Trigonometrie
Aufgaben und Lösungen
http://www.fersch.de

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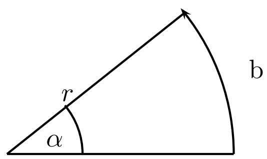
1 Gradmaß - Bogenmaß



$\alpha(^\circ)$	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\alpha(rad)$	0 0	$\frac{1}{6}\pi$ $0,5236$	$\frac{1}{4}\pi$ $0,7854$	$\frac{1}{3}\pi$ $1,0472$	$\frac{1}{2}\pi$ $1,5708$	$\frac{2}{3}\pi$ $2,0944$	$\frac{3}{4}\pi$ $2,3562$	$\frac{5}{6}\pi$ $2,618$	π $3,1416$

$\alpha(^\circ)$	210°	225°	240°	270°	300°	315°	330°	360°
$\alpha(rad)$	$\frac{7}{6}\pi$ $3,6652$	$\frac{5}{4}\pi$ $3,927$	$\frac{4}{3}\pi$ $4,1888$	$\frac{3}{2}\pi$ $4,7124$	$\frac{5}{3}\pi$ $5,236$	$\frac{7}{4}\pi$ $5,4978$	$\frac{11}{6}\pi$ $5,7596$	2π $6,2832$

Definiton Bogenmaß

Das Bogenmaß des Winkels x (rad), ist die Länge des Kreisbogens b durch Radius r.
$x = \frac{b}{r}$
Ist der Radius r=1 (Einheitskreis), ist das Bogenmaß des Winkels x (rad) die Länge des Kreisbogens b.
$x = b$

Umrechung Gradmaß - Bogenmaß

$\alpha = \frac{180}{\pi} \cdot x$
$x = \frac{\pi}{180} \cdot \alpha$
Kreiszahl π
α in Gradmaß $[^\circ]$
x in Bogemaß $[rad]$

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3,14$
$x = 1,57rad$
$\alpha = \frac{180}{\pi} \cdot 1,57rad$
$\alpha = 90^\circ$

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3,14$
$\alpha = 90^\circ$
$x = \frac{3,14}{180} \cdot 90^\circ$

$x = 1,57rad$

1.1 $\alpha = \frac{180}{\pi} \cdot x$

1.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Kreiszahl π $[]$
Bogenmaß x $[rad]$

Gesucht:
Winkel α $[^\circ]$

(1) $\pi = 3\frac{16}{113}$ $x = 1,57rad$
(2) $\pi = 3\frac{16}{113}$ $x = 0,785rad$
(3) $\pi = 3\frac{16}{113}$ $x = 3,93rad$
(4) $\pi = 3\frac{16}{113}$ $x = 2rad$
(5) $\pi = 3\frac{16}{113}$ $x = 1,57rad$
(6) $\pi = 3\frac{16}{113}$ $x = 1,57rad$

1.1.2 Lösungen

Aufgabe (1)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 1,57rad$
$\alpha = \frac{180}{\pi} \cdot 1,57rad$

$\alpha = 90°$

$phi =$	$alpha =$
$1,57rad$	$90°$
$1,57 \cdot 10^{3} mrad$	$5,4 \cdot 10^{3}{'}$
$90°$	$3,24 \cdot 10^{5}{''}$
$5,4 \cdot 10^{3}{'}$	$100gon$
$3,24 \cdot 10^{5}{''}{'}$	$1,57rad$

Aufgabe (2)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 0,785rad$
$\alpha = \frac{180}{\pi} \cdot 0,785rad$

$\alpha = 45°$

$phi =$	$alpha =$
$0,785rad$	$45°$
$785mrad$	$2,7 \cdot 10^{3}{'}$
$45°$	$1,62 \cdot 10^{5}{''}$
$2,7 \cdot 10^{3}{'}$	$50gon$
$1,62 \cdot 10^{5}{''}{'}$	$0,785rad$

Aufgabe (3)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 3,93rad$
$\alpha = \frac{180}{\pi} \cdot 3,93rad$

$\alpha = 225°$

$phi =$	$alpha =$
$3,93rad$	$225°$
$3,93 \cdot 10^{3} mrad$	$1,35 \cdot 10^{4}{'}$
$225°$	$8,1 \cdot 10^{5}{''}$
$1,35 \cdot 10^{4}{'}$	$250gon$
$8,1 \cdot 10^{5}{''}{'}$	$3,93rad$

Aufgabe (4)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 2rad$
$\alpha = \frac{180}{\pi} \cdot 2rad$

$\alpha = 115°$

$phi =$	$alpha =$
$2rad$	$115°$
$2 \cdot 10^{3} mrad$	$6,88 \cdot 10^{3}{'}$
$115°$	$4,13 \cdot 10^{5}{''}$
$6,88 \cdot 10^{3}{'}$	$127gon$
$4,13 \cdot 10^{5}{''}{'}$	$2rad$

Aufgabe (5)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 1,57rad$
$\alpha = \frac{180}{\pi} \cdot 1,57rad$

$\alpha = 90°$

$phi =$	$alpha =$
$1,57rad$	$90°$
$1,57 \cdot 10^{3} mrad$	$5,4 \cdot 10^{3}{'}$
$90°$	$3,24 \cdot 10^{5}{''}$
$5,4 \cdot 10^{3}{'}$	$100gon$
$3,24 \cdot 10^{5}{''}{'}$	$1,57rad$

Aufgabe (6)

$\alpha = \frac{180}{\pi} \cdot x$
$\pi = 3\frac{16}{113}$
$x = 1,57rad$
$\alpha = \frac{180}{\pi} \cdot 1,57rad$

$\alpha = 90°$

$phi =$	$alpha =$
$1,57rad$	$90°$
$1,57 \cdot 10^{3} mrad$	$5,4 \cdot 10^{3}{'}$
$90°$	$3,24 \cdot 10^{5}{''}$
$5,4 \cdot 10^{3}{'}$	$100gon$
$3,24 \cdot 10^{5}{''}{'}$	$1,57rad$

1.2 $x = \frac{\pi}{180} \cdot \alpha$

1.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Kreiszahl π []
Winkel α [°]

Gesucht:
Bogenmaß x [rad]

(1) $\pi = 3\frac{16}{113}$ $\alpha = 90°$
(2) $\pi = 3\frac{16}{113}$ $\alpha = 180°$
(3) $\pi = 3\frac{16}{113}$ $\alpha = 30°$
(4) $\pi = 3\frac{16}{113}$ $\alpha = 60°$
(5) $\pi = 3\frac{16}{113}$ $\alpha = 120°$
(6) $\pi = 3\frac{16}{113}$ $\alpha = 150°$
(7) $\pi = 3\frac{16}{113}$ $\alpha = 270°$

1.2.2 Lösungen

Aufgabe (1)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 90°$
$x = \frac{3\frac{16}{113}}{180} \cdot 90°$

$x = 1,57 rad$

$alpha =$	$phi =$
$90°$	$1,57 rad$
$5,4 \cdot 10^{3}$’	$1,57 \cdot 10^{3} mrad$
$3,24 \cdot 10^{5}$”	$90°$
$100 gon$	$5,4 \cdot 10^{3}$’
$1,57 rad$	$3,24 \cdot 10^{5}$”’

Aufgabe (2)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 180°$
$x = \frac{3\frac{16}{113}}{180} \cdot 180°$

$x = 3\frac{16}{113} rad$

$alpha =$	$phi =$
$180°$	$3\frac{16}{113} rad$
$1,08 \cdot 10^{4}$’	$3,14 \cdot 10^{3} mrad$
$6,48 \cdot 10^{5}$”	$180°$
$200 gon$	$1,08 \cdot 10^{4}$’
$3\frac{16}{113} rad$	$6,48 \cdot 10^{5}$”’

Aufgabe (3)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 30°$
$x = \frac{3\frac{16}{113}}{180} \cdot 30°$

$x = 0,524 rad$

$alpha =$	$phi =$
$30°$	$0,524 rad$
$1,8 \cdot 10^{3}$’	$524 mrad$
$1,08 \cdot 10^{5}$”	$30°$
$33\frac{1}{3} gon$	$1,8 \cdot 10^{3}$’
$0,524 rad$	$1,08 \cdot 10^{5}$”’

Aufgabe (4)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 60°$
$x = \frac{3\frac{16}{113}}{180} \cdot 60°$

$x = 1,05 rad$

$alpha =$	$phi =$
$60°$	$1,05 rad$
$3,6 \cdot 10^{3}$’	$1,05 \cdot 10^{3} mrad$
$2,16 \cdot 10^{5}$”	$60°$
$66\frac{2}{3} gon$	$3,6 \cdot 10^{3}$’
$1,05 rad$	$2,16 \cdot 10^{5}$”’

Aufgabe (5)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 120°$
$x = \frac{3\frac{16}{113}}{180} \cdot 120°$

$x = 2,09 rad$

$alpha =$	$phi =$
$120°$	$2,09 rad$
$7,2 \cdot 10^{3}$’	$2,09 \cdot 10^{3} mrad$
$4,32 \cdot 10^{5}$”	$120°$
$133\frac{1}{3} gon$	$7,2 \cdot 10^{3}$’
$2,09 rad$	$4,32 \cdot 10^{5}$”’

Aufgabe (6)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 150°$
$x = \frac{3\frac{16}{113}}{180} \cdot 150°$

$x = 2,62 rad$

$alpha =$	$phi =$
$150°$	$2,62 rad$
$9 \cdot 10^{3}$’	$2,62 \cdot 10^{3} mrad$
$5,4 \cdot 10^{5}$”	$150°$
$166\frac{2}{3} gon$	$9 \cdot 10^{3}$’
$2,62 rad$	$5,4 \cdot 10^{5}$”’

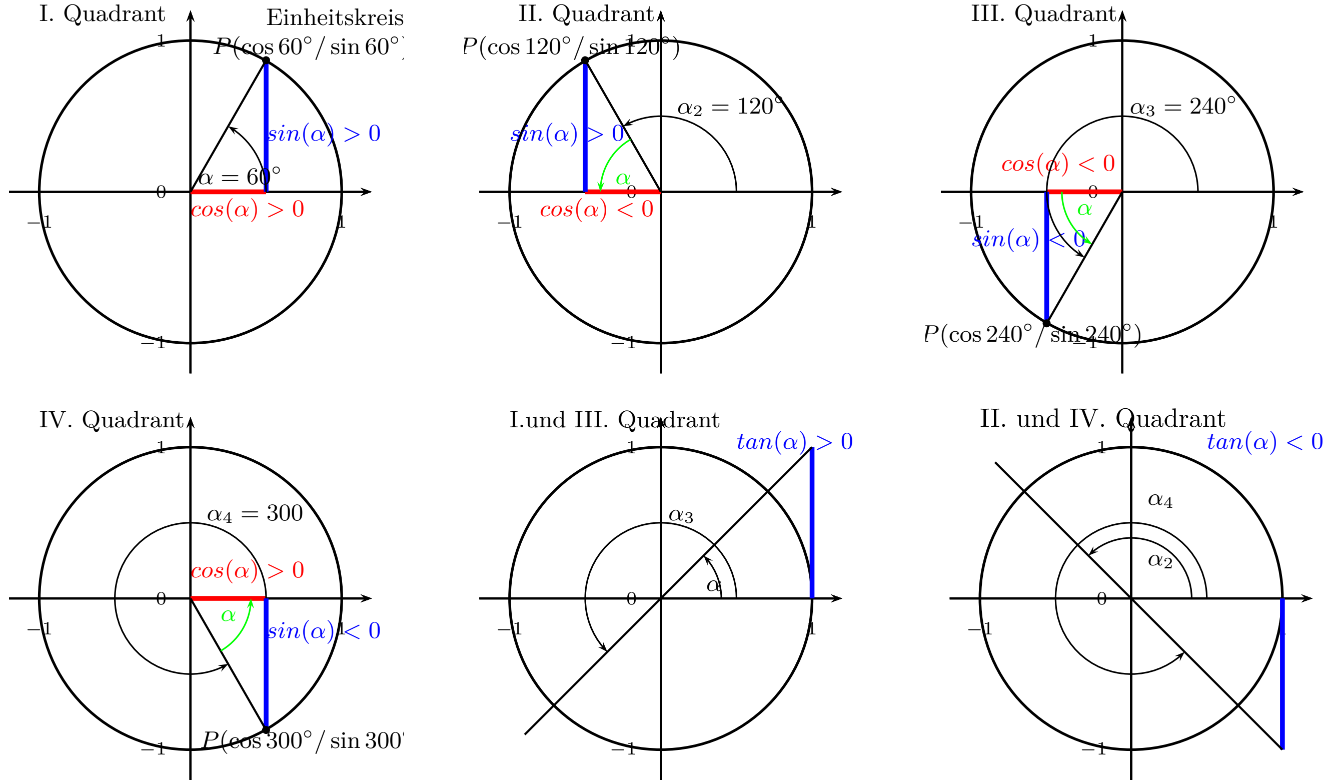
Aufgabe (7)

$x = \frac{\pi}{180} \cdot \alpha$
$\pi = 3\frac{16}{113}$
$\alpha = 270°$
$x = \frac{3\frac{16}{113}}{180} \cdot 270°$

$x = 4,71 rad$

$alpha =$	$phi =$
$270°$	$4,71 rad$
$1,62 \cdot 10^4$'	$4,71 \cdot 10^3 mrad$
$9,72 \cdot 10^5$"	$270°$
$300 gon$	$1,62 \cdot 10^4$'
$4,71 rad$	$9,72 \cdot 10^5$"'

2 Definition



$\alpha(°)$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$x(rad)$	$0°$	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\cos\alpha$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1
$\tan\alpha$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	$-$	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0

$\alpha(°)$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$x(rad)$	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
$\sin\alpha$	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0
$\cos\alpha$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\tan\alpha$	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	$-$	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0

Definition

Punkt auf dem Einheitskreis:
$P(cos\alpha/sin\alpha)$
Steigung :
$tan(\alpha) = \frac{sin(\alpha)}{cos(\alpha)} = m$

I. Quadrant: $\alpha = 60°$
$\cos(60°) = \frac{1}{2}$
$\sin(60°) = \frac{1}{2}\sqrt{2}$
$\tan(45°) = 1$
II. Quadrant: $\alpha_2 = 120°$
$\cos(120°) = \frac{1}{2}$
$\sin(120°) = -\frac{1}{2}\sqrt{2}$
$\tan(135°) = -1$
III. Quadrant: $\alpha_3 = 240°$
$\cos(210°) - \frac{1}{2}$
$\sin(210°) = -\frac{1}{2}\sqrt{2}$
$\tan(225°) = 1$
IV Quadrant: $\alpha_4 = 300°$
$\cos(300°) = -\frac{1}{2}$
$\sin(300°) = \frac{1}{2}\sqrt{2}$
$\tan(315°) = -1$

Komplementwinkel

$sin(90° - \alpha) = cos(\alpha)$
$cos(90° - \alpha) = sin(\alpha)$

$sin(90° - 30°) = sin(60°) = cos(30°)$
$cos(90° - 30°) = cos(60°) = sin(30°)$

Negative Winkel

$sin(-\alpha) = -sin(\alpha)$
$cos(-\alpha) = cos(\alpha)$
$tan(-\alpha) = \frac{1}{tan(\alpha)}$

