4	175
0	0	-6	-126
0	-5	4	-11

Zeilen vertauschen

x	y	z	
2	3	4	175
0	-5	4	-11
0	0	-6	-126

$z = \frac{-126}{-6} = 21$
$y \cdot (-5) + 4 \cdot 21 = (-11)$
$y = 19$
$x \cdot 2 + 3 \cdot 19 + 4 \cdot 21 = 175$
$x = 17$
$L = \{17/19/21\}$

Aufgabe (13)

$6x + 4y + 9z = 32$
$5x + 7y + 10z = 17$
$4x + 8y + 5z = 100$

x	y	z	
6	4	9	32
5	7	10	17
4	8	5	100

Zeile2 = Zeile2 · 6-Zeile1 · 5
$z2s1 = 5 \cdot 6 - 6 \cdot 5 = 0$
$z2s2 = 7 \cdot 6 - 4 \cdot 5 = 22$
$z2s3 = 10 \cdot 6 - 9 \cdot 5 = 15$
$z2s4 = 17 \cdot 6 - 32 \cdot 5 = -58$

x	y	z	
6	4	9	32
0	22	15	−58
4	8	5	100

Zeile3 = Zeile3 · 6-Zeile1 · 4
$z3s1 = 4 \cdot 6 - 6 \cdot 4 = 0$
$z3s2 = 8 \cdot 6 - 4 \cdot 4 = 32$
$z3s3 = 5 \cdot 6 - 9 \cdot 4 = -6$
$z3s4 = 100 \cdot 6 - 32 \cdot 4 = 472$

x	y	z	
6	4	9	32
0	22	15	−58
0	32	−6	472

Zeile3 = Zeile3 · 22-Zeile2 · 32
$z3s2 = 32 \cdot 22 - 22 \cdot 32 = 0$
$z3s3 = (-6) \cdot 22 - 15 \cdot 32 = -612$
$z3s4 = 472 \cdot 22 - (-58) \cdot 32 = 1,22 \cdot 10^4$

x	y	z	
6	4	9	32
0	22	15	−58
0	0	−612	$1,22 \cdot 10^4$

$z = \frac{1,22 \cdot 10^4}{-612} = -20$
$y \cdot 22 + 15 \cdot -20 = (-58)$
$y = 11$
$x \cdot 6 + 4 \cdot 11 + 9 \cdot (-20) = 32$
$x = 28$
$L = \{28/11/-20\}$

Aufgabe (14)

$x + y = 1$
$x + z = 6$
$y - z = 5$

x	y	z	
1	1	0	1
1	0	1	6
0	1	−1	5

Zeile2 = Zeile2 · 1-Zeile1 · 1
$z2s1 = 1 \cdot 1 - 1 \cdot 1 = 0$
$z2s2 = 0 \cdot 1 - 1 \cdot 1 = -1$
$z2s3 = 1 \cdot 1 - 0 \cdot 1 = 1$
$z2s4 = 6 \cdot 1 - 1 \cdot 1 = 5$

x	y	z	
1	1	0	1
0	−1	1	5
0	1	−1	5

Zeile3 = Zeile3 · (−1)-Zeile2 · 1
$z3s2 = 1 \cdot -1 - (-1) \cdot 1 = 0$
$z3s3 = (-1) \cdot -1 - 1 \cdot 1 = 0$
$z3s4 = 5 \cdot -1 - 5 \cdot 1 = -10$

x	y	z	
1	1	0	1
0	−1	1	5
0	0	0	−10

$L = \{\}$

Aufgabe (15)

$x - 2y + 3z = 9$
$3x + 8y + 9z = 5$
$2x + 3y + 6z = 7$

x	y	z	
1	−2	3	9
3	8	9	5
2	3	6	7

Zeile2 = Zeile2 · 1-Zeile1 · 3
$z2s1 = 3 \cdot 1 - 1 \cdot 3 = 0$
$z2s2 = 8 \cdot 1 - (-2) \cdot 3 = 14$
$z2s3 = 9 \cdot 1 - 3 \cdot 3 = 0$
$z2s4 = 5 \cdot 1 - 9 \cdot 3 = -22$

x	y	z	
1	−2	3	9
0	14	0	−22
2	3	6	7

Zeile3 = Zeile3 · 1-Zeile1 · 2
$z3s1 = 2 \cdot 1 - 1 \cdot 2 = 0$
$z3s2 = 3 \cdot 1 - (-2) \cdot 2 = 7$
$z3s3 = 6 \cdot 1 - 3 \cdot 2 = 0$
$z3s4 = 7 \cdot 1 - 9 \cdot 2 = -11$

x	y	z	
1	−2	3	9
0	14	0	−22
0	7	0	−11

Zeile3 = Zeile3 · 14-Zeile2 · 7
$z3s2 = 7 \cdot 14 - 14 \cdot 7 = 0$
$z3s3 = 0 \cdot 14 - 0 \cdot 7 = 0$
$z3s4 = (-11) \cdot 14 - (-22) \cdot 7 = 0$

x	y	z	
1	−2	3	9
0	14	0	−22
0	0	0	0

$L = unendlich$

Aufgabe (16)

$6x + 4y + 5z = 8$
$4x + 2y + 3z = 7$
$5x + 3y + 4z = 9$

x	y	z	
6	4	5	8
4	2	3	7
5	3	4	9

Zeile2 = Zeile2 · 6-Zeile1 · 4
$z2s1 = 4 \cdot 6 - 6 \cdot 4 = 0$
$z2s2 = 2 \cdot 6 - 4 \cdot 4 = -4$
$z2s3 = 3 \cdot 6 - 5 \cdot 4 = -2$
$z2s4 = 7 \cdot 6 - 8 \cdot 4 = 10$

x	y	z	
6	4	5	8
0	−4	−2	10
5	3	4	9

Zeile3 = Zeile3 · 6-Zeile1 · 5
$z3s1 = 5 \cdot 6 - 6 \cdot 5 = 0$
$z3s2 = 3 \cdot 6 - 4 \cdot 5 = -2$
$z3s3 = 4 \cdot 6 - 5 \cdot 5 = -1$
$z3s4 = 9 \cdot 6 - 8 \cdot 5 = 14$

x	y	z	
6	4	5	8
0	−4	−2	10
0	−2	−1	14

Zeile3 = Zeile3 · (−4)-Zeile2 · (−2)
$z3s2 = (-2) \cdot -4 - (-4) \cdot (-2) = 0$
$z3s3 = (-1) \cdot -4 - (-2) \cdot (-2) = 0$
$z3s4 = 14 \cdot -4 - 10 \cdot (-2) = -36$

x	y	z	
6	4	5	8
0	−4	−2	10
0	0	0	−36

$L = \{\}$

Aufgabe (17)

$x + 3y - 2z = 3$
$3x + 2y + z = 2$
$y + 3z = 5$

x	y	z	
1	3	−2	3
3	2	1	2
0	1	3	5

Zeile2 = Zeile2 · 1-Zeile1 · 3
$z2s1 = 3 \cdot 1 - 1 \cdot 3 = 0$
$z2s2 = 2 \cdot 1 - 3 \cdot 3 = -7$
$z2s3 = 1 \cdot 1 - (-2) \cdot 3 = 7$
$z2s4 = 2 \cdot 1 - 3 \cdot 3 = -7$

x	y	z	
1	3	−2	3
0	−7	7	−7
0	1	3	5

Zeile3 = Zeile3 · (−7)-Zeile2 · 1
$z3s2 = 1 \cdot -7 - (-7) \cdot 1 = 0$
$z3s3 = 3 \cdot -7 - 7 \cdot 1 = -28$
$z3s4 = 5 \cdot -7 - (-7) \cdot 1 = -28$

x	y	z	
1	3	−2	3
0	−7	7	−7
0	0	−28	−28

$z = \frac{-28}{-28} = 1$
$y \cdot (-7) + 7 \cdot 1 = (-7)$
$y = 2$
$x \cdot 1 + 3 \cdot 2 + (-2) \cdot 1 = 3$