$sin(-30°) = -sin(30°)$
$cos(-30°) = cos(30°)$
$tan(-30°) = \frac{1}{tan(30°)}$

2.1 $\sin\alpha - \cos\alpha - \tan\alpha$

2.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
y-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:
Winkel im Einheitskreis α [°]

(1) $\alpha = 45°$
(2) $\alpha = 135°$
(3) $\alpha = 225°$
(4) $\alpha = 315°$
(5) $\alpha = 30°$
(6) $\alpha = 150°$
(7) $\alpha = 210°$
(8) $\alpha = 330°$
(9) $\alpha = 90°$
(10) $\alpha = 180°$
(11) $\alpha = 270°$
(12) $\alpha = 360°$
(13) $\alpha = 180°$
(14) $\alpha = 270°$
(15) $\alpha = 180°$
(16) $\alpha = 270°$
(17) $\alpha = -90°$
(18) $\alpha = -90°$

(19) $\alpha = -90°$
(20) $\alpha = -90°$
(21) $\alpha = -90°$
(22) $\alpha = -90°$
(23) $\alpha = -90°$
(24) $\alpha = -90°$
(25) $\alpha = 90°$
(26) $\alpha = 180°$
(27) $\alpha = 270°$
(28) $\alpha = 45°$

2.1.2 Lösungen

Aufgabe (1)

$y = sin(45°)$
$y = 0,707$
$x = cos(45°)$
$x = 0,707$
$m = tan(45°)$
$m = 1$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$’
$1,62 \cdot 10^{5}$”
$50gon$
$0,785rad$

Aufgabe (2)

$y = sin(135°)$
$y = 0,707$
$x = cos(135°)$
$x = -0,707$
$m = tan(135°)$
$m = -1$

$alpha =$
$135°$
$8,1 \cdot 10^{3}$’
$4,86 \cdot 10^{5}$”
$150gon$
$2,36rad$

Aufgabe (3)

$y = sin(225°)$
$y = -0,707$
$x = cos(225°)$
$x = -0,707$
$m = tan(225°)$
$m = 1$

$alpha =$
$225°$
$1,35 \cdot 10^{4}$’
$8,1 \cdot 10^{5}$”
$250gon$
$3,93rad$

Aufgabe (4)

$y = sin(315°)$
$y = -0,707$
$x = cos(315°)$
$x = 0,707$
$m = tan(315°)$
$m = -1$

$alpha =$
$315°$
$1,89 \cdot 10^{4}$’
$1,13 \cdot 10^{6}$”
$350gon$
$5,5rad$

Aufgabe (5)

$y = sin(30°)$
$y = \frac{1}{2}$
$x = cos(30°)$
$x = 0,866$
$m = tan(30°)$
$m = 0,577$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$’
$1,08 \cdot 10^{5}$”
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (6)

$y = sin(150°)$
$y = \frac{1}{2}$
$x = cos(150°)$
$x = -0,866$
$m = tan(150°)$
$m = -0,577$

$alpha =$
$150°$
$9 \cdot 10^{3}$’
$5,4 \cdot 10^{5}$”
$166\frac{2}{3}gon$
$2,62rad$

Aufgabe (7)

$y = sin(210°)$
$y = -\frac{1}{2}$
$x = cos(210°)$
$x = -0,866$
$m = tan(210°)$
$m = 0,577$

$alpha =$
210°
$1,26 \cdot 10^{4}$'
$7,56 \cdot 10^{5}$"
$233\frac{1}{3}gon$
$3,67rad$

Aufgabe (8)

$y = sin(330°)$
$y = -\frac{1}{2}$
$x = cos(330°)$
$x = 0,866$
$m = tan(330°)$
$m = -0,577$

$alpha =$
330°
$1,98 \cdot 10^{4}$'
$1,19 \cdot 10^{6}$"
$366\frac{2}{3}gon$
$5,76rad$

Aufgabe (9)

$y = sin(90°)$
$y = 1$
$x = cos(90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(90°)$
$m = 6,19 \cdot 10^{14}$

$alpha =$
90°
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (10)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
180°
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (11)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
270°
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (12)

$y = sin(360°)$
$y = -6,46 \cdot 10^{-15}$
$x = cos(360°)$
$x = 1$
$m = tan(360°)$
$m = -6,46 \cdot 10^{-15}$

$alpha =$
360°
$2,16 \cdot 10^{4}$'
$1,3 \cdot 10^{6}$"
$400gon$
$6\frac{32}{113}rad$

Aufgabe (13)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$

$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
180°
$1,08 \cdot 10^{4}$’
$6,48 \cdot 10^{5}$”
$200gon$
$3\frac{16}{113}rad$

Aufgabe (14)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
270°
$1,62 \cdot 10^{4}$’
$9,72 \cdot 10^{5}$”
$300gon$
$4,71rad$

Aufgabe (15)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
180°
$1,08 \cdot 10^{4}$’
$6,48 \cdot 10^{5}$”
$200gon$
$3\frac{16}{113}rad$

Aufgabe (16)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
270°
$1,62 \cdot 10^{4}$’
$9,72 \cdot 10^{5}$”
$300gon$
$4,71rad$

Aufgabe (17)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
−90°
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (18)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
−90°
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (19)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (20)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (21)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (22)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (23)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (24)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (25)

$y = sin(90°)$
$y = 1$
$x = cos(90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(90°)$
$m = 6,19 \cdot 10^{14}$

$alpha =$
$90°$
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (26)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
$180°$
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (27)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
$270°$
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (28)

$y = sin(45°)$
$y = 0,707$
$x = cos(45°)$
$x = 0,707$
$m = tan(45°)$
$m = 1$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

2.2 $\sin\alpha = y$

2.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
y-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:$\alpha°$ $\quad 0 < \alpha < 360°$

(1) $y = 0$
(2) $y = 1$
(3) $y = -1$
(4) $y = \frac{1}{2}$
(5) $y = -\frac{1}{2}$
(6) $y = 0,866$
(7) $y = 0,707$
(8) $y = -0,866$
(9) $y = -0,707$
(10) $y = \frac{1}{5}$
(11) $y = -\frac{1}{5}$

2.2.2 Lösungen

Aufgabe (1)

$\sin\alpha = 0$
$\alpha_1 = 0°$

$alpha =$
0°
0'
0"
$0gon$
$0rad$

Aufgabe (2)

$\sin\alpha = 1$
$\alpha_1 = 90°$

$alpha =$
90°
$5,4 \cdot 10^3$'
$3,24 \cdot 10^5$"
$100gon$
$1,57rad$

Aufgabe (3)

$\sin\alpha = -1$
$\alpha_1 = 270°$

$alpha =$
90°
$5,4 \cdot 10^3$'
$3,24 \cdot 10^5$"
$100gon$
$1,57rad$

Aufgabe (4)

$\sin\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 30°$
II Quadrant: $\alpha_2 = 180° - 30° = 150°$

$alpha =$
30°
$1,8 \cdot 10^3$'
$1,08 \cdot 10^5$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (5)

$\sin\alpha = -\frac{1}{2}$
III Quadrant: $\alpha_1 = 180° + 30° = 210°$
IV Quadrant: $\alpha_2 = 360° - 30° = 330°$

$alpha =$
30°
$1,8 \cdot 10^3$'
$1,08 \cdot 10^5$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (6)

$\sin\alpha = 0,866$
I Quadrant: $\alpha_1 = 60°$
II Quadrant: $\alpha_2 = 180° - 60° = 120°$

$alpha =$
60°
$3,6 \cdot 10^3$'
$2,16 \cdot 10^5$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (7)

$\sin\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
II Quadrant: $\alpha_2 = 180° - 45° = 135°$

$alpha =$
45°
$2,7 \cdot 10^3$'
$1,62 \cdot 10^5$"
$50gon$
$0,785rad$

Aufgabe (8)

$\sin\alpha = -0,866$
III Quadrant: $\alpha_1 = 180° + 60° = 240°$
IV Quadrant: $\alpha_2 = 360° - 60° = 300°$

$alpha =$
60°
$3,6 \cdot 10^3$'
$2,16 \cdot 10^5$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (9)

$\sin\alpha = -0,707$
III Quadrant: $\alpha_1 = 180° + 45° = 225°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^3$’
$1,62 \cdot 10^5$”
$50gon$
$0,785rad$

Aufgabe (10)

$\sin\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,5°$
II Quadrant: $\alpha_2 = 180° - 11,5° = 168°$

$alpha =$
$11,5°$
692’
$4,15 \cdot 10^4$”
$12,8gon$
$0,201rad$

Aufgabe (11)

$\sin\alpha = -\frac{1}{5}$
III Quadrant: $\alpha_1 = 180° + 11,5° = 192°$
IV Quadrant: $\alpha_2 = 360° - 11,5° = 348°$

$alpha =$
$11,5°$
692’
$4,15 \cdot 10^4$”
$12,8gon$
$0,201rad$

2.3 $\cos\alpha = x$

2.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
x-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:$\alpha°$ $\quad 0 < \alpha < 360°$

(1) $x = 0$
(2) $x = 1$
(3) $x = -1$
(4) $x = \frac{1}{2}$
(5) $x = -\frac{1}{2}$
(6) $x = 0,866$
(7) $x = 0,707$
(8) $x = -0,866$
(9) $x = -0,707$
(10) $x = \frac{1}{5}$
(11) $x = -\frac{1}{5}$
(12) $x = 0,707$
(13) $x = \frac{1}{3}$
(14) $x = \frac{1}{3}$

2.3.2 Lösungen

Aufgabe (1)

$\cos\alpha = 0$

I Quadrant: $\alpha_1 = 90°$
IV Quadrant: $\alpha_2 = 360° - 90° = 270°$

$alpha =$
$90°$
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (2)

$\cos\alpha = 1$
$\alpha_1 = 0°$

$alpha =$
$0°$
0'
0"
$0gon$
$0rad$

Aufgabe (3)

$\cos\alpha = -1$
$\alpha_1 = 180°$

$alpha =$
$0°$
0'
0"
$0gon$
$0rad$

Aufgabe (4)

$\cos\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 60°$
IV Quadrant: $\alpha_2 = 360° - 60° = 300°$

$alpha =$
$60°$
$3,6 \cdot 10^{3}$'
$2,16 \cdot 10^{5}$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (5)

$\cos\alpha = -\frac{1}{2}$
II Quadrant: $\alpha_1 = 180° - 60° = 120°$
III Quadrant: $\alpha_2 = 180° + 60° = 240°$

$alpha =$
$60°$
$3,6 \cdot 10^{3}$'
$2,16 \cdot 10^{5}$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (6)

$\cos\alpha = 0,866$
I Quadrant: $\alpha_1 = 30°$
IV Quadrant: $\alpha_2 = 360° - 30° = 330°$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$'
$1,08 \cdot 10^{5}$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (7)

$\cos\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (8)

$\cos\alpha = -0,866$
II Quadrant: $\alpha_1 = 180° - 30° = 150°$
III Quadrant: $\alpha_2 = 180° + 30° = 210°$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$'
$1,08 \cdot 10^{5}$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (9)

$\cos\alpha = -0,707$
II Quadrant: $\alpha_1 = 180° - 45° = 135°$
III Quadrant: $\alpha_2 = 180° + 45° = 225°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (10)

$\cos\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 78,5°$
IV Quadrant: $\alpha_2 = 360° - 78,5° = 282°$

$alpha =$
$78,5°$
$4,71 \cdot 10^{3}$'
$2,82 \cdot 10^{5}$"
$87,2gon$
$1,37rad$

Aufgabe (11)

$\cos\alpha = -\frac{1}{5}$
II Quadrant: $\alpha_1 = 180° - 78,5° = 102°$
III Quadrant: $\alpha_2 = 180° + 78,5° = 258°$

$alpha =$
$78,5°$
$4,71 \cdot 10^{3}$'
$2,82 \cdot 10^{5}$"
$87,2gon$
$1,37rad$

Aufgabe (12)

$\cos\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (13)

$\cos\alpha = \frac{1}{3}$
I Quadrant: $\alpha_1 = 70,5°$
IV Quadrant: $\alpha_2 = 360° - 70,5° = 289°$

$alpha =$
$70,5°$
$4,23 \cdot 10^{3}$'
$2,54 \cdot 10^{5}$"
$78,4gon$
$1,23rad$

Aufgabe (14)

$\cos\alpha = \frac{1}{3}$
I Quadrant: $\alpha_1 = 70,5°$
IV Quadrant: $\alpha_2 = 360° - 70,5° = 289°$

$alpha =$
$70,5°$
$4,23 \cdot 10^{3}$'
$2,54 \cdot 10^{5}$"
$78,4gon$
$1,23rad$

2.4 $\tan\alpha = m$

2.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben: Steigung m
Gesucht:$\alpha°$ $\quad 0 < \alpha < 360°$

(1) $m = 3$
(2) $m = 2$
(3) $m = \frac{1}{2}$
(4) $m = 3$
(5) $m = -\frac{1}{5}$
(6) $m = \frac{1}{5}$
(7) $m = \frac{1}{5}$
(8) $m = 1$

2.4.2 Lösungen

Aufgabe (1)

$\tan\alpha = 3$
I Quadrant: $\alpha_1 = 71,6°$
III Quadrant: $\alpha_2 = 180° + 71,6° = 252°$

$alpha =$
$71,6°$
$4,29 \cdot 10^3$'
$2,58 \cdot 10^5$"
$79,5gon$
$1,25rad$

Aufgabe (2)

$\tan\alpha = 2$
I Quadrant: $\alpha_1 = 63,4°$
III Quadrant: $\alpha_2 = 180° + 63,4° = 243°$

$alpha =$
$63,4°$
$3,81 \cdot 10^3$'
$2,28 \cdot 10^5$"
$70,5gon$
$1,11rad$

Aufgabe (3)

$\tan\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 26,6°$
III Quadrant: $\alpha_2 = 180° + 26,6° = 207°$

$alpha =$
$26,6°$
$1,59 \cdot 10^3$'
$9,56 \cdot 10^4$"
$29,5gon$
$0,464rad$

Aufgabe (4)

$\tan\alpha = 3$
I Quadrant: $\alpha_1 = 71,6°$
III Quadrant: $\alpha_2 = 180° + 71,6° = 252°$

$alpha =$
$71,6°$
$4,29 \cdot 10^3$'
$2,58 \cdot 10^5$"
$79,5gon$
$1,25rad$

Aufgabe (5)

$\tan\alpha = -\frac{1}{5}$
II Quadrant: $\alpha_1 = 180° - 11,3° = 169°$
IV Quadrant: $\alpha_2 = 360° - 11,3° = 349°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^4$"
$12,6gon$
$0,197rad$

Aufgabe (6)

$\tan\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,3°$
III Quadrant: $\alpha_2 = 180° + 11,3° = 191°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^4$"
$12,6gon$
$0,197rad$

Aufgabe (7)

$\tan\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,3°$
III Quadrant: $\alpha_2 = 180° + 11,3° = 191°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^4$"
$12,6gon$
$0,197rad$

Aufgabe (8)