$x = -1$
$L = \{-1/2/1\}$

Aufgabe (18)

$4x + 6y + 8z = 0$
$5x + 6y + 67z = 8$
$8x + 87y + 6z = 6$

x	y	z	
4	6	8	0
5	6	67	8
8	87	6	6

Zeile2 = Zeile2 · 4-Zeile1 · 5
$z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0$
$z2s2 = 6 \cdot 4 - 6 \cdot 5 = -6$
$z2s3 = 67 \cdot 4 - 8 \cdot 5 = 228$
$z2s4 = 8 \cdot 4 - 0 \cdot 5 = 32$

x	y	z	
4	6	8	0
0	-6	228	32
8	87	6	6

Zeile3 = Zeile3 · 4-Zeile1 · 8
$z3s1 = 8 \cdot 4 - 4 \cdot 8 = 0$
$z3s2 = 87 \cdot 4 - 6 \cdot 8 = 300$
$z3s3 = 6 \cdot 4 - 8 \cdot 8 = -40$
$z3s4 = 6 \cdot 4 - 0 \cdot 8 = 24$

x	y	z	
4	6	8	0
0	-6	228	32
0	300	-40	24

Zeile3 = Zeile3 · (−6)-Zeile2 · 300
$z3s2 = 300 \cdot -6 - (-6) \cdot 300 = 0$
$z3s3 = (-40) \cdot -6 - 228 \cdot 300 = -6,82 \cdot 10^4$
$z3s4 = 24 \cdot -6 - 32 \cdot 300 = -9,74 \cdot 10^3$

x	y	z	
4	6	8	0
0	-6	228	32
0	0	$-6,82 \cdot 10^4$	$-9,74 \cdot 10^3$

$z = \frac{-9,74 \cdot 10^3}{-6,82 \cdot 10^4} = 0,143$
$y \cdot (-6) + 228 \cdot 0,143 = 32$
$y = 0,0991$
$x \cdot 4 + 6 \cdot 0,0991 + 8 \cdot 0,143 = 0$
$x = -0,435$
$L = \{-0,435/0,0991/0,143\}$

9 Gauß-Jordan-Algorithmus

$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$
$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$
$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$

Koeffizientenmatrix erstellen

x	y	z	
$a1$	$b1$	$c1$	$d1$
$a2$	$b2$	$c2$	$d2$
$a3$	$b3$	$c3$	$d3$

x	y	z	
$Zeile1Spalte1$	$z1s2$	$z1s3$	$z1s4$
$z2s1$	$z2s2$	$z2s3$	$z2s4$
$z3s1$	$z3s2$	$z3s3$	$z3s4$

Die Lösungsmenge ändert sich nicht durch:

- Multiplizieren oder Dividieren der Zeilen mit einer Zahl
- Addieren oder Subtrahieren der Zeilen
- Vertauschen der Zeilen

Ziel ist das Umformen in die Diagonalenform.

- Eindeutige Lösung

x	y	z	
$z1s1$	0	0	$z1s4$
0	$z2s3$	0	$z2s4$
0	0	$z3s3$	$z3s4$

$x = \frac{z1s4}{z1s1}$
$y = \frac{z2s4}{z2s3}$
$z = \frac{z3s3}{z3s4}$

- Keine Lösung

x	y	z	
$z1s1$	0	0	$z1s4$
0	$z2s3$	0	$z2s4$
0	0	0	$z3s4$

- Unendlich viele Lösungen

x	y	z	
$z1s1$	0	0	$z1s4$
0	$z2s3$	0	$z2s4$
0	0	0	0

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{12}{11}$
$z2s1 = 12 - 11 \cdot \frac{12}{11} = 0$
$z2s2 = 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11}$
$z2s3 = 5 - 4 \cdot \frac{12}{11} = \frac{7}{11}$
$z2s4 = 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
9	3	3	15

$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{11}$
$z3s1 = 9 - 11 \cdot \frac{9}{11} = 0$
$z3s2 = 3 - 13 \cdot \frac{9}{11} = -7\frac{7}{11}$
$z3s3 = 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11}$
$z3s4 = 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{13}{-\frac{2}{11}}$
$z1s2 = 13 - (-\frac{2}{11}) \cdot \frac{13}{-\frac{2}{11}} = 0$
$z1s3 = 4 - \frac{7}{11} \cdot \frac{13}{-\frac{2}{11}} = 49\frac{1}{2}$
$z1s4 = 37 - (-\frac{4}{11}) \cdot \frac{13}{-\frac{2}{11}} = 11$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}}$
$z3s2 = -7\frac{7}{11} - (-\frac{2}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$
$z3s3 = -\frac{3}{11} - \frac{7}{11} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = -27$
$z3s4 = -15\frac{3}{11} - (-\frac{4}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{49\frac{1}{2}}{-27}$
$z1s3 = 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0$
$z1s4 = 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{7}{11}}{-27}$
$z2s3 = \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0$
$z2s4 = -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11}$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	0	$-\frac{4}{11}$
0	0	-27	0

$x = \frac{11}{11} = 1$
$y = \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2$
$z = \frac{0}{-27} = 0$
$L = \{1/2/0\}$

9.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:

$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$
$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$
$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$

Gesucht:
x,y,z

(1)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(2)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(3)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x + 2y + -3z &= 7 \end{aligned}$$

(4)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(5)
$$\begin{aligned} 1x + 2 + 3z &= 4 \\ 2x + 3y + 2z &= 6 \\ 0x + 2y + 6z &= 0 \end{aligned}$$

(6)
$$\begin{aligned} -2x - 8 + 0z &= 1 \\ 1x + 4y + 0z &= -\tfrac{1}{2} \\ 8x - 2y + -1z &= 8 \end{aligned}$$

(7)
$$\begin{aligned} -2x + 2 + 4z &= 0 \\ 4x - \tfrac{1}{2}y + 2z &= 5 \\ 4x - 2y + -1z &= 8 \end{aligned}$$

(8)
$$\begin{aligned} 2x + 3 + -4z &= 16 \\ 4x + 9y + -1z &= 58 \\ 1x + 6y + 2z &= 34 \end{aligned}$$

(9)
$$\begin{aligned} 4x - 3 + 2z &= 10 \\ 5x + 6y + -7z &= 4 \\ 10x - 2y + -3z &= 7 \end{aligned}$$

(10)
$$\begin{aligned} 9x + 5 + 4z &= 13 \\ 6x + 3y + -5z &= 17 \\ 3x - 10y + 6z &= 23 \end{aligned}$$

(11)
$$\begin{aligned} 11x + 13 + 4z &= 37 \\ 12x + 14y + 5z &= 40 \\ 9x + 3y + 3z &= 15 \end{aligned}$$

(12)
$$\begin{aligned} 2x + 3 + 4z &= 175 \\ 4x + 6y + 5z &= 287 \\ 3x + 2y + 8z &= 257 \end{aligned}$$

(13)
$$\begin{aligned} 6x + 4 + 9z &= 32 \\ 5x + 7y + 10z &= 17 \\ 4x + 8y + 5z &= 100 \end{aligned}$$

(14)
$$\begin{aligned} 1x + 1 + 0z &= 1 \\ 1x + 0y + 1z &= 6 \\ 0x + 1y + -1z &= 5 \end{aligned}$$

(15)
$$\begin{aligned} 1x - 2 + 3z &= 9 \\ 3x + 8y + 9z &= 5 \\ 2x + 3y + 6z &= 7 \end{aligned}$$