$\tan\alpha = 1$
I Quadrant: $\alpha_1 = 45°$
III Quadrant: $\alpha_2 = 180° + 45° = 225°$

$alpha =$
$45°$
$2,7 \cdot 10^3$'
$1,62 \cdot 10^5$"
$50gon$
$0,785rad$

3 Quadrantenregel

α in Gradmaß

I. Quadrant $0° < \alpha < 90°$

$\sin(\alpha) > 0 \quad \cos(\alpha) > 0 \quad \tan(\alpha) > 0$

II. Quadrant $90° < \alpha_2 < 180°$

$\sin(\alpha_2) > 0 \quad \cos(\alpha_2) < 0 \quad \tan(\alpha_2) < 0$

$\alpha_2 = 180° - \alpha$

$sin(180° - \alpha) = sin(\alpha)$

$cos(180° - \alpha) = -cos(\alpha)$

$tan(180° - \alpha) = -tan(\alpha)$

III. Quadrant $180° < \alpha_3 < 270°$

$\sin(\alpha_3) < 0 \quad \cos(\alpha_3) < 0 \quad \tan(\alpha_3) > 0$

$\alpha_3 = 180° + \alpha$

$sin(180° + \alpha) = -sin(\alpha)$

$cos(180° + \alpha) = -cos(\alpha)$

$tan(180° + \alpha) = tan(\alpha)$

IV. Quadrant $270° < \alpha_4 < 360°$

$\sin(\alpha_4) < 0 \quad \cos(\alpha_4) > 0 \quad \tan(\alpha_4) < 0$

$\alpha_4 = 360° - \alpha$

$sin(360° - \alpha) = -sin(\alpha)$

$cos(360° - \alpha) = cos(\alpha)$

$tan(360° - \alpha) = -tan(\alpha)$

$\sin\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 30°$
II Quadrant: $\alpha_2 = 180° - 30° = 150°$
$\sin\alpha = -\frac{1}{2}$
III Quadrant: $\alpha_1 = 180° + 30° = 210°$
IV Quadrant: $\alpha_2 = 360° - 30° = 330°$
$\cos\alpha = \frac{1}{2}\sqrt{2}$
I Quadrant: $\alpha_1 = 45°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$
$\cos\alpha = -\frac{1}{2}\sqrt{2}$
II Quadrant: $\alpha_1 = 180° - 45° = 135°$
III Quadrant: $\alpha_2 = 180° + 45° = 225°$

x in Bogenmaß

I. Quadrant $0 < x < \frac{\pi}{2}$

$\sin(x) > 0 \quad \cos(x) > 0 \quad \tan(x) > 0$

II. Quadrant $\frac{\pi}{2} < x_2 < \pi$

$\sin(x_2) > 0 \quad \cos(x_2) < 0 \quad \tan(x_2) < 0$

$x_2 = \pi - x$

$sin(\pi - x) = sin(x)$

$cos(\pi - x) = -cos(x)$

$tan(\pi - x) = -tan(x)$

III. Quadrant $\pi < x_3 < \frac{3\pi}{2}$

$\sin(x_3) < 0 \quad \cos(x_3) < 0 \quad \tan(x_3) > 0$

$x_3 = \pi + x$

$sin(\pi + x) = -sin(x)$

$cos(\pi + x) = -cos(x)$

$tan(\pi + x) = tan(x)$

IV. Quadrant $\frac{3\pi}{2} < x_4 < 2\pi$

$\sin(x_4) < 0 \quad \cos(x_4) > 0 \quad \tan(x_4) < 0$

$x_4 = 2\pi - x$

$sin(2\pi - x) = -sin(x)$

$cos(2\pi - x) = cos(x)$

$tan(2\pi - x) = -tan(x)$

3.1 $\sin\alpha - \cos\alpha - \tan\alpha$

3.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
y-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:
Winkel im Einheitskreis α [°]

(1) $\alpha = 45°$
(2) $\alpha = 135°$
(3) $\alpha = 225°$
(4) $\alpha = 315°$
(5) $\alpha = 30°$
(6) $\alpha = 150°$
(7) $\alpha = 210°$
(8) $\alpha = 330°$
(9) $\alpha = 90°$
(10) $\alpha = 180°$
(11) $\alpha = 270°$
(12) $\alpha = 360°$
(13) $\alpha = 180°$
(14) $\alpha = 270°$
(15) $\alpha = 180°$
(16) $\alpha = 270°$
(17) $\alpha = -90°$
(18) $\alpha = -90°$
(19) $\alpha = -90°$
(20) $\alpha = -90°$
(21) $\alpha = -90°$
(22) $\alpha = -90°$
(23) $\alpha = -90°$
(24) $\alpha = -90°$
(25) $\alpha = 90°$
(26) $\alpha = 180°$
(27) $\alpha = 270°$
(28) $\alpha = 45°$

3.1.2 Lösungen

Aufgabe (1)

$y = sin(45°)$
$y = 0,707$
$x = cos(45°)$
$x = 0,707$
$m = tan(45°)$
$m = 1$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50 gon$
$0,785 rad$

Aufgabe (2)

$y = sin(135°)$
$y = 0,707$
$x = cos(135°)$
$x = -0,707$
$m = tan(135°)$
$m = -1$

$alpha =$
$135°$
$8,1 \cdot 10^{3}$'
$4,86 \cdot 10^{5}$"
$150 gon$
$2,36 rad$

Aufgabe (3)

$y = sin(225°)$
$y = -0,707$
$x = cos(225°)$
$x = -0,707$
$m = tan(225°)$
$m = 1$

$alpha =$
$225°$
$1,35 \cdot 10^{4}$'
$8,1 \cdot 10^{5}$"
$250 gon$
$3,93 rad$

Aufgabe (4)

$y = sin(315°)$
$y = -0,707$
$x = cos(315°)$
$x = 0,707$
$m = tan(315°)$
$m = -1$

$alpha =$
$315°$
$1,89 \cdot 10^{4}$'
$1,13 \cdot 10^{6}$"
$350 gon$
$5,5 rad$

Aufgabe (5)

$y = sin(30°)$
$y = \frac{1}{2}$
$x = cos(30°)$
$x = 0,866$
$m = tan(30°)$
$m = 0,577$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$'
$1,08 \cdot 10^{5}$"
$33\frac{1}{3} gon$
$0,524 rad$

Aufgabe (6)

$y = sin(150°)$
$y = \frac{1}{2}$
$x = cos(150°)$
$x = -0,866$
$m = tan(150°)$
$m = -0,577$

$alpha =$
$150°$
$9 \cdot 10^{3}$'
$5,4 \cdot 10^{5}$"
$166\frac{2}{3} gon$
$2,62 rad$

Aufgabe (7)

$y = sin(210°)$
$y = -\frac{1}{2}$
$x = cos(210°)$
$x = -0,866$
$m = tan(210°)$
$m = 0,577$

$alpha =$
$210°$
$1,26 \cdot 10^{4}$'
$7,56 \cdot 10^{5}$"
$233\frac{1}{3}gon$
$3,67rad$

Aufgabe (8)

$y = sin(330°)$
$y = -\frac{1}{2}$
$x = cos(330°)$
$x = 0,866$
$m = tan(330°)$
$m = -0,577$

$alpha =$
$330°$
$1,98 \cdot 10^{4}$'
$1,19 \cdot 10^{6}$"
$366\frac{2}{3}gon$
$5,76rad$

Aufgabe (9)

$y = sin(90°)$
$y = 1$
$x = cos(90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(90°)$
$m = 6,19 \cdot 10^{14}$

$alpha =$
$90°$
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (10)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
$180°$
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (11)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
$270°$
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (12)

$y = sin(360°)$
$y = -6,46 \cdot 10^{-15}$
$x = cos(360°)$
$x = 1$
$m = tan(360°)$
$m = -6,46 \cdot 10^{-15}$

$alpha =$
$360°$
$2,16 \cdot 10^{4}$'
$1,3 \cdot 10^{6}$"
$400gon$
$6\frac{32}{113}rad$

Aufgabe (13)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$

$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
$180°$
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (14)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
$270°$
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (15)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
$180°$
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (16)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
$270°$
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (17)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$'
$-3,24 \cdot 10^{5}$"
$-100gon$
$-1,57rad$

Aufgabe (18)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$'
$-3,24 \cdot 10^{5}$"
$-100gon$
$-1,57rad$

Aufgabe (19)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (20)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (21)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (22)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (23)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (24)

$y = sin(-90°)$
$y = -1$
$x = cos(-90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(-90°)$
$m = -6,19 \cdot 10^{14}$

$alpha =$
$-90°$
$-5,4 \cdot 10^{3}$’
$-3,24 \cdot 10^{5}$”
$-100gon$
$-1,57rad$

Aufgabe (25)

$y = sin(90°)$
$y = 1$
$x = cos(90°)$
$x = 1,62 \cdot 10^{-15}$
$m = tan(90°)$
$m = 6,19 \cdot 10^{14}$

$alpha =$
$90°$
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (26)

$y = sin(180°)$
$y = 3,23 \cdot 10^{-15}$
$x = cos(180°)$
$x = -1$
$m = tan(180°)$
$m = -3,23 \cdot 10^{-15}$

$alpha =$
$180°$
$1,08 \cdot 10^{4}$'
$6,48 \cdot 10^{5}$"
$200gon$
$3\frac{16}{113}rad$

Aufgabe (27)

$y = sin(270°)$
$y = -1$
$x = cos(270°)$
$x = -4,62 \cdot 10^{-15}$
$m = tan(270°)$
$m = 2,16 \cdot 10^{14}$

$alpha =$
$270°$
$1,62 \cdot 10^{4}$'
$9,72 \cdot 10^{5}$"
$300gon$
$4,71rad$

Aufgabe (28)

$y = sin(45°)$
$y = 0,707$
$x = cos(45°)$
$x = 0,707$
$m = tan(45°)$
$m = 1$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

3.2 $\sin \alpha = y$

3.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
y-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:$\alpha°$ $0 < \alpha < 360°$

(1) $y = 0$
(2) $y = 1$
(3) $y = -1$
(4) $y = \frac{1}{2}$
(5) $y = -\frac{1}{2}$
(6) $y = 0,866$
(7) $y = 0,707$
(8) $y = -0,866$
(9) $y = -0,707$
(10) $y = \frac{1}{5}$
(11) $y = -\frac{1}{5}$

3.2.2 Lösungen

Aufgabe (1)

$\sin\alpha = 0$
$\alpha_1 = 0°$

$alpha =$
0°
0’
0”
$0gon$
$0rad$

Aufgabe (2)

$\sin\alpha = 1$
$\alpha_1 = 90°$

$alpha =$
90°
$5,4 \cdot 10^{3}$’
$3,24 \cdot 10^{5}$”
$100gon$
$1,57rad$

Aufgabe (3)

$\sin\alpha = -1$
$\alpha_1 = 270°$

$alpha =$
90°
$5,4 \cdot 10^{3}$’
$3,24 \cdot 10^{5}$”
$100gon$
$1,57rad$

Aufgabe (4)

$\sin\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 30°$
II Quadrant: $\alpha_2 = 180° - 30° = 150°$

$alpha =$
30°
$1,8 \cdot 10^{3}$’
$1,08 \cdot 10^{5}$”
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (5)

$\sin\alpha = -\frac{1}{2}$
III Quadrant: $\alpha_1 = 180° + 30° = 210°$
IV Quadrant: $\alpha_2 = 360° - 30° = 330°$

$alpha =$
30°
$1,8 \cdot 10^{3}$’
$1,08 \cdot 10^{5}$”
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (6)

$\sin\alpha = 0,866$
I Quadrant: $\alpha_1 = 60°$
II Quadrant: $\alpha_2 = 180° - 60° = 120°$

$alpha =$
60°
$3,6 \cdot 10^{3}$’
$2,16 \cdot 10^{5}$”
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (7)

$\sin\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
II Quadrant: $\alpha_2 = 180° - 45° = 135°$

$alpha =$
45°
$2,7 \cdot 10^{3}$’
$1,62 \cdot 10^{5}$”
$50gon$
$0,785rad$

Aufgabe (8)

$\sin\alpha = -0,866$
III Quadrant: $\alpha_1 = 180° + 60° = 240°$
IV Quadrant: $\alpha_2 = 360° - 60° = 300°$

$alpha =$
60°
$3,6 \cdot 10^{3}$’
$2,16 \cdot 10^{5}$”
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (9)

$\sin\alpha = -0,707$
III Quadrant: $\alpha_1 = 180° + 45° = 225°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$’
$1,62 \cdot 10^{5}$”
$50gon$
$0,785rad$

Aufgabe (10)

$\sin\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,5°$
II Quadrant: $\alpha_2 = 180° - 11,5° = 168°$

$alpha =$
$11,5°$
692’
$4,15 \cdot 10^{4}$”
$12,8gon$
$0,201rad$

Aufgabe (11)

$\sin\alpha = -\frac{1}{5}$
III Quadrant: $\alpha_1 = 180° + 11,5° = 192°$
IV Quadrant: $\alpha_2 = 360° - 11,5° = 348°$

$alpha =$
$11,5°$
692’
$4,15 \cdot 10^{4}$”
$12,8gon$
$0,201rad$

3.3 $\cos\alpha = x$

3.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
x-Wert des Punktes P(x;y) auf dem Einheitskreis
Gesucht:$\alpha°$ $0 < \alpha < 360°$

(1) $x = 0$
(2) $x = 1$
(3) $x = -1$
(4) $x = \frac{1}{2}$
(5) $x = -\frac{1}{2}$
(6) $x = 0,866$
(7) $x = 0,707$
(8) $x = -0,866$
(9) $x = -0,707$
(10) $x = \frac{1}{5}$
(11) $x = -\frac{1}{5}$
(12) $x = 0,707$
(13) $x = \frac{1}{3}$
(14) $x = \frac{1}{3}$

3.3.2 Lösungen

Aufgabe (1)

$\cos\alpha = 0$

I Quadrant: $\alpha_1 = 90°$
IV Quadrant: $\alpha_2 = 360° - 90° = 270°$

$alpha =$
$90°$
$5,4 \cdot 10^{3}$'
$3,24 \cdot 10^{5}$"
$100gon$
$1,57rad$

Aufgabe (2)

$\cos\alpha = 1$
$\alpha_1 = 0°$

$alpha =$
$0°$
0'
0"
$0gon$
$0rad$

Aufgabe (3)

$\cos\alpha = -1$
$\alpha_1 = 180°$

$alpha =$
$0°$
0'
0"
$0gon$
$0rad$

Aufgabe (4)

$\cos\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 60°$
IV Quadrant: $\alpha_2 = 360° - 60° = 300°$

$alpha =$
$60°$
$3,6 \cdot 10^{3}$'
$2,16 \cdot 10^{5}$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (5)

$\cos\alpha = -\frac{1}{2}$
II Quadrant: $\alpha_1 = 180° - 60° = 120°$
III Quadrant: $\alpha_2 = 180° + 60° = 240°$

$alpha =$
$60°$
$3,6 \cdot 10^{3}$'
$2,16 \cdot 10^{5}$"
$66\frac{2}{3}gon$
$1,05rad$

Aufgabe (6)