(16)
$$\begin{aligned} 1x + 3 + -2z &= 3 \\ 3x + 2y + 1z &= 2 \\ 0x + 1y + 3z &= 5 \end{aligned}$$

9.2 Lösungen

Aufgabe (1)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

Zeile2 = Zeile2-Zeile1 $\cdot \frac{12}{11}$
$z2s1 = 12 - 11 \cdot \frac{12}{11} = 0$
$z2s2 = 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11}$
$z2s3 = 5 - 4 \cdot \frac{12}{11} = \frac{7}{11}$
$z2s4 = 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
9	3	3	15

Zeile3 = Zeile3-Zeile1 $\cdot \frac{9}{11}$
$z3s1 = 9 - 11 \cdot \frac{9}{11} = 0$
$z3s2 = 3 - 13 \cdot \frac{9}{11} = -7\frac{7}{11}$
$z3s3 = 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11}$
$z3s4 = 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

Zeile1 = Zeile1-Zeile2 $\cdot \frac{13}{-\frac{2}{11}}$
$z1s2 = 13 - (-\frac{2}{11}) \cdot \frac{13}{-\frac{2}{11}} = 0$
$z1s3 = 4 - \frac{7}{11} \cdot \frac{13}{-\frac{2}{11}} = 49\frac{1}{2}$
$z1s4 = 37 - (-\frac{4}{11}) \cdot \frac{13}{-\frac{2}{11}} = 11$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}}$
$z3s2 = -7\frac{7}{11} - (-\frac{2}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$
$z3s3 = -\frac{3}{11} - \frac{7}{11} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = -27$
$z3s4 = -15\frac{3}{11} - (-\frac{4}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

Zeile1 = Zeile1-Zeile3 $\cdot \frac{49\frac{1}{2}}{-27}$
$z1s3 = 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0$
$z1s4 = 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

Zeile2 = Zeile2-Zeile3 $\cdot \frac{\frac{7}{11}}{-27}$
$z2s3 = \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0$
$z2s4 = -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11}$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	0	$-\frac{4}{11}$
0	0	-27	0

$x = \frac{11}{11} = 1$
$y = \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2$
$z = \frac{0}{-27} = 0$
$L = \{1/2/0\}$

Aufgabe (2)

$9x + 5y + 4z = 13$
$6x + 3y - 5z = 17$
$3x - 10y + 6z = 23$

x	y	z	
9	5	4	13
6	3	-5	17
3	-10	6	23

Zeile2 = Zeile2-Zeile1 $\cdot \frac{6}{9}$
$z2s1 = 6 - 9 \cdot \frac{6}{9} = 0$
$z2s2 = 3 - 5 \cdot \frac{6}{9} = -\frac{1}{3}$
$z2s3 = -5 - 4 \cdot \frac{6}{9} = -7\frac{2}{3}$
$z2s4 = 17 - 13 \cdot \frac{6}{9} = 8\frac{1}{3}$

x	y	z	
9	5	4	13
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
3	-10	6	23

Zeile3 = Zeile3-Zeile1 $\cdot \frac{3}{9}$
$z3s1 = 3 - 9 \cdot \frac{3}{9} = 0$
$z3s2 = -10 - 5 \cdot \frac{3}{9} = -11\frac{2}{3}$
$z3s3 = 6 - 4 \cdot \frac{3}{9} = 4\frac{2}{3}$
$z3s4 = 23 - 13 \cdot \frac{3}{9} = 18\frac{2}{3}$

x	y	z	
9	5	4	13
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	$-11\frac{2}{3}$	$4\frac{2}{3}$	$18\frac{2}{3}$

Zeile1 = Zeile1-Zeile2 $\cdot \frac{5}{-\frac{1}{3}}$
$z1s2 = 5 - (-\frac{1}{3}) \cdot \frac{5}{-\frac{1}{3}} = 0$
$z1s3 = 4 - (-7\frac{2}{3}) \cdot \frac{5}{-\frac{1}{3}} = -111$
$z1s4 = 13 - 8\frac{1}{3} \cdot \frac{5}{-\frac{1}{3}} = 138$

x	y	z	
9	0	-111	138
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	$-11\frac{2}{3}$	$4\frac{2}{3}$	$18\frac{2}{3}$

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}}$
$z3s2 = -11\frac{2}{3} - (-\frac{1}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 0$
$z3s3 = 4\frac{2}{3} - (-7\frac{2}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 273$
$z3s4 = 18\frac{2}{3} - 8\frac{1}{3} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = -273$

x	y	z	
9	0	-111	138
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	0	273	-273

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-111}{273}$
$z1s3 = -111 - 273 \cdot \frac{-111}{273} = 0$
$z1s4 = 138 - (-273) \cdot \frac{-111}{273} = 27$

x	y	z	
9	0	0	27
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	0	273	-273

Zeile2 = Zeile2-Zeile3 $\cdot \frac{-7\frac{2}{3}}{273}$
$z2s3 = -7\frac{2}{3} - 273 \cdot \frac{-7\frac{2}{3}}{273} = 0$
$z2s4 = 8\frac{1}{3} - (-273) \cdot \frac{-7\frac{2}{3}}{273} = \frac{2}{3}$

x	y	z	
9	0	0	27
0	$-\frac{1}{3}$	0	$\frac{2}{3}$
0	0	273	-273

$x = \frac{27}{9} = 3$
$y = \frac{\frac{2}{3}}{-\frac{1}{3}} = -2$
$z = \frac{-273}{273} = -1$
$L = \{3/-2/-1\}$

Aufgabe (3)

$4x - 3y + 2z = 10$
$5x + 6y - 7z = 4$
$10x + 2y - 3z = 7$

x	y	z	
4	-3	2	10
5	6	-7	4
10	2	-3	7

Zeile2 = Zeile2-Zeile1 $\cdot \frac{5}{4}$
$z2s1 = 5 - 4 \cdot \frac{5}{4} = 0$
$z2s2 = 6 - (-3) \cdot \frac{5}{4} = 9\frac{3}{4}$
$z2s3 = -7 - 2 \cdot \frac{5}{4} = -9\frac{1}{2}$
$z2s4 = 4 - 10 \cdot \frac{5}{4} = -8\frac{1}{2}$

x	y	z	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
10	2	-3	7

Zeile3 = Zeile3-Zeile1 $\cdot \frac{10}{4}$
$z3s1 = 10 - 4 \cdot \frac{10}{4} = 0$
$z3s2 = 2 - (-3) \cdot \frac{10}{4} = 9\frac{1}{2}$
$z3s3 = -3 - 2 \cdot \frac{10}{4} = -8$
$z3s4 = 7 - 10 \cdot \frac{10}{4} = -18$

x	y	z	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$9\frac{1}{2}$	-8	-18

Zeile1 = Zeile1-Zeile2 $\cdot \frac{-3}{9\frac{3}{4}}$
$z1s2 = -3 - 9\frac{3}{4} \cdot \frac{-3}{9\frac{3}{4}} = 0$
$z1s3 = 2 - (-9\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = -\frac{12}{13}$
$z1s4 = 10 - (-8\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = 7\frac{5}{13}$

x	y	z	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$9\frac{1}{2}$	-8	-18

Zeile3 = Zeile3-Zeile2 $\cdot \frac{9\frac{1}{2}}{9\frac{3}{4}}$
$z3s2 = 9\frac{1}{2} - 9\frac{3}{4} \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = 0$
$z3s3 = -8 - (-9\frac{1}{2}) \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = 1\frac{10}{39}$
$z3s4 = -18 - (-8\frac{1}{2}) \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = -9\frac{28}{39}$

x	y	z	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$1\frac{10}{39}$	$-9\frac{28}{39}$

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-\frac{12}{13}}{1\frac{10}{39}}$