$\cos\alpha = 0,866$
I Quadrant: $\alpha_1 = 30°$
IV Quadrant: $\alpha_2 = 360° - 30° = 330°$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$'
$1,08 \cdot 10^{5}$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (7)

$\cos\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (8)

$\cos\alpha = -0,866$
II Quadrant: $\alpha_1 = 180° - 30° = 150°$
III Quadrant: $\alpha_2 = 180° + 30° = 210°$

$alpha =$
$30°$
$1,8 \cdot 10^{3}$'
$1,08 \cdot 10^{5}$"
$33\frac{1}{3}gon$
$0,524rad$

Aufgabe (9)

$\cos\alpha = -0,707$
II Quadrant: $\alpha_1 = 180° - 45° = 135°$
III Quadrant: $\alpha_2 = 180° + 45° = 225°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (10)

$\cos\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 78,5°$
IV Quadrant: $\alpha_2 = 360° - 78,5° = 282°$

$alpha =$
$78,5°$
$4,71 \cdot 10^{3}$'
$2,82 \cdot 10^{5}$"
$87,2gon$
$1,37rad$

Aufgabe (11)

$\cos\alpha = -\frac{1}{5}$
II Quadrant: $\alpha_1 = 180° - 78,5° = 102°$
III Quadrant: $\alpha_2 = 180° + 78,5° = 258°$

$alpha =$
$78,5°$
$4,71 \cdot 10^{3}$'
$2,82 \cdot 10^{5}$"
$87,2gon$
$1,37rad$

Aufgabe (12)

$\cos\alpha = 0,707$
I Quadrant: $\alpha_1 = 45°$
IV Quadrant: $\alpha_2 = 360° - 45° = 315°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50gon$
$0,785rad$

Aufgabe (13)

$\cos\alpha = \frac{1}{3}$
I Quadrant: $\alpha_1 = 70,5°$
IV Quadrant: $\alpha_2 = 360° - 70,5° = 289°$

$alpha =$
$70,5°$
$4,23 \cdot 10^{3}$'
$2,54 \cdot 10^{5}$"
$78,4gon$
$1,23rad$

Aufgabe (14)

$\cos\alpha = \frac{1}{3}$
I Quadrant: $\alpha_1 = 70,5°$
IV Quadrant: $\alpha_2 = 360° - 70,5° = 289°$

$alpha =$
$70,5°$
$4,23 \cdot 10^{3}$'
$2,54 \cdot 10^{5}$"
$78,4gon$
$1,23rad$

3.4 $\tan\alpha = m$

3.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben: Steigung m
Gesucht:$\alpha°$ $\quad 0 < \alpha < 360°$

(1) $m = 3$
(2) $m = 2$
(3) $m = \frac{1}{2}$
(4) $m = 3$
(5) $m = -\frac{1}{5}$
(6) $m = \frac{1}{5}$
(7) $m = \frac{1}{5}$
(8) $m = 1$

3.4.2 Lösungen

Aufgabe (1)

$\tan\alpha = 3$
I Quadrant: $\alpha_1 = 71,6°$
III Quadrant: $\alpha_2 = 180° + 71,6° = 252°$

$alpha =$
$71,6°$
$4,29 \cdot 10^{3}$'
$2,58 \cdot 10^{5}$"
$79,5 gon$
$1,25 rad$

Aufgabe (2)

$\tan\alpha = 2$
I Quadrant: $\alpha_1 = 63,4°$
III Quadrant: $\alpha_2 = 180° + 63,4° = 243°$

$alpha =$
$63,4°$
$3,81 \cdot 10^{3}$'
$2,28 \cdot 10^{5}$"
$70,5 gon$
$1,11 rad$

Aufgabe (3)

$\tan\alpha = \frac{1}{2}$
I Quadrant: $\alpha_1 = 26,6°$
III Quadrant: $\alpha_2 = 180° + 26,6° = 207°$

$alpha =$
$26,6°$
$1,59 \cdot 10^{3}$'
$9,56 \cdot 10^{4}$"
$29,5 gon$
$0,464 rad$

Aufgabe (4)

$\tan\alpha = 3$
I Quadrant: $\alpha_1 = 71,6°$
III Quadrant: $\alpha_2 = 180° + 71,6° = 252°$

$alpha =$
$71,6°$
$4,29 \cdot 10^{3}$'
$2,58 \cdot 10^{5}$"
$79,5 gon$
$1,25 rad$

Aufgabe (5)

$\tan\alpha = -\frac{1}{5}$
II Quadrant: $\alpha_1 = 180° - 11,3° = 169°$
IV Quadrant: $\alpha_2 = 360° - 11,3° = 349°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^{4}$"
$12,6 gon$
$0,197 rad$

Aufgabe (6)

$\tan\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,3°$
III Quadrant: $\alpha_2 = 180° + 11,3° = 191°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^{4}$"
$12,6 gon$
$0,197 rad$

Aufgabe (7)

$\tan\alpha = \frac{1}{5}$
I Quadrant: $\alpha_1 = 11,3°$
III Quadrant: $\alpha_2 = 180° + 11,3° = 191°$

$alpha =$
$11,3°$
679'
$4,07 \cdot 10^{4}$"
$12,6 gon$
$0,197 rad$

Aufgabe (8)

$\tan\alpha = 1$
I Quadrant: $\alpha_1 = 45°$
III Quadrant: $\alpha_2 = 180° + 45° = 225°$

$alpha =$
$45°$
$2,7 \cdot 10^{3}$'
$1,62 \cdot 10^{5}$"
$50 gon$
$0,785 rad$

4 Umrechnungen

tan - sin - cos

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\sin\alpha = \tan\alpha \cdot \cos\alpha$

$\cos\alpha = \frac{\sin\alpha}{\tan\alpha}$

sin - cos

$sin^2\alpha + cos^2\alpha = 1$

$sin\alpha = \sqrt{1 - cos^2\alpha}$

$cos\alpha = \sqrt{1 - sin^2\alpha}$

Additionstheoreme

$sin(\alpha + \beta) = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta$

$sin(\alpha - \beta) = sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta$

$cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta$

$cos(\alpha - \beta) = cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta$

$tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha \cdot tan\beta}$

$tan(\alpha - \beta) = \frac{tan\alpha - tan\beta}{1 + tan\alpha \cdot tan\beta}$

$sin2\alpha = 2 \cdot sin\alpha \cdot cos\alpha$

$cos2\alpha = 2 \cdot cos^2\alpha - 1 = cos^2\alpha - sin^2\alpha$

$tan2\alpha = \frac{2 \cdot tan\alpha}{1 - tan^2\alpha}$

4.1 $sin\alpha = \sqrt{1 - cos^2\alpha}$

4.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α [°]

Gesucht:
Sinus alpha $sin\alpha$ []

(1) $\alpha = 30°$
(2) $\alpha = 60°$
(3) $\alpha = 45°$
(4) $\alpha = 90°$
(5) $\alpha = 120°$

4.1.2 Lösungen

Aufgabe (1)

$sin\alpha = \sqrt{1 - cos^2\alpha}$
$\alpha = 30°$
$sin30° = \sqrt{1 - cos^2 30°}$

$sin\alpha = \frac{1}{2}$

$alpha =$	$sinalpha =$
$30°$	$\frac{1}{2}rad$
$1,8 \cdot 10^{3}$'	$500mrad$
$1,08 \cdot 10^{5}$"	$28,6°$
$33\frac{1}{3}gon$	$1,72 \cdot 10^{3}$'
$0,524rad$	$1,03 \cdot 10^{5}$"'

Aufgabe (2)

$sin\alpha = \sqrt{1 - cos^2\alpha}$
$\alpha = 60°$
$sin60° = \sqrt{1 - cos^2 60°}$

$sin\alpha = 0,866$

$alpha =$	$sinalpha =$
$60°$	$0,866rad$
$3,6 \cdot 10^{3}$'	$866mrad$
$2,16 \cdot 10^{5}$"	$49,6°$
$66\frac{2}{3}gon$	$2,98 \cdot 10^{3}$'
$1,05rad$	$1,79 \cdot 10^{5}$"'

Aufgabe (3)

$sin\alpha = \sqrt{1 - cos^2\alpha}$
$\alpha = 45°$
$sin45° = \sqrt{1 - cos^2 45°}$

$sin\alpha = 0,707$

$alpha =$	$sinalpha =$
$45°$	$0,707rad$
$2,7 \cdot 10^{3}$'	$707mrad$
$1,62 \cdot 10^{5}$"	$40,5°$
$50gon$	$2,43 \cdot 10^{3}$'
$0,785rad$	$1,46 \cdot 10^{5}$"'

Aufgabe (4)

$sin\alpha = \sqrt{1 - cos^2\alpha}$
$\alpha = 90°$
$sin90° = \sqrt{1 - cos^2 90°}$

$sin\alpha = 1$

$alpha =$	$sinalpha =$
$90°$	$1rad$
$5,4 \cdot 10^{3}$'	$10^{3}mrad$
$3,24 \cdot 10^{5}$"	$57,3°$
$100gon$	$3,44 \cdot 10^{3}$'
$1,57rad$	$2,06 \cdot 10^{5}$"'

Aufgabe (5)

$sin\alpha = \sqrt{1 - cos^2\alpha}$
$\alpha = 120°$
$sin120° = \sqrt{1 - cos^2 120°}$

$sin\alpha = 0,866$

$alpha =$	$sinalpha =$
$120°$	$0,866rad$
$7,2 \cdot 10^{3}$'	$866mrad$
$4,32 \cdot 10^{5}$"	$49,6°$
$133\frac{1}{3}gon$	$2,98 \cdot 10^{3}$'
$2,09rad$	$1,79 \cdot 10^{5}$"'

4.2 $cos\alpha = \sqrt{1 - sin^2\alpha}$

4.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α [°]

Gesucht:
Kosinus alpha $cos\alpha$ []

(1) $\alpha = 30°$
(2) $\alpha = 60°$
(3) $\alpha = 45°$
(4) $\alpha = 90°$
(5) $\alpha = 120°$

4.2.2 Lösungen

Aufgabe (1)

$cos\alpha = \sqrt{1 - sin^2\alpha}$
$\alpha = 30°$
$cos\alpha = \sqrt{1 - sin^2 30°}$

$cos\alpha = 0,866$

$alpha =$	$cosalpha =$
$30°$	$0,866rad$
$1,8 \cdot 10^{3}$'	$866mrad$
$1,08 \cdot 10^{5}$"	$49,6°$
$33\frac{1}{3}gon$	$2,98 \cdot 10^{3}$'
$0,524rad$	$1,79 \cdot 10^{5}$"'

Aufgabe (2)

$cos\alpha = \sqrt{1 - sin^2\alpha}$
$\alpha = 60°$
$cos\alpha = \sqrt{1 - sin^2 60°}$

$cos\alpha = \frac{1}{2}$

$alpha =$	$cosalpha =$
$60°$	$\frac{1}{2}rad$
$3,6 \cdot 10^{3}$'	$500mrad$
$2,16 \cdot 10^{5}$"	$28,6°$
$66\frac{2}{3}gon$	$1,72 \cdot 10^{3}$'
$1,05rad$	$1,03 \cdot 10^{5}$"'

Aufgabe (3)

$cos\alpha = \sqrt{1 - sin^2\alpha}$
$\alpha = 45°$
$cos\alpha = \sqrt{1 - sin^2 45°}$

$cos\alpha = 0,707$

$alpha =$	$cosalpha =$
$45°$	$0,707rad$
$2,7 \cdot 10^{3}$'	$707mrad$
$1,62 \cdot 10^{5}$"	$40,5°$
$50gon$	$2,43 \cdot 10^{3}$'
$0,785rad$	$1,46 \cdot 10^{5}$"'

Aufgabe (4)

$cos\alpha = \sqrt{1 - sin^2\alpha}$
$\alpha = 90°$
$cos\alpha = \sqrt{1 - sin^2 90°}$

$cos\alpha = 0$

$alpha =$	$cosalpha =$
$90°$	$0rad$
$5,4 \cdot 10^{3}$'	$0mrad$
$3,24 \cdot 10^{5}$"	$0°$
$100gon$	0'
$1,57rad$	0"'

Aufgabe (5)

$cos\alpha = \sqrt{1 - sin^2\alpha}$
$\alpha = 120°$
$cos\alpha = \sqrt{1 - sin^2 120°}$

$cos\alpha = \frac{1}{2}$

$alpha =$	$cosalpha =$
$120°$	$\frac{1}{2}rad$
$7,2 \cdot 10^{3}$'	$500mrad$
$4,32 \cdot 10^{5}$"	$28,6°$
$133\frac{1}{3}gon$	$1,72 \cdot 10^{3}$'
$2,09rad$	$1,03 \cdot 10^{5}$"'

4.3 $tan\alpha = \frac{sin\alpha}{cos\alpha}$

4.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α [°]

Gesucht:
Tangens alpha $tan\alpha$ []

(1) $\alpha = 5°$
(2) $\alpha = 20°$
(3) $\alpha = 30°$
(4) $\alpha = 45°$
(5) $\alpha = 60°$

4.3.2 Lösungen

Aufgabe (1)

$tan\alpha = \frac{sin\alpha}{cos\alpha}$
$\alpha = 5°$
$tan\alpha = \frac{sin5°}{cos5°}$

$tan\alpha = 0,0875$

$alpha =$	$Tanalpha =$
5°	$0,0875rad$
300'	$87,5mrad$
$1,8 \cdot 10^4$"	$5,01°$
$5\frac{5}{9}gon$	301'
$0,0873rad$	$1,8 \cdot 10^4$"'

Aufgabe (2)

$tan\alpha = \frac{sin\alpha}{cos\alpha}$
$\alpha = 20°$
$tan\alpha = \frac{sin20°}{cos20°}$

$tan\alpha = 0,364$

$alpha =$	$Tanalpha =$
20°	$0,364rad$
$1,2 \cdot 10^3$'	$364mrad$
$7,2 \cdot 10^4$"	$20,9°$
$22\frac{2}{9}gon$	$1,25 \cdot 10^3$'
$0,349rad$	$7,51 \cdot 10^4$"'

Aufgabe (3)

$tan\alpha = \frac{sin\alpha}{cos\alpha}$
$\alpha = 30°$
$tan\alpha = \frac{sin30°}{cos30°}$

$tan\alpha = 0,577$

$alpha =$	$Tanalpha =$
30°	$0,577rad$
$1,8 \cdot 10^3$'	$577mrad$
$1,08 \cdot 10^5$"	$33,1°$
$33\frac{1}{3}gon$	$1,98 \cdot 10^3$'
$0,524rad$	$1,19 \cdot 10^5$"'

Aufgabe (4)

$tan\alpha = \frac{sin\alpha}{cos\alpha}$
$\alpha = 45°$
$tan\alpha = \frac{sin45°}{cos45°}$

$tan\alpha = 1$

$alpha =$	$Tanalpha =$
45°	$1rad$
$2,7 \cdot 10^3$'	10^3mrad
$1,62 \cdot 10^5$"	$57,3°$
$50gon$	$3,44 \cdot 10^3$'
$0,785rad$	$2,06 \cdot 10^5$"'

Aufgabe (5)

$tan\alpha = \frac{sin\alpha}{cos\alpha}$
$\alpha = 60°$
$tan\alpha = \frac{sin60°}{cos60°}$

$tan\alpha = 1,73$

$alpha =$	$Tanalpha =$
60°	$1,73rad$
$3,6 \cdot 10^3$'	$1,73 \cdot 10^3mrad$
$2,16 \cdot 10^5$"	$99,2°$
$66\frac{2}{3}gon$	$5,95 \cdot 10^3$'
$1,05rad$	$3,57 \cdot 10^5$"'

4.4 $sin\alpha = tan\alpha \cdot cos\alpha$

4.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α [°]

Gesucht:
Sinus alpha $sin\alpha$ []

(1) $\alpha = 15°$
(2) $\alpha = 30°$
(3) $\alpha = 60°$
(4) $\alpha = 45°$

4.4.2 Lösungen

Aufgabe (1)

$sin\alpha = tan\alpha \cdot cos\alpha$
$\alpha = 15°$
$sin\alpha = tan15° \cdot cos15°$

$sin\alpha = 0,259$

$alpha =$	$sinalpha =$
$15°$	$0,259rad$
900'	$259mrad$
$5,4 \cdot 10^{4}$"	$14,8°$
$16\frac{2}{3}gon$	890'
$0,262rad$	$5,34 \cdot 10^{4}$"'

Aufgabe (2)

$sin\alpha = tan\alpha \cdot cos\alpha$
$\alpha = 30°$
$sin\alpha = tan30° \cdot cos30°$

$sin\alpha = \frac{1}{2}$

$alpha =$	$sinalpha =$
$30°$	$\frac{1}{2}rad$
$1,8 \cdot 10^{3}$'	$500mrad$
$1,08 \cdot 10^{5}$"	$28,6°$
$33\frac{1}{3}gon$	$1,72 \cdot 10^{3}$'
$0,524rad$	$1,03 \cdot 10^{5}$"'

Aufgabe (3)

$sin\alpha = tan\alpha \cdot cos\alpha$
$\alpha = 60°$
$sin\alpha = tan60° \cdot cos60°$

$sin\alpha = 0,866$

$alpha =$	$sinalpha =$
$60°$	$0,866rad$
$3,6 \cdot 10^{3}$'	$866mrad$
$2,16 \cdot 10^{5}$"	$49,6°$
$66\frac{2}{3}gon$	$2,98 \cdot 10^{3}$'
$1,05rad$	$1,79 \cdot 10^{5}$"'

Aufgabe (4)

$sin\alpha = tan\alpha \cdot cos\alpha$
$\alpha = 45°$
$sin\alpha = tan45° \cdot cos45°$

$sin\alpha = 0,707$

$alpha =$	$sinalpha =$
$45°$	$0,707rad$
$2,7 \cdot 10^{3}$'	$707mrad$
$1,62 \cdot 10^{5}$"	$40,5°$
$50gon$	$2,43 \cdot 10^{3}$'
$0,785rad$	$1,46 \cdot 10^{5}$"'

4.5 $cos\alpha = \frac{sin\alpha}{tan\alpha}$

4.5.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α [°]

Gesucht:
Kosinus alpha $cos\alpha$ []

(1) $\alpha = 15°$
(2) $\alpha = 30°$
(3) $\alpha = 60°$
(4) $\alpha = 45°$

4.5.2 Lösungen

Aufgabe (1)

$cos\alpha = \frac{sin\alpha}{tan\alpha}$
$\alpha = 15°$
$cos\alpha = \frac{sin15°}{tan15°}$

$cos\alpha = 0,966$

$alpha =$	$cosalpha =$
15°	$0,966rad$
900'	$966mrad$
$5,4 \cdot 10^{4}$"	$55,3°$
$16\frac{2}{3}gon$	$3,32 \cdot 10^{3}$'
$0,262rad$	$1,99 \cdot 10^{5}$"'

Aufgabe (2)

$cos\alpha = \frac{sin\alpha}{tan\alpha}$
$\alpha = 30°$
$cos\alpha = \frac{sin30°}{tan30°}$

$cos\alpha = 0,866$

$alpha =$	$cosalpha =$
30°	$0,866rad$
$1,8 \cdot 10^{3}$'	$866mrad$
$1,08 \cdot 10^{5}$"	$49,6°$
$33\frac{1}{3}gon$	$2,98 \cdot 10^{3}$'
$0,524rad$	$1,79 \cdot 10^{5}$"'

Aufgabe (3)

$cos\alpha = \frac{sin\alpha}{tan\alpha}$
$\alpha = 60°$
$cos\alpha = \frac{sin60°}{tan60°}$

$cos\alpha = \frac{1}{2}$

$alpha =$	$cosalpha =$
60°	$\frac{1}{2}rad$
$3,6 \cdot 10^{3}$'	$500mrad$
$2,16 \cdot 10^{5}$"	$28,6°$
$66\frac{2}{3}gon$	$1,72 \cdot 10^{3}$'
$1,05rad$	$1,03 \cdot 10^{5}$"'

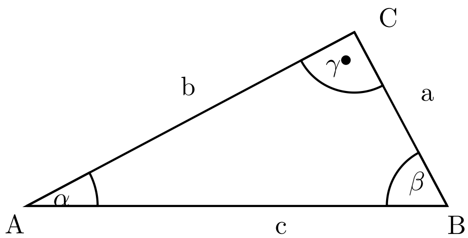
Aufgabe (4)

$cos\alpha = \frac{sin\alpha}{tan\alpha}$
$\alpha = 45°$
$cos\alpha = \frac{sin45°}{tan45°}$

$cos\alpha = 0,707$

$alpha =$	$cosalpha =$
45°	$0,707rad$
$2,7 \cdot 10^{3}$'	$707mrad$
$1,62 \cdot 10^{5}$"	$40,5°$
$50gon$	$2,43 \cdot 10^{3}$'
$0,785rad$	$1,46 \cdot 10^{5}$"'

5 Rechtwinkliges Dreieck



$sin\alpha = \frac{a}{c} \qquad sin\alpha = \frac{\text{Gegenkathete}}{\text{Hypotenuse}}$

c	Hypotenuse	m
a	Gegenkathete zu α	m
α	Winkel	$^\circ$

$a = sin\alpha \cdot c \qquad c = \frac{a}{sin\alpha}$

$cos\alpha = \frac{b}{c} \qquad cos\alpha = \frac{\text{Ankathete}}{\text{Hypotenuse}}$

c	Hypotenuse	m
b	Ankathete zu α	m
α	Winkel	$^\circ$

$b = cos\alpha \cdot c \qquad c = \frac{b}{cos\alpha}$

$tan\alpha = \frac{a}{b} \qquad tan\alpha = \frac{\text{Gegenkathete}}{\text{Ankathete}}$

b	Ankathete zu α	m
a	Gegenkathete zu α	m
α	Winkel	$^\circ$

$a = tan\alpha \cdot b \qquad b = \frac{a}{tan\alpha}$

5.1 $sin\alpha = \frac{a}{c}$

5.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Hypotenuse c $[m]$
Gegenkathete zu α a $[m]$

Gesucht:
Winkel α $[^\circ]$

(1) $c = 9m \qquad a = 6m$
(2) $c = 3m \qquad a = 2m$
(3) $c = 4,24m \qquad a = 3m$
(4) $c = 2\frac{1}{2}m \qquad a = 1m$
(5) $c = \frac{3}{10}m \qquad a = \frac{1}{10}m$
(6) $c = 6m \qquad a = 5m$
(7) $c = 2m \qquad a = 1,73m$

5.1.2 Lösungen

Aufgabe (1)

$sin\alpha = \frac{a}{c}$
$c = 9m$
$a = 6m$
$sin\alpha = \frac{6m}{9m}$

$\alpha = 41,8°$

$c =$	$a =$	$alpha =$
$9m$	$6m$	$41,8°$
$90dm$	$60dm$	$2,51 \cdot 10^{3}{}'$
$900cm$	$600cm$	$1,51 \cdot 10^{5}{}''$
$9 \cdot 10^{3}mm$	$6 \cdot 10^{3}mm$	$46,5gon$
$9 \cdot 10^{6}\mu m$	$6 \cdot 10^{6}\mu m$	$0,73rad$

Aufgabe (2)

$sin\alpha = \frac{a}{c}$
$c = 3m$
$a = 2m$
$sin\alpha = \frac{2m}{3m}$

$\alpha = 41,8°$

$c =$	$a =$	$alpha =$
$3m$	$2m$	$41,8°$
$30dm$	$20dm$	$2,51 \cdot 10^{3}{}'$
$300cm$	$200cm$	$1,51 \cdot 10^{5}{}''$
$3 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$	$46,5gon$
$3 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$	$0,73rad$

Aufgabe (3)

$sin\alpha = \frac{a}{c}$
$c = 4,24m$
$a = 3m$
$sin\alpha = \frac{3m}{4,24m}$

$\alpha = 45°$

$c =$	$a =$	$alpha =$
$4,24m$	$3m$	$45°$
$42,4dm$	$30dm$	$2,7 \cdot 10^{3}{}'$
$424cm$	$300cm$	$1,62 \cdot 10^{5}{}''$
$4,24 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$50gon$
$4,24 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$0,785rad$

Aufgabe (4)

$sin\alpha = \frac{a}{c}$
$c = 2\frac{1}{2}m$
$a = 1m$
$sin\alpha = \frac{1m}{2\frac{1}{2}m}$

$\alpha = 23,6°$

$c =$	$a =$	$alpha =$
$2\frac{1}{2}m$	$1m$	$23,6°$
$25dm$	$10dm$	$1,41 \cdot 10^{3}{}'$
$250cm$	$100cm$	$8,49 \cdot 10^{4}{}''$
$2,5 \cdot 10^{3}mm$	$10^{3}mm$	$26,2gon$
$2,5 \cdot 10^{6}\mu m$	$10^{6}\mu m$	$0,412rad$

Aufgabe (5)

$sin\alpha = \frac{a}{c}$
$c = \frac{3}{10}m$
$a = \frac{1}{10}m$
$sin\alpha = \frac{\frac{1}{10}m}{\frac{3}{10}m}$

$\alpha = 19,5°$

$c =$	$a =$	$alpha =$
$\frac{3}{10}m$	$\frac{1}{10}m$	$19,5°$
$3dm$	$1dm$	$1,17 \cdot 10^{3}{}'$
$30cm$	$10cm$	$7,01 \cdot 10^{4}{}''$
$300mm$	$100mm$	$21,6gon$
$3 \cdot 10^{5}\mu m$	$10^{5}\mu m$	$0,34rad$

Aufgabe (6)

$sin\alpha = \frac{a}{c}$
$c = 6m$
$a = 5m$
$sin\alpha = \frac{5m}{6m}$

$\alpha = 56,4°$

$c =$	$a =$	$alpha =$
$6m$	$5m$	$56,4°$
$60dm$	$50dm$	$3,39 \cdot 10^{3}{}'$
$600cm$	$500cm$	$2,03 \cdot 10^{5}{}''$
$6 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$62,7gon$
$6 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$0,985rad$

Aufgabe (7)

$sin\alpha = \frac{a}{c}$
$c = 2m$
$a = 1,73m$
$sin\alpha = \frac{1,73m}{2m}$

$\alpha = 60°$

$c =$	$a =$	$alpha =$
$2m$	$1,73m$	$60°$
$20dm$	$17,3dm$	$3,6 \cdot 10^{3}$ '
$200cm$	$173cm$	$2,16 \cdot 10^{5}$ "
$2 \cdot 10^{3}mm$	$1,73 \cdot 10^{3}mm$	$66\frac{2}{3}gon$
$2 \cdot 10^{6}\mu m$	$1,73 \cdot 10^{6}\mu m$	$1,05rad$

5.2 $a = sin\alpha \cdot c$

5.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[°]$
Hypotenuse c $[m]$

Gesucht:
Gegenkathete zu α a $[m]$

(1) $\alpha = 30°$ $c = 4m$
(2) $\alpha = 45°$ $c = 5m$
(3) $\alpha = 30°$ $c = 2m$
(4) $\alpha = 30°$ $c = 4\frac{1}{2}m$
(5) $\alpha = 60°$ $c = 1\frac{1}{5}m$
(6) $\alpha = 20°$ $c = 6\frac{1}{2}m$

5.2.2 Lösungen

Aufgabe (1)

$a = sin\alpha \cdot c$
$\alpha = 30°$
$c = 4m$
$a = sin30° \cdot 4m$

$a = 2m$

$alpha =$	$c =$	$a =$
$30°$	$4m$	$2m$
$1,8 \cdot 10^{3}$'	$40dm$	$20dm$
$1,08 \cdot 10^{5}$"	$400cm$	$200cm$
$33\frac{1}{3}gon$	$4 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$
$0,524rad$	$4 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$

Aufgabe (2)

$a = sin\alpha \cdot c$
$\alpha = 45°$
$c = 5m$
$a = sin45° \cdot 5m$

$a = 3,54m$

$alpha =$	$c =$	$a =$
$45°$	$5m$	$3,54m$
$2,7 \cdot 10^{3}$'	$50dm$	$35,4dm$
$1,62 \cdot 10^{5}$"	$500cm$	$354cm$
$50gon$	$5 \cdot 10^{3}mm$	$3,54 \cdot 10^{3}mm$
$0,785rad$	$5 \cdot 10^{6}\mu m$	$3,54 \cdot 10^{6}\mu m$

Aufgabe (3)

$a = sin\alpha \cdot c$
$\alpha = 30°$
$c = 2m$
$a = sin30° \cdot 2m$

$a = 1m$

$alpha =$	$c =$	$a =$
$30°$	$2m$	$1m$
$1,8 \cdot 10^{3}$'	$20dm$	$10dm$
$1,08 \cdot 10^{5}$"	$200cm$	$100cm$
$33\frac{1}{3}gon$	$2 \cdot 10^{3}mm$	$10^{3}mm$
$0,524rad$	$2 \cdot 10^{6}\mu m$	$10^{6}\mu m$

Aufgabe (4)

$a = sin\alpha \cdot c$
$\alpha = 30°$
$c = 4\frac{1}{2}m$
$a = sin30° \cdot 4\frac{1}{2}m$

$a = 2\frac{1}{4}m$

$alpha =$	$c =$	$a =$
$30°$	$4\frac{1}{2}m$	$2\frac{1}{4}m$
$1,8 \cdot 10^{3}$'	$45dm$	$22\frac{1}{2}dm$
$1,08 \cdot 10^{5}$"	$450cm$	$225cm$
$33\frac{1}{3}gon$	$4,5 \cdot 10^{3}mm$	$2,25 \cdot 10^{3}mm$
$0,524rad$	$4,5 \cdot 10^{6}\mu m$	$2,25 \cdot 10^{6}\mu m$

Aufgabe (5)

$a = sin\alpha \cdot c$
$\alpha = 60°$
$c = 1\frac{1}{5}m$
$a = sin60° \cdot 1\frac{1}{5}m$