$z1s3 = -\frac{12}{13} - 1\frac{10}{39} \cdot \frac{-\frac{12}{13}}{1\frac{10}{39}} = 0$

$z1s4 = 7\frac{5}{13} - (-9\frac{28}{39}) \cdot \frac{-\frac{12}{13}}{1\frac{10}{39}} = \frac{12}{49}$

x	y	z	
4	0	0	$\frac{12}{49}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$1\frac{10}{39}$	$-9\frac{28}{39}$

Zeile2 = Zeile2-Zeile3 $\cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}}$

$z2s3 = -9\frac{1}{2} - 1\frac{10}{39} \cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}} = 0$

$z2s4 = -8\frac{1}{2} - (-9\frac{28}{39}) \cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}} = -81\frac{48}{49}$

x	y	z	
4	0	0	$\frac{12}{49}$
0	$9\frac{3}{4}$	0	$-81\frac{48}{49}$
0	0	$1\frac{10}{39}$	$-9\frac{28}{39}$

$x = \frac{\frac{12}{49}}{4} = \frac{3}{49}$

$y = \frac{-81\frac{48}{49}}{9\frac{3}{4}} = -8\frac{20}{49}$

$z = \frac{-9\frac{28}{39}}{1\frac{10}{39}} = -7\frac{36}{49}$

$L = \{\frac{3}{49} / -8\frac{20}{49} / -7\frac{36}{49}\}$

Aufgabe (4)

$2x + 3y - 4z = 16$

$4x + 9y - z = 58$

$x + 6y + 2z = 34$

x	y	z	
2	3	-4	16
4	9	-1	58
1	6	2	34

Zeile2 = Zeile2-Zeile1 $\cdot \frac{4}{2}$

$z2s1 = 4 - 2 \cdot \frac{4}{2} = 0$

$z2s2 = 9 - 3 \cdot \frac{4}{2} = 3$

$z2s3 = -1 - (-4) \cdot \frac{4}{2} = 7$

$z2s4 = 58 - 16 \cdot \frac{4}{2} = 26$

x	y	z	
2	3	-4	16
0	3	7	26
1	6	2	34

Zeile3 = Zeile3-Zeile1 $\cdot \frac{1}{2}$

$z3s1 = 1 - 2 \cdot \frac{1}{2} = 0$

$z3s2 = 6 - 3 \cdot \frac{1}{2} = 4\frac{1}{2}$

$z3s3 = 2 - (-4) \cdot \frac{1}{2} = 4$

$z3s4 = 34 - 16 \cdot \frac{1}{2} = 26$

x	y	z	
2	3	-4	16
0	3	7	26
0	$4\frac{1}{2}$	4	26

Zeile1 = Zeile1-Zeile2 $\cdot \frac{3}{3}$

$z1s2 = 3 - 3 \cdot \frac{3}{3} = 0$

$z1s3 = -4 - 7 \cdot \frac{3}{3} = -11$

$z1s4 = 16 - 26 \cdot \frac{3}{3} = -10$

x	y	z	
2	0	-11	-10
0	3	7	26
0	$4\frac{1}{2}$	4	26

Zeile3 = Zeile3-Zeile2 $\cdot \frac{4\frac{1}{2}}{3}$

$z3s2 = 4\frac{1}{2} - 3 \cdot \frac{4\frac{1}{2}}{3} = 0$

$z3s3 = 4 - 7 \cdot \frac{4\frac{1}{2}}{3} = -6\frac{1}{2}$

$z3s4 = 26 - 26 \cdot \frac{4\frac{1}{2}}{3} = -13$

x	y	z	
2	0	-11	-10
0	3	7	26
0	0	$-6\frac{1}{2}$	-13

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-11}{-6\frac{1}{2}}$

$z1s3 = -11 - (-6\frac{1}{2}) \cdot \frac{-11}{-6\frac{1}{2}} = 0$

$z1s4 = -10 - (-13) \cdot \frac{-11}{-6\frac{1}{2}} = 12$

x	y	z	
2	0	0	12
0	3	7	26
0	0	$-6\frac{1}{2}$	-13

Zeile2 = Zeile2-Zeile3 $\cdot \frac{7}{-6\frac{1}{2}}$

$z2s3 = 7 - (-6\frac{1}{2}) \cdot \frac{7}{-6\frac{1}{2}} = 0$

$z2s4 = 26 - (-13) \cdot \frac{7}{-6\frac{1}{2}} = 12$

x	y	z	
2	0	0	12
0	3	0	12
0	0	$-6\frac{1}{2}$	-13

$x = \frac{12}{2} = 6$

$y = \frac{12}{3} = 4$

$z = \frac{-13}{-6\frac{1}{2}} = 2$

$L = \{6/4/2\}$

Aufgabe (5)

$x + 2y + 3z = 4$
$2x + 3y + 2z = 6$
$2y + 6z = 0$

x	y	z	
1	2	3	4
2	3	2	6
0	2	6	0

Zeile2 = Zeile2-Zeile1 $\cdot \frac{2}{1}$
$z2s1 = 2 - 1 \cdot \frac{2}{1} = 0$
$z2s2 = 3 - 2 \cdot \frac{2}{1} = -1$
$z2s3 = 2 - 3 \cdot \frac{2}{1} = -4$
$z2s4 = 6 - 4 \cdot \frac{2}{1} = -2$

x	y	z	
1	2	3	4
0	-1	-4	-2
0	2	6	0

Zeile1 = Zeile1-Zeile2 $\cdot \frac{2}{-1}$
$z1s2 = 2 - (-1) \cdot \frac{2}{-1} = 0$
$z1s3 = 3 - (-4) \cdot \frac{2}{-1} = -5$
$z1s4 = 4 - (-2) \cdot \frac{2}{-1} = 0$

x	y	z	
1	0	-5	0
0	-1	-4	-2
0	2	6	0

Zeile3 = Zeile3-Zeile2 $\cdot \frac{2}{-1}$
$z3s2 = 2 - (-1) \cdot \frac{2}{-1} = 0$
$z3s3 = 6 - (-4) \cdot \frac{2}{-1} = -2$
$z3s4 = 0 - (-2) \cdot \frac{2}{-1} = -4$

x	y	z	
1	0	-5	0
0	-1	-4	-2
0	0	-2	-4

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-5}{-2}$
$z1s3 = -5 - (-2) \cdot \frac{-5}{-2} = 0$
$z1s4 = 0 - (-4) \cdot \frac{-5}{-2} = 10$

x	y	z	
1	0	0	10
0	-1	-4	-2
0	0	-2	-4

Zeile2 = Zeile2-Zeile3 $\cdot \frac{-4}{-2}$
$z2s3 = -4 - (-2) \cdot \frac{-4}{-2} = 0$
$z2s4 = -2 - (-4) \cdot \frac{-4}{-2} = 6$

x	y	z	
1	0	0	10
0	-1	0	6
0	0	-2	-4

$x = \frac{10}{1} = 10$
$y = \frac{6}{-1} = -6$
$z = \frac{-4}{-2} = 2$
$L = \{10/-6/2\}$

Aufgabe (6)

$-2x - 8y = 1$
$x + 4y = -\frac{1}{2}$
$8x - 2y - z = 8$

x	y	z	
-2	-8	0	1
1	4	0	$-\frac{1}{2}$
8	-2	-1	8

Zeile2 = Zeile2-Zeile1 $\cdot \frac{1}{-2}$
$z2s1 = 1 - (-2) \cdot \frac{1}{-2} = 0$
$z2s2 = 4 - (-8) \cdot \frac{1}{-2} = 0$
$z2s3 = 0 - 0 \cdot \frac{1}{-2} = 0$
$z2s4 = -\frac{1}{2} - 1 \cdot \frac{1}{-2} = 0$

x	y	z	
-2	-8	0	1
0	0	0	0
8	-2	-1	8

Zeile3 = Zeile3-Zeile1 $\cdot \frac{8}{-2}$
$z3s1 = 8 - (-2) \cdot \frac{8}{-2} = 0$
$z3s2 = -2 - (-8) \cdot \frac{8}{-2} = -34$
$z3s3 = -1 - 0 \cdot \frac{8}{-2} = -1$
$z3s4 = 8 - 1 \cdot \frac{8}{-2} = 12$

x	y	z	
-2	-8	0	1
0	0	0	0
0	-34	-1	12

Zeilen vertauschen

x	y	z	
-2	-8	0	1
0	-34	-1	12
0	0	0	0

Zeile1 = Zeile1-Zeile2 $\cdot \frac{-8}{-34}$
$z1s2 = -8 - (-34) \cdot \frac{-8}{-34} = 0$
$z1s3 = 0 - (-1) \cdot \frac{-8}{-34} = \frac{4}{17}$
$z1s4 = 1 - 12 \cdot \frac{-8}{-34} = -1\frac{14}{17}$

x	y	z	
-2	0	$\frac{4}{17}$	$-1\frac{14}{17}$
0	-34	-1	12
0	0	0	0

$L = unendlich$

Aufgabe (7)

$-2x + 2y + 4z = 0$
$4x - \frac{1}{2}y + 2z = 5$
$4x - 2y - z = 8$

x	y	z	
-2	2	4	0
4	$-\frac{1}{2}$	2	5
4	-2	-1	8

Zeile2 = Zeile2-Zeile1 $\cdot \frac{4}{-2}$
$z2s1 = 4 - (-2) \cdot \frac{4}{-2} = 0$
$z2s2 = -\frac{1}{2} - 2 \cdot \frac{4}{-2} = 3\frac{1}{2}$
$z2s3 = 2 - 4 \cdot \frac{4}{-2} = 10$
$z2s4 = 5 - 0 \cdot \frac{4}{-2} = 5$

x	y	z	
-2	2	4	0
0	$3\frac{1}{2}$	10	5
4	-2	-1	8

Zeile3 = Zeile3-Zeile1 $\cdot \frac{4}{-2}$
$z3s1 = 4 - (-2) \cdot \frac{4}{-2} = 0$
$z3s2 = -2 - 2 \cdot \frac{4}{-2} = 2$
$z3s3 = -1 - 4 \cdot \frac{4}{-2} = 7$
$z3s4 = 8 - 0 \cdot \frac{4}{-2} = 8$

x	y	z	
-2	2	4	0
0	$3\frac{1}{2}$	10	5
0	2	7	8

Zeile1 = Zeile1-Zeile2 $\cdot \frac{2}{3\frac{1}{2}}$
$z1s2 = 2 - 3\frac{1}{2} \cdot \frac{2}{3\frac{1}{2}} = 0$
$z1s3 = 4 - 10 \cdot \frac{2}{3\frac{1}{2}} = -1\frac{5}{7}$
$z1s4 = 0 - 5 \cdot \frac{2}{3\frac{1}{2}} = -2\frac{6}{7}$

x	y	z	
-2	0	$-1\frac{5}{7}$	$-2\frac{6}{7}$
0	$3\frac{1}{2}$	10	5
0	2	7	8

Zeile3 = Zeile3-Zeile2 $\cdot \frac{2}{3\frac{1}{2}}$
$z3s2 = 2 - 3\frac{1}{2} \cdot \frac{2}{3\frac{1}{2}} = 0$
$z3s3 = 7 - 10 \cdot \frac{2}{3\frac{1}{2}} = 1\frac{2}{7}$
$z3s4 = 8 - 5 \cdot \frac{2}{3\frac{1}{2}} = 5\frac{1}{7}$

x	y	z	
-2	0	$-1\frac{5}{7}$	$-2\frac{6}{7}$
0	$3\frac{1}{2}$	10	5
0	0	$1\frac{2}{7}$	$5\frac{1}{7}$

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}}$
$z1s3 = -1\frac{5}{7} - 1\frac{2}{7} \cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}} = 0$
$z1s4 = -2\frac{6}{7} - 5\frac{1}{7} \cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}} = 4$

x	y	z	
-2	0	0	4
0	$3\frac{1}{2}$	10	5
0	0	$1\frac{2}{7}$	$5\frac{1}{7}$

Zeile2 = Zeile2-Zeile3 $\cdot \frac{10}{1\frac{2}{7}}$
$z2s3 = 10 - 1\frac{2}{7} \cdot \frac{10}{1\frac{2}{7}} = 0$
$z2s4 = 5 - 5\frac{1}{7} \cdot \frac{10}{1\frac{2}{7}} = -35$

x	y	z	
-2	0	0	4
0	$3\frac{1}{2}$	0	-35
0	0	$1\frac{2}{7}$	$5\frac{1}{7}$

$x = \frac{4}{-2} = -2$
$y = \frac{-35}{3\frac{1}{2}} = -10$
$z = \frac{5\frac{1}{7}}{1\frac{2}{7}} = 4$
$L = \{-2/-10/4\}$

Aufgabe (8)

$2x + 3y - 4z = 16$
$4x + 9y - z = 58$
$x + 6y + 2z = 34$

x	y	z	
2	3	-4	16
4	9	-1	58
1	6	2	34

Zeile2 = Zeile2-Zeile1 $\cdot \frac{4}{2}$
$z2s1 = 4 - 2 \cdot \frac{4}{2} = 0$
$z2s2 = 9 - 3 \cdot \frac{4}{2} = 3$
$z2s3 = -1 - (-4) \cdot \frac{4}{2} = 7$
$z2s4 = 58 - 16 \cdot \frac{4}{2} = 26$

x	y	z	
2	3	-4	16
0	3	7	26
1	6	2	34

Zeile3 = Zeile3-Zeile1 $\cdot \frac{1}{2}$
$z3s1 = 1 - 2 \cdot \frac{1}{2} = 0$
$z3s2 = 6 - 3 \cdot \frac{1}{2} = 4\frac{1}{2}$
$z3s3 = 2 - (-4) \cdot \frac{1}{2} = 4$
$z3s4 = 34 - 16 \cdot \frac{1}{2} = 26$

x	y	z	
2	3	-4	16
0	3	7	26
0	$4\frac{1}{2}$	4	26

Zeile1 = Zeile1-Zeile2 $\cdot \frac{3}{3}$
$z1s2 = 3 - 3 \cdot \frac{3}{3} = 0$
$z1s3 = -4 - 7 \cdot \frac{3}{3} = -11$
$z1s4 = 16 - 26 \cdot \frac{3}{3} = -10$

x	y	z	
2	0	-11	-10
0	3	7	26
0	$4\frac{1}{2}$	4	26

Zeile3 = Zeile3-Zeile2 $\cdot \frac{4\frac{1}{2}}{3}$
$z3s2 = 4\frac{1}{2} - 3 \cdot \frac{4\frac{1}{2}}{3} = 0$
$z3s3 = 4 - 7 \cdot \frac{4\frac{1}{2}}{3} = -6\frac{1}{2}$
$z3s4 = 26 - 26 \cdot \frac{4\frac{1}{2}}{3} = -13$

x	y	z	
2	0	-11	-10
0	3	7	26
0	0	$-6\frac{1}{2}$	-13

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-11}{-6\frac{1}{2}}$
$z1s3 = -11 - (-6\frac{1}{2}) \cdot \frac{-11}{-6\frac{1}{2}} = 0$
$z1s4 = -10 - (-13) \cdot \frac{-11}{-6\frac{1}{2}} = 12$

x	y	z	
2	0	0	12
0	3	7	26
0	0	$-6\frac{1}{2}$	-13

Zeile2 = Zeile2-Zeile3 $\cdot \frac{7}{-6\frac{1}{2}}$
$z2s3 = 7 - (-6\frac{1}{2}) \cdot \frac{7}{-6\frac{1}{2}} = 0$
$z2s4 = 26 - (-13) \cdot \frac{7}{-6\frac{1}{2}} = 12$

x	y	z	
2	0	0	12
0	3	0	12
0	0	$-6\frac{1}{2}$	-13

$x = \frac{12}{2} = 6$
$y = \frac{12}{3} = 4$
$z = \frac{-13}{-6\frac{1}{2}} = 2$
$L = \{6/4/2\}$