$a = 1,04m$

$alpha =$	$c =$	$a =$
$60°$	$1\frac{1}{5}m$	$1,04m$
$3,6 \cdot 10^{3}$'	$12dm$	$10,4dm$
$2,16 \cdot 10^{5}$"	$120cm$	$104cm$
$66\frac{2}{3}gon$	$1,2 \cdot 10^{3}mm$	$1,04 \cdot 10^{3}mm$
$1,05rad$	$1,2 \cdot 10^{6}\mu m$	$1,04 \cdot 10^{6}\mu m$

Aufgabe (6)

$a = sin\alpha \cdot c$
$\alpha = 20°$
$c = 6\frac{1}{2}m$
$a = sin20° \cdot 6\frac{1}{2}m$

$a = 2,22m$

$alpha =$	$c =$	$a =$
$20°$	$6\frac{1}{2}m$	$2,22m$
$1,2 \cdot 10^{3}$'	$65dm$	$22,2dm$
$7,2 \cdot 10^{4}$"	$650cm$	$222cm$
$22\frac{2}{9}gon$	$6,5 \cdot 10^{3}mm$	$2,22 \cdot 10^{3}mm$
$0,349rad$	$6,5 \cdot 10^{6}\mu m$	$2,22 \cdot 10^{6}\mu m$

5.3 $c = \frac{a}{sin\alpha}$

5.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[^\circ]$
Gegenkathete zu α a $[m]$

Gesucht:
Hypotenuse c $[m]$

(1) $\alpha = 50^\circ$ $a = 7m$
(2) $\alpha = 20^\circ$ $a = 8m$
(3) $\alpha = 30^\circ$ $a = \frac{1}{5}m$
(4) $\alpha = 30^\circ$ $a = 3m$
(5) $\alpha = 70^\circ$ $a = 34m$

5.3.2 Lösungen

Aufgabe (1)

$c = \frac{a}{sin\alpha}$
$\alpha = 50°$
$a = 7m$
$c = \frac{7m}{sin50°}$

$c = 9,14m$

$alpha =$	$a =$	$c =$
$50°$	$7m$	$9,14m$
$3 \cdot 10^{3}$'	$70dm$	$91,4dm$
$1,8 \cdot 10^{5}$"	$700cm$	$914cm$
$55\frac{5}{9}gon$	$7 \cdot 10^{3}mm$	$9,14 \cdot 10^{3}mm$
$0,873rad$	$7 \cdot 10^{6}\mu m$	$9,14 \cdot 10^{6}\mu m$

Aufgabe (2)

$c = \frac{a}{sin\alpha}$
$\alpha = 20°$
$a = 8m$
$c = \frac{8m}{sin20°}$

$c = 23,4m$

$alpha =$	$a =$	$c =$
$20°$	$8m$	$23,4m$
$1,2 \cdot 10^{3}$'	$80dm$	$234dm$
$7,2 \cdot 10^{4}$"	$800cm$	$2,34 \cdot 10^{3}cm$
$22\frac{2}{9}gon$	$8 \cdot 10^{3}mm$	$2,34 \cdot 10^{4}mm$
$0,349rad$	$8 \cdot 10^{6}\mu m$	$2,34 \cdot 10^{7}\mu m$

Aufgabe (3)

$c = \frac{a}{sin\alpha}$
$\alpha = 30°$
$a = \frac{1}{5}m$
$c = \frac{\frac{1}{5}m}{sin30°}$

$c = \frac{2}{5}m$

$alpha =$	$a =$	$c =$
$30°$	$\frac{1}{5}m$	$\frac{2}{5}m$
$1,8 \cdot 10^{3}$'	$2dm$	$4dm$
$1,08 \cdot 10^{5}$"	$20cm$	$40cm$
$33\frac{1}{3}gon$	$200mm$	$400mm$
$0,524rad$	$2 \cdot 10^{5}\mu m$	$4 \cdot 10^{5}\mu m$

Aufgabe (4)

$c = \frac{a}{sin\alpha}$
$\alpha = 30°$
$a = 3m$
$c = \frac{3m}{sin30°}$

$c = 6m$

$alpha =$	$a =$	$c =$
$30°$	$3m$	$6m$
$1,8 \cdot 10^{3}$'	$30dm$	$60dm$
$1,08 \cdot 10^{5}$"	$300cm$	$600cm$
$33\frac{1}{3}gon$	$3 \cdot 10^{3}mm$	$6 \cdot 10^{3}mm$
$0,524rad$	$3 \cdot 10^{6}\mu m$	$6 \cdot 10^{6}\mu m$

Aufgabe (5)

$c = \frac{a}{sin\alpha}$
$\alpha = 70°$
$a = 34m$
$c = \frac{34m}{sin70°}$

$c = 36,2m$

$alpha =$	$a =$	$c =$
$70°$	$34m$	$36,2m$
$4,2 \cdot 10^{3}$'	$340dm$	$362dm$
$2,52 \cdot 10^{5}$"	$3,4 \cdot 10^{3}cm$	$3,62 \cdot 10^{3}cm$
$77\frac{7}{9}gon$	$3,4 \cdot 10^{4}mm$	$3,62 \cdot 10^{4}mm$
$1,22rad$	$3,4 \cdot 10^{7}\mu m$	$3,62 \cdot 10^{7}\mu m$

5.4 $cos\alpha = \frac{b}{c}$

5.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Hypotenuse c $[m]$
Ankathete zu α b $[m]$

Gesucht:
Winkel α $[°]$

(1) $c = 9m$ $b = 6m$
(2) $c = 3m$ $b = 2m$
(3) $c = 4,24m$ $b = 3m$
(4) $c = 2\frac{1}{2}m$ $b = 1m$
(5) $c = 2\frac{3}{10}m$ $b = 1m$
(6) $c = 6m$ $b = 5m$
(7) $c = 2m$ $b = 1,73m$

5.4.2 Lösungen

Aufgabe (1)

$cos\alpha = \frac{b}{c}$
$c = 9m$
$b = 6m$
$cos\alpha = \frac{6m}{9m}$

$\alpha = 48,2°$

$c =$	$b =$	$alpha =$
$9m$	$6m$	$48,2°$
$90dm$	$60dm$	$2,89 \cdot 10^{3}{}'$
$900cm$	$600cm$	$1,73 \cdot 10^{5}{}''$
$9 \cdot 10^{3}mm$	$6 \cdot 10^{3}mm$	$53,5gon$
$9 \cdot 10^{6}\mu m$	$6 \cdot 10^{6}\mu m$	$0,841rad$

Aufgabe (2)

$cos\alpha = \frac{b}{c}$
$c = 3m$
$b = 2m$
$cos\alpha = \frac{2m}{3m}$

$\alpha = 48,2°$

$c =$	$b =$	$alpha =$
$3m$	$2m$	$48,2°$
$30dm$	$20dm$	$2,89 \cdot 10^{3}{}'$
$300cm$	$200cm$	$1,73 \cdot 10^{5}{}''$
$3 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$	$53,5gon$
$3 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$	$0,841rad$

Aufgabe (3)

$cos\alpha = \frac{b}{c}$
$c = 4,24m$
$b = 3m$
$cos\alpha = \frac{3m}{4,24m}$

$\alpha = 45°$

$c =$	$b =$	$alpha =$
$4,24m$	$3m$	$45°$
$42,4dm$	$30dm$	$2,7 \cdot 10^{3}{}'$
$424cm$	$300cm$	$1,62 \cdot 10^{5}{}''$
$4,24 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$50gon$
$4,24 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$0,785rad$

Aufgabe (4)

$cos\alpha = \frac{b}{c}$
$c = 2\frac{1}{2}m$
$b = 1m$
$cos\alpha = \frac{1m}{2\frac{1}{2}m}$

$\alpha = 66,4°$

$c =$	$b =$	$alpha =$
$2\frac{1}{2}m$	$1m$	$66,4°$
$25dm$	$10dm$	$3,99 \cdot 10^{3}{}'$
$250cm$	$100cm$	$2,39 \cdot 10^{5}{}''$
$2,5 \cdot 10^{3}mm$	$10^{3}mm$	$73,8gon$
$2,5 \cdot 10^{6}\mu m$	$10^{6}\mu m$	$1,16rad$

Aufgabe (5)

$cos\alpha = \frac{b}{c}$
$c = 2\frac{3}{10}m$
$b = 1m$
$cos\alpha = \frac{1m}{2\frac{3}{10}m}$

$\alpha = 64,2°$

$c =$	$b =$	$alpha =$
$2\frac{3}{10}m$	$1m$	$64,2°$
$23dm$	$10dm$	$3,85 \cdot 10^{3}{}'$
$230cm$	$100cm$	$2,31 \cdot 10^{5}{}''$
$2,3 \cdot 10^{3}mm$	$10^{3}mm$	$71,4gon$
$2,3 \cdot 10^{6}\mu m$	$10^{6}\mu m$	$1,12rad$

Aufgabe (6)

$cos\alpha = \frac{b}{c}$
$c = 6m$
$b = 5m$
$cos\alpha = \frac{5m}{6m}$

$\alpha = 33,6°$

$c =$	$b =$	$alpha =$
$6m$	$5m$	$33,6°$
$60dm$	$50dm$	$2,01 \cdot 10^{3}{}'$
$600cm$	$500cm$	$1,21 \cdot 10^{5}{}''$
$6 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$37,3gon$
$6 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$0,586rad$

Aufgabe (7)

$cos\alpha = \frac{b}{c}$
$c = 2m$
$b = 1,73m$
$cos\alpha = \frac{1,73m}{2m}$

$\alpha = 30°$

$c =$	$b =$	$alpha =$
$2m$	$1,73m$	$30°$
$20dm$	$17,3dm$	$1,8 \cdot 10^{3}$'
$200cm$	$173cm$	$1,08 \cdot 10^{5}$"
$2 \cdot 10^{3}mm$	$1,73 \cdot 10^{3}mm$	$33\frac{1}{3}gon$
$2 \cdot 10^{6}\mu m$	$1,73 \cdot 10^{6}\mu m$	$0,524rad$

5.5 $b = cos\alpha \cdot c$

5.5.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[^\circ]$
Hypotenuse c $[m]$

Gesucht:
Ankathete zu α b $[m]$

(1) $\alpha = 30^\circ$ $c = 4m$
(2) $\alpha = 45^\circ$ $c = 5m$
(3) $\alpha = 30^\circ$ $c = 2m$
(4) $\alpha = 30^\circ$ $c = 4\frac{1}{2}m$
(5) $\alpha = 60^\circ$ $c = 1\frac{1}{5}m$
(6) $\alpha = 20^\circ$ $c = 6\frac{1}{2}m$

5.5.2 Lösungen

Aufgabe (1)

$b = cos\alpha \cdot c$
$\alpha = 30°$
$c = 4m$
$b = cos30° \cdot 4m$

$b = 3,46m$

$alpha =$	$c =$	$b =$
$30°$	$4m$	$3,46m$
$1,8 \cdot 10^{3}$'	$40dm$	$34,6dm$
$1,08 \cdot 10^{5}$"	$400cm$	$346cm$
$33\frac{1}{3}gon$	$4 \cdot 10^{3}mm$	$3,46 \cdot 10^{3}mm$
$0,524rad$	$4 \cdot 10^{6}\mu m$	$3,46 \cdot 10^{6}\mu m$

Aufgabe (2)

$b = cos\alpha \cdot c$
$\alpha = 45°$
$c = 5m$
$b = cos45° \cdot 5m$

$b = 3,54m$

$alpha =$	$c =$	$b =$
$45°$	$5m$	$3,54m$
$2,7 \cdot 10^{3}$'	$50dm$	$35,4dm$
$1,62 \cdot 10^{5}$"	$500cm$	$354cm$
$50gon$	$5 \cdot 10^{3}mm$	$3,54 \cdot 10^{3}mm$
$0,785rad$	$5 \cdot 10^{6}\mu m$	$3,54 \cdot 10^{6}\mu m$

Aufgabe (3)

$b = cos\alpha \cdot c$
$\alpha = 30°$
$c = 2m$
$b = cos30° \cdot 2m$

$b = 1,73m$

$alpha =$	$c =$	$b =$
$30°$	$2m$	$1,73m$
$1,8 \cdot 10^{3}$'	$20dm$	$17,3dm$
$1,08 \cdot 10^{5}$"	$200cm$	$173cm$
$33\frac{1}{3}gon$	$2 \cdot 10^{3}mm$	$1,73 \cdot 10^{3}mm$
$0,524rad$	$2 \cdot 10^{6}\mu m$	$1,73 \cdot 10^{6}\mu m$

Aufgabe (4)

$b = cos\alpha \cdot c$
$\alpha = 30°$
$c = 4\frac{1}{2}m$
$b = cos30° \cdot 4\frac{1}{2}m$

$b = 3,9m$

$alpha =$	$c =$	$b =$
$30°$	$4\frac{1}{2}m$	$3,9m$
$1,8 \cdot 10^{3}$'	$45dm$	$39dm$
$1,08 \cdot 10^{5}$"	$450cm$	$390cm$
$33\frac{1}{3}gon$	$4,5 \cdot 10^{3}mm$	$3,9 \cdot 10^{3}mm$
$0,524rad$	$4,5 \cdot 10^{6}\mu m$	$3,9 \cdot 10^{6}\mu m$

Aufgabe (5)

$b = cos\alpha \cdot c$
$\alpha = 60°$
$c = 1\frac{1}{5}m$
$b = cos60° \cdot 1\frac{1}{5}m$

$b = \frac{3}{5}m$

$alpha =$	$c =$	$b =$
$60°$	$1\frac{1}{5}m$	$\frac{3}{5}m$
$3,6 \cdot 10^{3}$'	$12dm$	$6dm$
$2,16 \cdot 10^{5}$"	$120cm$	$60cm$
$66\frac{2}{3}gon$	$1,2 \cdot 10^{3}mm$	$600mm$
$1,05rad$	$1,2 \cdot 10^{6}\mu m$	$6 \cdot 10^{5}\mu m$

Aufgabe (6)

$b = cos\alpha \cdot c$
$\alpha = 20°$
$c = 6\frac{1}{2}m$
$b = cos20° \cdot 6\frac{1}{2}m$

$b = 6,11m$

$alpha =$	$c =$	$b =$
$20°$	$6\frac{1}{2}m$	$6,11m$
$1,2 \cdot 10^{3}$'	$65dm$	$61,1dm$
$7,2 \cdot 10^{4}$"	$650cm$	$611cm$
$22\frac{2}{9}gon$	$6,5 \cdot 10^{3}mm$	$6,11 \cdot 10^{3}mm$
$0,349rad$	$6,5 \cdot 10^{6}\mu m$	$6,11 \cdot 10^{6}\mu m$

5.6 $c = \frac{b}{cos\alpha}$

5.6.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[°]$
Ankathete zu α b $[m]$

Gesucht:
Hypotenuse c $[m]$

(1) $\alpha = 50°$ $b = 7m$
(2) $\alpha = 20°$ $b = 8m$
(3) $\alpha = 30°$ $b = \frac{1}{5}m$
(4) $\alpha = 30°$ $b = 3m$
(5) $\alpha = 70°$ $b = 34m$

5.6.2 Lösungen

Aufgabe (1)

$c = \frac{b}{cos\alpha}$
$\alpha = 50°$
$b = 7m$
$c = \frac{7m}{cos50°}$

$c = 10,9m$

$alpha =$	$b =$	$c =$
$50°$	$7m$	$10,9m$
$3 \cdot 10^{3}$'	$70dm$	$109dm$
$1,8 \cdot 10^{5}$"	$700cm$	$1,09 \cdot 10^{3}cm$
$55\frac{5}{9}gon$	$7 \cdot 10^{3}mm$	$1,09 \cdot 10^{4}mm$
$0,873rad$	$7 \cdot 10^{6}\mu m$	$1,09 \cdot 10^{7}\mu m$

Aufgabe (2)

$c = \frac{b}{cos\alpha}$
$\alpha = 20°$
$b = 8m$
$c = \frac{8m}{cos20°}$

$c = 8,51m$

$alpha =$	$b =$	$c =$
$20°$	$8m$	$8,51m$
$1,2 \cdot 10^{3}$'	$80dm$	$85,1dm$
$7,2 \cdot 10^{4}$"	$800cm$	$851cm$
$22\frac{2}{9}gon$	$8 \cdot 10^{3}mm$	$8,51 \cdot 10^{3}mm$
$0,349rad$	$8 \cdot 10^{6}\mu m$	$8,51 \cdot 10^{6}\mu m$

Aufgabe (3)

$c = \frac{b}{cos\alpha}$
$\alpha = 30°$
$b = \frac{1}{5}m$
$c = \frac{\frac{1}{5}m}{cos30°}$

$c = 0,231m$

$alpha =$	$b =$	$c =$
$30°$	$\frac{1}{5}m$	$0,231m$
$1,8 \cdot 10^{3}$'	$2dm$	$2,31dm$
$1,08 \cdot 10^{5}$"	$20cm$	$23,1cm$
$33\frac{1}{3}gon$	$200mm$	$231mm$
$0,524rad$	$2 \cdot 10^{5}\mu m$	$2,31 \cdot 10^{5}\mu m$

Aufgabe (4)

$c = \frac{b}{cos\alpha}$
$\alpha = 30°$
$b = 3m$
$c = \frac{3m}{cos30°}$

$c = 3,46m$

$alpha =$	$b =$	$c =$
$30°$	$3m$	$3,46m$
$1,8 \cdot 10^{3}$'	$30dm$	$34,6dm$
$1,08 \cdot 10^{5}$"	$300cm$	$346cm$
$33\frac{1}{3}gon$	$3 \cdot 10^{3}mm$	$3,46 \cdot 10^{3}mm$
$0,524rad$	$3 \cdot 10^{6}\mu m$	$3,46 \cdot 10^{6}\mu m$

Aufgabe (5)

$c = \frac{b}{cos\alpha}$
$\alpha = 70°$
$b = 34m$
$c = \frac{34m}{cos70°}$

$c = 99,4m$

$alpha =$	$b =$	$c =$
$70°$	$34m$	$99,4m$
$4,2 \cdot 10^{3}$'	$340dm$	$994dm$
$2,52 \cdot 10^{5}$"	$3,4 \cdot 10^{3}cm$	$9,94 \cdot 10^{3}cm$
$77\frac{7}{9}gon$	$3,4 \cdot 10^{4}mm$	$9,94 \cdot 10^{4}mm$
$1,22rad$	$3,4 \cdot 10^{7}\mu m$	$9,94 \cdot 10^{7}\mu m$

5.7 $tan\alpha = \frac{a}{b}$

5.7.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Ankathete zu α b $[m]$
Gegenkathete zu α a $[m]$

Gesucht:
Winkel α $[°]$

(1) $b = 7m$ $a = 8m$
(2) $b = 8m$ $a = 5m$
(3) $b = 3m$ $a = 4m$
(4) $b = 3m$ $a = 3m$
(5) $b = 4m$ $a = 2m$
(6) $b = 6\frac{1}{5}m$ $a = 3\frac{4}{5}m$

5.7.2 Lösungen

Aufgabe (1)

$tan\alpha = \frac{a}{b}$
$b = 7m$
$a = 8m$
$tan\alpha = \frac{8m}{7m}$

$\alpha = 48,8°$

$b =$	$a =$	$alpha =$
$7m$	$8m$	$48,8°$
$70dm$	$80dm$	$2,93 \cdot 10^{3}$'
$700cm$	$800cm$	$1,76 \cdot 10^{5}$"
$7 \cdot 10^{3}mm$	$8 \cdot 10^{3}mm$	$54,2gon$
$7 \cdot 10^{6}\mu m$	$8 \cdot 10^{6}\mu m$	$0,852rad$

Aufgabe (2)

$tan\alpha = \frac{a}{b}$
$b = 8m$
$a = 5m$
$tan\alpha = \frac{5m}{8m}$

$\alpha = 32°$

$b =$	$a =$	$alpha =$
$8m$	$5m$	$32°$
$80dm$	$50dm$	$1,92 \cdot 10^{3}$'
$800cm$	$500cm$	$1,15 \cdot 10^{5}$"
$8 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$35,6gon$
$8 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$0,559rad$

Aufgabe (3)

$tan\alpha = \frac{a}{b}$
$b = 3m$
$a = 4m$
$tan\alpha = \frac{4m}{3m}$

$\alpha = 53,1°$

$b =$	$a =$	$alpha =$
$3m$	$4m$	$53,1°$
$30dm$	$40dm$	$3,19 \cdot 10^{3}$'
$300cm$	$400cm$	$1,91 \cdot 10^{5}$"
$3 \cdot 10^{3}mm$	$4 \cdot 10^{3}mm$	$59gon$
$3 \cdot 10^{6}\mu m$	$4 \cdot 10^{6}\mu m$	$0,927rad$

Aufgabe (4)

$tan\alpha = \frac{a}{b}$
$b = 3m$
$a = 3m$
$tan\alpha = \frac{3m}{3m}$

$\alpha = 45°$

$b =$	$a =$	$alpha =$
$3m$	$3m$	$45°$
$30dm$	$30dm$	$2,7 \cdot 10^{3}$'
$300cm$	$300cm$	$1,62 \cdot 10^{5}$"
$3 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$50gon$
$3 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$0,785rad$

Aufgabe (5)

$tan\alpha = \frac{a}{b}$
$b = 4m$
$a = 2m$
$tan\alpha = \frac{2m}{4m}$

$\alpha = 26,6°$

$b =$	$a =$	$alpha =$
$4m$	$2m$	$26,6°$
$40dm$	$20dm$	$1,59 \cdot 10^{3}$'
$400cm$	$200cm$	$9,56 \cdot 10^{4}$"
$4 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$	$29,5gon$
$4 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$	$0,464rad$

Aufgabe (6)

$tan\alpha = \frac{a}{b}$
$b = 6\frac{1}{5}m$
$a = 3\frac{4}{5}m$
$tan\alpha = \frac{3\frac{4}{5}m}{6\frac{1}{5}m}$

$\alpha = 31,5°$

$b =$	$a =$	$alpha =$
$6\frac{1}{5}m$	$3\frac{4}{5}m$	$31,5°$
$62dm$	$38dm$	$1,89 \cdot 10^{3}$'
$620cm$	$380cm$	$1,13 \cdot 10^{5}$"
$6,2 \cdot 10^{3}mm$	$3,8 \cdot 10^{3}mm$	$35gon$
$6,2 \cdot 10^{6}\mu m$	$3,8 \cdot 10^{6}\mu m$	$0,55rad$

5.8 $a = tan\alpha \cdot b$

5.8.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[°]$
Ankathete zu α b $[m]$

Gesucht:
Gegenkathete zu α a $[m]$

(1) $\alpha = 30°$ $b = 4m$
(2) $\alpha = 45°$ $b = 5m$
(3) $\alpha = 30°$ $b = 2m$
(4) $\alpha = 30°$ $b = 4\frac{1}{2}m$
(5) $\alpha = 60°$ $b = 1\frac{1}{5}m$
(6) $\alpha = 20°$ $b = 6\frac{1}{2}m$

5.8.2 Lösungen

Aufgabe (1)

$a = tan\alpha \cdot b$
$\alpha = 30°$
$b = 4m$
$a = tan30° \cdot 4m$

$a = 2,31m$

$alpha =$	$b =$	$a =$
$30°$	$4m$	$2,31m$
$1,8 \cdot 10^{3}{}'$	$40dm$	$23,1dm$
$1,08 \cdot 10^{5}{}''$	$400cm$	$231cm$
$33\frac{1}{3}gon$	$4 \cdot 10^{3}mm$	$2,31 \cdot 10^{3}mm$
$0,524rad$	$4 \cdot 10^{6}\mu m$	$2,31 \cdot 10^{6}\mu m$

Aufgabe (2)

$a = tan\alpha \cdot b$
$\alpha = 45°$
$b = 5m$
$a = tan45° \cdot 5m$

$a = 5m$

$alpha =$	$b =$	$a =$
$45°$	$5m$	$5m$
$2,7 \cdot 10^{3}{}'$	$50dm$	$50dm$
$1,62 \cdot 10^{5}{}''$	$500cm$	$500cm$
$50gon$	$5 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$
$0,785rad$	$5 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$

Aufgabe (3)

$a = tan\alpha \cdot b$
$\alpha = 30°$
$b = 2m$
$a = tan30° \cdot 2m$

$a = 1,15m$

$alpha =$	$b =$	$a =$
$30°$	$2m$	$1,15m$
$1,8 \cdot 10^{3}{}'$	$20dm$	$11,5dm$
$1,08 \cdot 10^{5}{}''$	$200cm$	$115cm$
$33\frac{1}{3}gon$	$2 \cdot 10^{3}mm$	$1,15 \cdot 10^{3}mm$
$0,524rad$	$2 \cdot 10^{6}\mu m$	$1,15 \cdot 10^{6}\mu m$

Aufgabe (4)

$a = tan\alpha \cdot b$
$\alpha = 30°$
$b = 4\frac{1}{2}m$
$a = tan30° \cdot 4\frac{1}{2}m$

$a = 2,6m$

$alpha =$	$b =$	$a =$
$30°$	$4\frac{1}{2}m$	$2,6m$
$1,8 \cdot 10^{3}{}'$	$45dm$	$26dm$
$1,08 \cdot 10^{5}{}''$	$450cm$	$260cm$
$33\frac{1}{3}gon$	$4,5 \cdot 10^{3}mm$	$2,6 \cdot 10^{3}mm$
$0,524rad$	$4,5 \cdot 10^{6}\mu m$	$2,6 \cdot 10^{6}\mu m$

Aufgabe (5)

$a = tan\alpha \cdot b$
$\alpha = 60°$
$b = 1\frac{1}{5}m$
$a = tan60° \cdot 1\frac{1}{5}m$

$a = 2,08m$

$alpha =$	$b =$	$a =$
$60°$	$1\frac{1}{5}m$	$2,08m$
$3,6 \cdot 10^{3}{}'$	$12dm$	$20,8dm$
$2,16 \cdot 10^{5}{}''$	$120cm$	$208cm$
$66\frac{2}{3}gon$	$1,2 \cdot 10^{3}mm$	$2,08 \cdot 10^{3}mm$
$1,05rad$	$1,2 \cdot 10^{6}\mu m$	$2,08 \cdot 10^{6}\mu m$

Aufgabe (6)

$a = tan\alpha \cdot b$
$\alpha = 20°$
$b = 6\frac{1}{2}m$
$a = tan20° \cdot 6\frac{1}{2}m$

$a = 2,37m$

$alpha =$	$b =$	$a =$
$20°$	$6\frac{1}{2}m$	$2,37m$
$1,2 \cdot 10^{3}{}'$	$65dm$	$23,7dm$
$7,2 \cdot 10^{4}{}''$	$650cm$	$237cm$
$22\frac{2}{9}gon$	$6,5 \cdot 10^{3}mm$	$2,37 \cdot 10^{3}mm$
$0,349rad$	$6,5 \cdot 10^{6}\mu m$	$2,37 \cdot 10^{6}\mu m$

5.9 $b = \frac{a}{tan\alpha}$

5.9.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[°]$
Gegenkathete zu α a $[m]$

Gesucht:
Ankathete zu α b $[m]$

(1) $\alpha = 30°$ $a = 4m$
(2) $\alpha = 45°$ $a = 5m$
(3) $\alpha = 30°$ $a = 2m$
(4) $\alpha = 30°$ $a = 4\frac{1}{2}m$
(5) $\alpha = 60°$ $a = 1\frac{1}{5}m$
(6) $\alpha = 20°$ $a = 6\frac{1}{2}m$

5.9.2 Lösungen

Aufgabe (1)

$b = \frac{a}{tan\alpha}$
$\alpha = 30°$
$a = 4m$
$b = \frac{4m}{tan30°}$

$b = 6,93m$

$alpha =$	$a =$	$b =$
$30°$	$4m$	$6,93m$
$1,8 \cdot 10^{3}$'	$40dm$	$69,3dm$
$1,08 \cdot 10^{5}$"	$400cm$	$693cm$
$33\frac{1}{3}gon$	$4 \cdot 10^{3}mm$	$6,93 \cdot 10^{3}mm$
$0,524rad$	$4 \cdot 10^{6}\mu m$	$6,93 \cdot 10^{6}\mu m$

Aufgabe (2)

$b = \frac{a}{tan\alpha}$
$\alpha = 45°$
$a = 5m$
$b = \frac{5m}{tan45°}$

$b = 5m$

$alpha =$	$a =$	$b =$
$45°$	$5m$	$5m$
$2,7 \cdot 10^{3}$'	$50dm$	$50dm$
$1,62 \cdot 10^{5}$"	$500cm$	$500cm$
$50gon$	$5 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$
$0,785rad$	$5 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$

Aufgabe (3)

$b = \frac{a}{tan\alpha}$
$\alpha = 30°$
$a = 2m$
$b = \frac{2m}{tan30°}$

$b = 3,46m$

$alpha =$	$a =$	$b =$
$30°$	$2m$	$3,46m$
$1,8 \cdot 10^{3}$'	$20dm$	$34,6dm$
$1,08 \cdot 10^{5}$"	$200cm$	$346cm$
$33\frac{1}{3}gon$	$2 \cdot 10^{3}mm$	$3,46 \cdot 10^{3}mm$
$0,524rad$	$2 \cdot 10^{6}\mu m$	$3,46 \cdot 10^{6}\mu m$

Aufgabe (4)

$b = \frac{a}{tan\alpha}$
$\alpha = 30°$
$a = 4\frac{1}{2}m$
$b = \frac{4\frac{1}{2}m}{tan30°}$

$b = 7,79m$

$alpha =$	$a =$	$b =$
$30°$	$4\frac{1}{2}m$	$7,79m$
$1,8 \cdot 10^{3}$'	$45dm$	$77,9dm$
$1,08 \cdot 10^{5}$"	$450cm$	$779cm$
$33\frac{1}{3}gon$	$4,5 \cdot 10^{3}mm$	$7,79 \cdot 10^{3}mm$
$0,524rad$	$4,5 \cdot 10^{6}\mu m$	$7,79 \cdot 10^{6}\mu m$

Aufgabe (5)