Aufgabe (9)

$4x - 3y + 2z = 10$
$5x + 6y - 7z = 4$
$10x - 2y - 3z = 7$

x	y	z	
4	-3	2	10
5	6	-7	4
10	-2	-3	7

Zeile2 = Zeile2-Zeile1 $\cdot \frac{5}{4}$
$z2s1 = 5 - 4 \cdot \frac{5}{4} = 0$
$z2s2 = 6 - (-3) \cdot \frac{5}{4} = 9\frac{3}{4}$
$z2s3 = -7 - 2 \cdot \frac{5}{4} = -9\frac{1}{2}$
$z2s4 = 4 - 10 \cdot \frac{5}{4} = -8\frac{1}{2}$

x	y	z	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
10	-2	-3	7

Zeile3 = Zeile3-Zeile1 $\cdot \frac{10}{4}$
$z3s1 = 10 - 4 \cdot \frac{10}{4} = 0$
$z3s2 = -2 - (-3) \cdot \frac{10}{4} = 5\frac{1}{2}$
$z3s3 = -3 - 2 \cdot \frac{10}{4} = -8$
$z3s4 = 7 - 10 \cdot \frac{10}{4} = -18$

x	y	z	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$5\frac{1}{2}$	-8	-18

Zeile1 = Zeile1-Zeile2 $\cdot \frac{-3}{9\frac{3}{4}}$
$z1s2 = -3 - 9\frac{3}{4} \cdot \frac{-3}{9\frac{3}{4}} = 0$
$z1s3 = 2 - (-9\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = -\frac{12}{13}$
$z1s4 = 10 - (-8\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = 7\frac{5}{13}$

x	y	z	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$5\frac{1}{2}$	-8	-18

Zeile3 = Zeile3-Zeile2 $\cdot \frac{5\frac{1}{2}}{9\frac{3}{4}}$

$z3s2 = 5\frac{1}{2} - 9\frac{3}{4} \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = 0$

$z3s3 = -8 - (-9\frac{1}{2}) \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = -2\frac{25}{39}$

$z3s4 = -18 - (-8\frac{1}{2}) \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = -13\frac{8}{39}$

x	y	z	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}}$

$z1s3 = -\frac{12}{13} - (-2\frac{25}{39}) \cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}} = 0$

$z1s4 = 7\frac{5}{13} - (-13\frac{8}{39}) \cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}} = 12$

x	y	z	
4	0	0	12
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

Zeile2 = Zeile2-Zeile3 $\cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}}$

$z2s3 = -9\frac{1}{2} - (-2\frac{25}{39}) \cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}} = 0$

$z2s4 = -8\frac{1}{2} - (-13\frac{8}{39}) \cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}} = 39$

x	y	z	
4	0	0	12
0	$9\frac{3}{4}$	0	39
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

$x = \frac{12}{4} = 3$

$y = \frac{39}{9\frac{3}{4}} = 4$

$z = \frac{-13\frac{8}{39}}{-2\frac{25}{39}} = 5$

$L = \{3/4/5\}$

Aufgabe (10)

$9x + 5y + 4z = 13$

$6x + 3y - 5z = 17$

$3x - 10y + 6z = 23$

x	y	z	
9	5	4	13
6	3	-5	17
3	-10	6	23

Zeile2 = Zeile2-Zeile1 $\cdot \frac{6}{9}$

$z2s1 = 6 - 9 \cdot \frac{6}{9} = 0$

$z2s2 = 3 - 5 \cdot \frac{6}{9} = -\frac{1}{3}$

$z2s3 = -5 - 4 \cdot \frac{6}{9} = -7\frac{2}{3}$

$z2s4 = 17 - 13 \cdot \frac{6}{9} = 8\frac{1}{3}$

x	y	z	
9	5	4	13
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
3	-10	6	23

Zeile3 = Zeile3-Zeile1 $\cdot \frac{3}{9}$

$z3s1 = 3 - 9 \cdot \frac{3}{9} = 0$

$z3s2 = -10 - 5 \cdot \frac{3}{9} = -11\frac{2}{3}$

$z3s3 = 6 - 4 \cdot \frac{3}{9} = 4\frac{2}{3}$

$z3s4 = 23 - 13 \cdot \frac{3}{9} = 18\frac{2}{3}$

x	y	z	
9	5	4	13
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	$-11\frac{2}{3}$	$4\frac{2}{3}$	$18\frac{2}{3}$

Zeile1 = Zeile1-Zeile2 $\cdot \frac{5}{-\frac{1}{3}}$

$z1s2 = 5 - (-\frac{1}{3}) \cdot \frac{5}{-\frac{1}{3}} = 0$

$z1s3 = 4 - (-7\frac{2}{3}) \cdot \frac{5}{-\frac{1}{3}} = -111$

$z1s4 = 13 - 8\frac{1}{3} \cdot \frac{5}{-\frac{1}{3}} = 138$

x	y	z	
9	0	-111	138
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	$-11\frac{2}{3}$	$4\frac{2}{3}$	$18\frac{2}{3}$

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}}$

$z3s2 = -11\frac{2}{3} - (-\frac{1}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 0$

$z3s3 = 4\frac{2}{3} - (-7\frac{2}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 273$

$z3s4 = 18\frac{2}{3} - 8\frac{1}{3} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = -273$

x	y	z	
9	0	-111	138
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	0	273	-273

Zeile1 = Zeile1-Zeile3 $\cdot \frac{-111}{273}$

$z1s3 = -111 - 273 \cdot \frac{-111}{273} = 0$

$z1s4 = 138 - (-273) \cdot \frac{-111}{273} = 27$

x	y	z	
9	0	0	27
0	$-\frac{1}{3}$	$-7\frac{2}{3}$	$8\frac{1}{3}$
0	0	273	-273

Zeile2 = Zeile2-Zeile3 $\cdot \frac{-7\frac{2}{3}}{273}$

$z2s3 = -7\frac{2}{3} - 273 \cdot \frac{-7\frac{2}{3}}{273} = 0$

$z2s4 = 8\frac{1}{3} - (-273) \cdot \frac{-7\frac{2}{3}}{273} = \frac{2}{3}$

x	y	z	
9	0	0	27
0	$-\frac{1}{3}$	0	$\frac{2}{3}$
0	0	273	-273

$x = \frac{27}{9} = 3$
$y = \frac{\frac{2}{3}}{-\frac{1}{3}} = -2$
$z = \frac{-273}{273} = -1$
$L = \{3/-2/-1\}$

Aufgabe (11)

$11x + 13y + 4z = 37$
$12x + 14y + 5z = 40$
$9x + 3y + 3z = 15$

x	y	z	
11	13	4	37
12	14	5	40
9	3	3	15

Zeile2 = Zeile2-Zeile1 $\cdot \frac{12}{11}$
$z2s1 = 12 - 11 \cdot \frac{12}{11} = 0$
$z2s2 = 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11}$
$z2s3 = 5 - 4 \cdot \frac{12}{11} = \frac{7}{11}$
$z2s4 = 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
9	3	3	15

Zeile3 = Zeile3-Zeile1 $\cdot \frac{9}{11}$
$z3s1 = 9 - 11 \cdot \frac{9}{11} = 0$
$z3s2 = 3 - 13 \cdot \frac{9}{11} = -7\frac{7}{11}$
$z3s3 = 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11}$
$z3s4 = 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11}$

x	y	z	
11	13	4	37
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

Zeile1 = Zeile1-Zeile2 $\cdot \frac{13}{-\frac{2}{11}}$
$z1s2 = 13 - (-\frac{2}{11}) \cdot \frac{13}{-\frac{2}{11}} = 0$
$z1s3 = 4 - \frac{7}{11} \cdot \frac{13}{-\frac{2}{11}} = 49\frac{1}{2}$
$z1s4 = 37 - (-\frac{4}{11}) \cdot \frac{13}{-\frac{2}{11}} = 11$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

Zeile3 = Zeile3-Zeile2 $\cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}}$
$z3s2 = -7\frac{7}{11} - (-\frac{2}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$
$z3s3 = -\frac{3}{11} - \frac{7}{11} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = -27$
$z3s4 = -15\frac{3}{11} - (-\frac{4}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0$

x	y	z	
11	0	$49\frac{1}{2}$	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

Zeile1 = Zeile1-Zeile3 $\cdot \frac{49\frac{1}{2}}{-27}$
$z1s3 = 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0$
$z1s4 = 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
0	0	-27	0

Zeile2 = Zeile2-Zeile3 $\cdot \frac{\frac{7}{11}}{-27}$
$z2s3 = \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0$
$z2s4 = -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11}$

x	y	z	
11	0	0	11
0	$-\frac{2}{11}$	0	$-\frac{4}{11}$
0	0	-27	0

$x = \frac{11}{11} = 1$
$y = \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2$
$z = \frac{0}{-27} = 0$
$L = \{1/2/0\}$

Aufgabe (12)

$2x + 3y + 4z = 175$
$4x + 6y + 5z = 287$
$3x + 2y + 8z = 257$

x	y	z	
2	3	4	175
4	6	5	287
3	2	8	257

Zeile2 = Zeile2-Zeile1 $\cdot \frac{4}{2}$
$z2s1 = 4 - 2 \cdot \frac{4}{2} = 0$
$z2s2 = 6 - 3 \cdot \frac{4}{2} = 0$
$z2s3 = 5 - 4 \cdot \frac{4}{2} = -3$
$z2s4 = 287 - 175 \cdot \frac{4}{2} = -63$

x	y	z	
2	3	4	175
0	0	-3	-63
3	2	8	257

Zeile3 = Zeile3-Zeile1 $\cdot \frac{3}{2}$
$z3s1 = 3 - 2 \cdot \frac{3}{2} = 0$
$z3s2 = 2 - 3 \cdot \frac{3}{2} = -2\frac{1}{2}$
$z3s3 = 8 - 4 \cdot \frac{3}{2} = 2$
$z3s4 = 257 - 175 \cdot \frac{3}{2} = -5\frac{1}{2}$

x	y	z	
2	3	4	175
0	0	-3	-63
0	$-2\frac{1}{2}$	2	$-5\frac{1}{2}$

Zeilen vertauschen

x	y	z	
2	3	4	175
0	$-2\frac{1}{2}$	2	$-5\frac{1}{2}$
0	0	-3	-63

Zeile1 = Zeile1-Zeile2 $\cdot \frac{3}{-2\frac{1}{2}}$
$z1s2 = 3 - (-2\frac{1}{2}) \cdot \frac{3}{-2\frac{1}{2}} = 0$
$z1s3 = 4 - 2 \cdot \frac{3}{-2\frac{1}{2}} = 6\frac{2}{5}$
$z1s4 = 175 - (-5\frac{1}{2}) \cdot \frac{3}{-2\frac{1}{2}} = 168\frac{2}{5}$

x	y	z	
2	0	$6\frac{2}{5}$	$168\frac{2}{5}$
0	$-2\frac{1}{2}$	2	$-5\frac{1}{2}$
0	0	-3	-63

Zeile1 = Zeile1-Zeile3 $\cdot \frac{6\frac{2}{5}}{-3}$
$z1s3 = 6\frac{2}{5} - (-3) \cdot \frac{6\frac{2}{5}}{-3} = 0$
$z1s4 = 168\frac{2}{5} - (-63) \cdot \frac{6\frac{2}{5}}{-3} = 34$

x	y	z	
2	0	0	34
0	$-2\frac{1}{2}$	2	$-5\frac{1}{2}$
0	0	-3	-63

Zeile2 = Zeile2-Zeile3 $\cdot \frac{2}{-3}$
$z2s3 = 2 - (-3) \cdot \frac{2}{-3} = 0$
$z2s4 = -5\frac{1}{2} - (-63) \cdot \frac{2}{-3} = -47\frac{1}{2}$

x	y	z	
2	0	0	34
0	$-2\frac{1}{2}$	0	$-47\frac{1}{2}$
0	0	-3	-63

$x = \frac{34}{2} = 17$
$y = \frac{-47\frac{1}{2}}{-2\frac{1}{2}} = 19$
$z = \frac{-63}{-3} = 21$
$L = \{17/19/21\}$

Aufgabe (13)

$6x + 4y + 9z = 32$
$5x + 7y + 10z = 17$
$4x + 8y + 5z = 100$

x	y	z	
6	4	9	32
5	7	10	17
4	8	5	100

Zeile2 = Zeile2-Zeile1 $\cdot \frac{5}{6}$
$z2s1 = 5 - 6 \cdot \frac{5}{6} = 0$
$z2s2 = 7 - 4 \cdot \frac{5}{6} = 3\frac{2}{3}$
$z2s3 = 10 - 9 \cdot \frac{5}{6} = 2\frac{1}{2}$
$z2s4 = 17 - 32 \cdot \frac{5}{6} = -9\frac{2}{3}$

x	y	z	
6	4	9	32
0	$3\frac{2}{3}$	$2\frac{1}{2}$	$-9\frac{2}{3}$
4	8	5	100

Zeile3 = Zeile3-Zeile1 $\cdot \frac{4}{6}$
$z3s1 = 4 - 6 \cdot \frac{4}{6} = 0$
$z3s2 = 8 - 4 \cdot \frac{4}{6} = 5\frac{1}{3}$
$z3s3 = 5 - 9 \cdot \frac{4}{6} = -1$
$z3s4 = 100 - 32 \cdot \frac{4}{6} = 78\frac{2}{3}$

x	y	z	
6	4	9	32
0	$3\frac{2}{3}$	$2\frac{1}{2}$	$-9\frac{2}{3}$
0	$5\frac{1}{3}$	-1	$78\frac{2}{3}$

Zeile1 = Zeile1-Zeile2 $\cdot \frac{4}{3\frac{2}{3}}$
$z1s2 = 4 - 3\frac{2}{3} \cdot \frac{4}{3\frac{2}{3}} = 0$
$z1s3 = 9 - 2\frac{1}{2} \cdot \frac{4}{3\frac{2}{3}} = 6\frac{3}{11}$
$z1s4 = 32 - (-9\frac{2}{3}) \cdot \frac{4}{3\frac{2}{3}} = 42\frac{6}{11}$

x	y	z	
6	0	$6\frac{3}{11}$	$42\frac{6}{11}$
0	$3\frac{2}{3}$	$2\frac{1}{2}$	$-9\frac{2}{3}$
0	$5\frac{1}{3}$	-1	$78\frac{2}{3}$

Zeile3 = Zeile3-Zeile2 $\cdot \frac{5\frac{1}{3}}{3\frac{2}{3}}$

$z3s2 = 5\frac{1}{3} - 3\frac{2}{3} \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = 0$

$z3s3 = -1 - 2\frac{1}{2} \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = -4\frac{7}{11}$

$z3s4 = 78\frac{2}{3} - (-9\frac{2}{3}) \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = 92\frac{8}{11}$

x	y	z	
6	0	$6\frac{3}{11}$	$42\frac{6}{11}$
0	$3\frac{2}{3}$	$2\frac{1}{2}$	$-9\frac{2}{3}$
0	0	$-4\frac{7}{11}$	$92\frac{8}{11}$

Zeile1 = Zeile1-Zeile3 $\cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}}$

$z1s3 = 6\frac{3}{11} - (-4\frac{7}{11}) \cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}} = 0$

$z1s4 = 42\frac{6}{11} - 92\frac{8}{11} \cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}} = 168$

x	y	z	
6	0	0	168
0	$3\frac{2}{3}$	$2\frac{1}{2}$	$-9\frac{2}{3}$
0	0	$-4\frac{7}{11}$	$92\frac{8}{11}$

Zeile2 = Zeile2-Zeile3 $\cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}}$

$z2s3 = 2\frac{1}{2} - (-4\frac{7}{11}) \cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}} = 0$

$z2s4 = -9\frac{2}{3} - 92\frac{8}{11} \cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}} = 40\frac{1}{3}$

x	y	z	
6	0	0	168
0	$3\frac{2}{3}$	0	$40\frac{1}{3}$
0	0	$-4\frac{7}{11}$	$92\frac{8}{11}$

$x = \frac{168}{6} = 28$

$y = \frac{40\frac{1}{3}}{3\frac{2}{3}} = 11$

$z = \frac{92\frac{8}{11}}{-4\frac{7}{11}} = -20$

$L = \{28/11/-20\}$

Aufgabe (14)

$x + y = 1$

$x + z = 6$

$y - z = 5$

x	y	z	
1	1	0	1
1	0	1	6
0	1	-1	5

Zeile2 = Zeile2-Zeile1 $\cdot \frac{1}{1}$

$z2s1 = 1 - 1 \cdot \frac{1}{1} = 0$

$z2s2 = 0 - 1 \cdot \frac{1}{1} = -1$

$z2s3 = 1 - 0 \cdot \frac{1}{1} = 1$

$z2s4 = 6 - 1 \cdot \frac{1}{1} = 5$

x	y	z	
1	1	0	1
0	-1	1	5
0	1	-1	5

Zeile1 = Zeile1-Zeile2 $\cdot \frac{1}{-1}$

$z1s2 = 1 - (-1) \cdot \frac{1}{-1} = 0$

$z1s3 = 0 - 1 \cdot \frac{1}{-1} = 1$

$z1s4 = 1 - 5 \cdot \frac{1}{-1} = 6$

x	y	z	
1	0	1	6
0	-1	1	5
0	1	-1	5

Zeile3 = Zeile3-Zeile2 $\cdot \frac{1}{-1}$

$z3s2 = 1 - (-1) \cdot \frac{1}{-1} = 0$

$z3s3 = -1 - 1 \cdot \frac{1}{-1} = 0$

$z3s4 = 5 - 5 \cdot \frac{1}{-1} = 10$

x	y	z	
1	0	1	6
0	-1	1	5
0	0	0	10

$L = \{\}$

Aufgabe (15)

$x - 2y + 3z = 9$

$3x + 8y + 9z = 5$

$2x + 3y + 6z = 7$

x	y	z	
1	-2	3	9
3	8	9	5
2	3	6	7

Zeile2 = Zeile2-Zeile1 $\cdot \frac{3}{1}$

$z2s1 = 3 - 1 \cdot \frac{3}{1} = 0$

$z2s2 = 8 - (-2) \cdot \frac{3}{1} = 14$

$z2s3 = 9 - 3 \cdot \frac{3}{1} = 0$

$z2s4 = 5 - 9 \cdot \frac{3}{1} = -22$

x	y	z	
1	-2	3	9
0	14	0	-22
2	3	6	7

Zeile3 = Zeile3-Zeile1 $\cdot \frac{2}{1}$
$z3s1 = 2 - 1 \cdot \frac{2}{1} = 0$
$z3s2 = 3 - (-2) \cdot \frac{2}{1} = 7$
$z3s3 = 6 - 3 \cdot \frac{2}{1} = 0$
$z3s4 = 7 - 9 \cdot \frac{2}{1} = -11$

x	y	z	
1	-2	3	9
0	14	0	-22
0	7	0	-11

Zeile1 = Zeile1-Zeile2 $\cdot \frac{-2}{14}$
$z1s2 = -2 - 14 \cdot \frac{-2}{14} = 0$
$z1s3 = 3 - 0 \cdot \frac{-2}{14} = 3$
$z1s4 = 9 - (-22) \cdot \frac{-2}{14} = 5\frac{6}{7}$

x	y	z	
1	0	3	$5\frac{6}{7}$
0	14	0	-22
0	7	0	-11

Zeile3 = Zeile3-Zeile2 $\cdot \frac{7}{14}$
$z3s2 = 7 - 14 \cdot \frac{7}{14} = 0$
$z3s3 = 0 - 0 \cdot \frac{7}{14} = 0$
$z3s4 = -11 - (-22) \cdot \frac{7}{14} = 0$

x	y	z	
1	0	3	$5\frac{6}{7}$
0	14	0	-22
0	0	0	0

$L = unendlich$

Aufgabe (16)

$x + 3y - 2z = 3$
$3x + 2y + z = 2$
$y + 3z = 5$

x	y	z	
1	3	-2	3
3	2	1	2
0	1	3	5

Zeile2 = Zeile2-Zeile1 $\cdot \frac{3}{1}$
$z2s1 = 3 - 1 \cdot \frac{3}{1} = 0$
$z2s2 = 2 - 3 \cdot \frac{3}{1} = -7$
$z2s3 = 1 - (-2) \cdot \frac{3}{1} = 7$
$z2s4 = 2 - 3 \cdot \frac{3}{1} = -7$

x	y	z	
1	3	-2	3
0	-7	7	-7
0	1	3	5

Zeile1 = Zeile1-Zeile2 $\cdot \frac{3}{-7}$
$z1s2 = 3 - (-7) \cdot \frac{3}{-7} = 0$
$z1s3 = -2 - 7 \cdot \frac{3}{-7} = 1$
$z1s4 = 3 - (-7) \cdot \frac{3}{-7} = 0$

x	y	z	
1	0	1	0
0	-7	7	-7
0	1	3	5

Zeile3 = Zeile3-Zeile2 $\cdot \frac{1}{-7}$
$z3s2 = 1 - (-7) \cdot \frac{1}{-7} = 0$
$z3s3 = 3 - 7 \cdot \frac{1}{-7} = 4$
$z3s4 = 5 - (-7) \cdot \frac{1}{-7} = 4$

x	y	z	
1	0	1	0
0	-7	7	-7
0	0	4	4

Zeile1 = Zeile1-Zeile3 $\cdot \frac{1}{4}$
$z1s3 = 1 - 4 \cdot \frac{1}{4} = 0$
$z1s4 = 0 - 4 \cdot \frac{1}{4} = -1$

x	y	z	
1	0	0	-1
0	-7	7	-7
0	0	4	4

Zeile2 = Zeile2-Zeile3 $\cdot \frac{7}{4}$
$z2s3 = 7 - 4 \cdot \frac{7}{4} = 0$
$z2s4 = -7 - 4 \cdot \frac{7}{4} = -14$

x	y	z	
1	0	0	-1
0	-7	0	-14
0	0	4	4

$x = \frac{-1}{1} = -1$
$y = \frac{-14}{-7} = 2$
$z = \frac{4}{4} = 1$
$L = \{-1/2/1\}$