$b = \frac{a}{tan\alpha}$
$\alpha = 60°$
$a = 1\frac{1}{5}m$
$b = \frac{1\frac{1}{5}m}{tan60°}$

$b = 0,693m$

$alpha =$	$a =$	$b =$
$60°$	$1\frac{1}{5}m$	$0,693m$
$3,6 \cdot 10^{3}$'	$12dm$	$6,93dm$
$2,16 \cdot 10^{5}$"	$120cm$	$69,3cm$
$66\frac{2}{3}gon$	$1,2 \cdot 10^{3}mm$	$693mm$
$1,05rad$	$1,2 \cdot 10^{6}\mu m$	$6,93 \cdot 10^{5}\mu m$

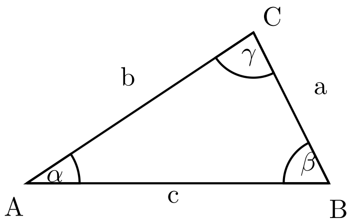
Aufgabe (6)

$b = \frac{a}{tan\alpha}$
$\alpha = 20°$
$a = 6\frac{1}{2}m$
$b = \frac{6\frac{1}{2}m}{tan20°}$

$b = 17,9m$

$alpha =$	$a =$	$b =$
$20°$	$6\frac{1}{2}m$	$17,9m$
$1,2 \cdot 10^{3}$'	$65dm$	$179dm$
$7,2 \cdot 10^{4}$"	$650cm$	$1,79 \cdot 10^{3}cm$
$22\frac{2}{9}gon$	$6,5 \cdot 10^{3}mm$	$1,79 \cdot 10^{4}mm$
$0,349rad$	$6,5 \cdot 10^{6}\mu m$	$1,79 \cdot 10^{7}\mu m$

6 Sinussatz



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$$
$$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : b$$
$$\sin\alpha = \frac{a \cdot \sin\beta}{b}$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$$
$$a = \frac{b \cdot \sin\alpha}{\sin\beta}$$
$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$
$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\sin\alpha = \frac{a \cdot \sin\beta}{b} \qquad \sin\alpha = \frac{a \cdot \sin\gamma}{c}$$
$$\sin\beta = \frac{b \cdot \sin\alpha}{a} \qquad \sin\beta = \frac{b \cdot \sin\gamma}{c}$$
$$\sin\gamma = \frac{c \cdot \sin\alpha}{a} \qquad \sin\gamma = \frac{c \cdot \sin\beta}{b}$$
$$a = \frac{b \cdot \sin\alpha}{\sin\beta} \qquad a = \frac{c \cdot \sin\alpha}{\sin\gamma}$$
$$b = \frac{a \cdot \sin\beta}{\sin\alpha} \qquad b = \frac{c \cdot \sin\beta}{\sin\gamma}$$
$$c = \frac{a \cdot \sin\gamma}{\sin\alpha} \qquad c = \frac{b \cdot \sin\gamma}{\sin\beta}$$

6.1 $a = \frac{b \cdot sin\alpha}{sin\beta}$

6.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel β $[°]$
Winkel α $[°]$
Länge der Seite b $[m]$

Gesucht:
Länge der Seite a $[m]$

(1) $\beta = 45°$ $\alpha = 30°$ $b = 3m$
(2) $\beta = 50°$ $\alpha = 45°$ $b = 7m$
(3) $\beta = 120°$ $\alpha = 30°$ $b = 5m$
(4) $\beta = 150°$ $\alpha = 30°$ $b = 7\frac{2}{5}m$
(5) $\beta = 45°$ $\alpha = 135°$ $b = 7\frac{2}{5}m$
(6) $\beta = 45°$ $\alpha = 30°$ $b = 24m$
(7) $\beta = 20°$ $\alpha = 50°$ $b = \frac{2}{5}m$

6.1.2 Lösungen

Aufgabe (1)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 45°$
$\alpha = 30°$
$b = 3m$
$a = \frac{3m \cdot sin30°}{sin45°}$

$a = 2,12m$

$beta =$	$alpha =$	$b =$	$a =$
$45°$	$30°$	$3m$	$2,12m$
$2,7 \cdot 10^{3}$'	$1,8 \cdot 10^{3}$'	$30dm$	$21,2dm$
$1,62 \cdot 10^{5}$"	$1,08 \cdot 10^{5}$"	$300cm$	$212cm$
$50gon$	$33\frac{1}{3}gon$	$3 \cdot 10^{3}mm$	$2,12 \cdot 10^{3}mm$
$0,785rad$	$0,524rad$	$3 \cdot 10^{6}\mu m$	$2,12 \cdot 10^{6}\mu m$

Aufgabe (2)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 50°$
$\alpha = 45°$
$b = 7m$
$a = \frac{7m \cdot sin45°}{sin50°}$

$a = 6,46m$

$beta =$	$alpha =$	$b =$	$a =$
$50°$	$45°$	$7m$	$6,46m$
$3 \cdot 10^{3}$'	$2,7 \cdot 10^{3}$'	$70dm$	$64,6dm$
$1,8 \cdot 10^{5}$"	$1,62 \cdot 10^{5}$"	$700cm$	$646cm$
$55\frac{5}{9}gon$	$50gon$	$7 \cdot 10^{3}mm$	$6,46 \cdot 10^{3}mm$
$0,873rad$	$0,785rad$	$7 \cdot 10^{6}\mu m$	$6,46 \cdot 10^{6}\mu m$

Aufgabe (3)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 120°$
$\alpha = 30°$
$b = 5m$
$a = \frac{5m \cdot sin30°}{sin120°}$

$a = 2,89m$

$beta =$	$alpha =$	$b =$	$a =$
$120°$	$30°$	$5m$	$2,89m$
$7,2 \cdot 10^{3}$'	$1,8 \cdot 10^{3}$'	$50dm$	$28,9dm$
$4,32 \cdot 10^{5}$"	$1,08 \cdot 10^{5}$"	$500cm$	$289cm$
$133\frac{1}{3}gon$	$33\frac{1}{3}gon$	$5 \cdot 10^{3}mm$	$2,89 \cdot 10^{3}mm$
$2,09rad$	$0,524rad$	$5 \cdot 10^{6}\mu m$	$2,89 \cdot 10^{6}\mu m$

Aufgabe (4)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 150°$
$\alpha = 30°$
$b = 7\frac{2}{5}m$
$a = \frac{7\frac{2}{5}m \cdot sin30°}{sin150°}$

$a = 7\frac{2}{5}m$

$beta =$	$alpha =$	$b =$	$a =$
$150°$	$30°$	$7\frac{2}{5}m$	$7\frac{2}{5}m$
$9 \cdot 10^{3}$'	$1,8 \cdot 10^{3}$'	$74dm$	$74dm$
$5,4 \cdot 10^{5}$"	$1,08 \cdot 10^{5}$"	$740cm$	$740cm$
$166\frac{2}{3}gon$	$33\frac{1}{3}gon$	$7,4 \cdot 10^{3}mm$	$7,4 \cdot 10^{3}mm$
$2,62rad$	$0,524rad$	$7,4 \cdot 10^{6}\mu m$	$7,4 \cdot 10^{6}\mu m$

Aufgabe (5)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 45°$
$\alpha = 135°$
$b = 7\frac{2}{5}m$
$a = \frac{7\frac{2}{5}m \cdot sin135°}{sin45°}$

$a = 7\frac{2}{5}m$

$beta =$	$alpha =$	$b =$	$a =$
$45°$	$135°$	$7\frac{2}{5}m$	$7\frac{2}{5}m$
$2,7 \cdot 10^{3}$'	$8,1 \cdot 10^{3}$'	$74dm$	$74dm$
$1,62 \cdot 10^{5}$"	$4,86 \cdot 10^{5}$"	$740cm$	$740cm$
$50gon$	$150gon$	$7,4 \cdot 10^{3}mm$	$7,4 \cdot 10^{3}mm$
$0,785rad$	$2,36rad$	$7,4 \cdot 10^{6}\mu m$	$7,4 \cdot 10^{6}\mu m$

Aufgabe (6)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 45°$
$\alpha = 30°$
$b = 24m$
$a = \frac{24m \cdot sin30°}{sin45°}$

$a = 17m$

$beta =$	$alpha =$	$b =$	$a =$
$45°$	$30°$	$24m$	$17m$
$2,7 \cdot 10^{3}$'	$1,8 \cdot 10^{3}$'	$240dm$	$170dm$
$1,62 \cdot 10^{5}$"	$1,08 \cdot 10^{5}$"	$2,4 \cdot 10^{3}cm$	$1,7 \cdot 10^{3}cm$
$50gon$	$33\frac{1}{3}gon$	$2,4 \cdot 10^{4}mm$	$1,7 \cdot 10^{4}mm$
$0,785rad$	$0,524rad$	$2,4 \cdot 10^{7}\mu m$	$1,7 \cdot 10^{7}\mu m$

Aufgabe (7)

$a = \frac{b \cdot sin\alpha}{sin\beta}$
$\beta = 20°$
$\alpha = 50°$
$b = \frac{2}{5}m$
$a = \frac{\frac{2}{5}m \cdot sin50°}{sin20°}$

$a = 0,896m$

$beta =$	$alpha =$	$b =$	$a =$
$20°$	$50°$	$\frac{2}{5}m$	$0,896m$
$1,2 \cdot 10^{3}$'	$3 \cdot 10^{3}$'	$4dm$	$8,96dm$
$7,2 \cdot 10^{4}$"	$1,8 \cdot 10^{5}$"	$40cm$	$89,6cm$
$22\frac{2}{9}gon$	$55\frac{5}{9}gon$	$400mm$	$896mm$
$0,349rad$	$0,873rad$	$4 \cdot 10^{5}\mu m$	$8,96 \cdot 10^{5}\mu m$

6.2 $sin\alpha = \frac{a \cdot sin\beta}{b}$

6.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Länge der Seite b $[m]$
Länge der Seite a $[m]$
Winkel β $[°]$

Gesucht:
Winkel α $[°]$

(1) $b = 4m$ $a = 3m$ $\beta = 80°$
(2) $b = 8m$ $a = 2m$ $\beta = 50°$
(3) $b = 3m$ $a = 3m$ $\beta = 60°$
(4) $b = 5m$ $a = 2m$ $\beta = 40°$
(5) $b = 15m$ $a = 2m$ $\beta = 120°$

6.2.2 Lösungen

Aufgabe (1)

$sin\alpha = \frac{a \cdot sin\beta}{b}$
$b = 4m$
$a = 3m$
$\beta = 80°$
$sin\alpha = \frac{3m \cdot sin80°}{4m}$

$0 < \alpha < 90° \qquad \alpha_1 = 47,6°$
$90° < \alpha < 180° \qquad \alpha_2 = 180° - 47,6°$
$\alpha_2 = 132$

$b =$	$a =$	$beta =$	$alpha =$
$4m$	$3m$	$80°$	$47,6°$
$40dm$	$30dm$	$4,8 \cdot 10^{3}$'	$2,86 \cdot 10^{3}$'
$400cm$	$300cm$	$2,88 \cdot 10^{5}$"	$1,71 \cdot 10^{5}$"
$4 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$88\frac{8}{9}gon$	$52,9gon$
$4 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$1,4rad$	$0,831rad$

Aufgabe (2)

$sin\alpha = \frac{a \cdot sin\beta}{b}$
$b = 8m$
$a = 2m$
$\beta = 50°$
$sin\alpha = \frac{2m \cdot sin50°}{8m}$

$0 < \alpha < 90° \qquad \alpha_1 = 11\frac{5}{122}°$
$90° < \alpha < 180° \qquad \alpha_2 = 180° - 11\frac{5}{122}°$
$\alpha_2 = 168\frac{117}{122}$

$b =$	$a =$	$beta =$	$alpha =$
$8m$	$2m$	$50°$	$11\frac{5}{122}°$
$80dm$	$20dm$	$3 \cdot 10^{3}$'	662'
$800cm$	$200cm$	$1,8 \cdot 10^{5}$"	$3,97 \cdot 10^{4}$"
$8 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$	$55\frac{5}{9}gon$	$12,3gon$
$8 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$	$0,873rad$	$0,193rad$

Aufgabe (3)

$sin\alpha = \frac{a \cdot sin\beta}{b}$
$b = 3m$
$a = 3m$
$\beta = 60°$
$sin\alpha = \frac{3m \cdot sin60°}{3m}$

$0 < \alpha < 90° \qquad \alpha_1 = 60°$
$90° < \alpha < 180° \qquad \alpha_2 = 180° - 60°$
$\alpha_2 = 120$

$b =$	$a =$	$beta =$	$alpha =$
$3m$	$3m$	$60°$	$60°$
$30dm$	$30dm$	$3,6 \cdot 10^{3}$'	$3,6 \cdot 10^{3}$'
$300cm$	$300cm$	$2,16 \cdot 10^{5}$"	$2,16 \cdot 10^{5}$"
$3 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$66\frac{2}{3}gon$	$66\frac{2}{3}gon$
$3 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$1,05rad$	$1,05rad$

Aufgabe (4)

$sin\alpha = \frac{a \cdot sin\beta}{b}$
$b = 5m$
$a = 2m$
$\beta = 40°$
$sin\alpha = \frac{2m \cdot sin40°}{5m}$

$0 < \alpha < 90° \qquad \alpha_1 = 14,9°$
$90° < \alpha < 180° \qquad \alpha_2 = 180° - 14,9°$
$\alpha_2 = 165$

$b =$	$a =$	$beta =$	$alpha =$
$5m$	$2m$	$40°$	$14,9°$
$50dm$	$20dm$	$2,4 \cdot 10^{3}$'	894'
$500cm$	$200cm$	$1,44 \cdot 10^{5}$"	$5,36 \cdot 10^{4}$"
$5 \cdot 10^{3}mm$	$2 \cdot 10^{3}mm$	$44\frac{4}{9}gon$	$16,6gon$
$5 \cdot 10^{6}\mu m$	$2 \cdot 10^{6}\mu m$	$0,698rad$	$0,26rad$

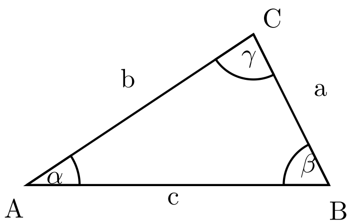
Aufgabe (5)

$sin\alpha = \frac{a \cdot sin\beta}{b}$
$b = 15m$
$a = 2m$
$\beta = 120°$
$sin\alpha = \frac{2m \cdot sin120°}{15m}$

$0 < \alpha < 90° \qquad \alpha_1 = 6,63°$
$90° < \alpha < 180° \qquad \alpha_2 = 180° - 6,63°$
$\alpha_2 = 173$

$b =$	$a =$	$beta =$	$alpha =$
$15m$	$2m$	$120°$	$6,63°$
$150dm$	$20dm$	$7,2 \cdot 10^{3}$'	398'
$1,5 \cdot 10^{3}cm$	$200cm$	$4,32 \cdot 10^{5}$"	$2,39 \cdot 10^{4}$"
$1,5 \cdot 10^{4}mm$	$2 \cdot 10^{3}mm$	$133\frac{1}{3}gon$	$7,37gon$
$1,5 \cdot 10^{7}\mu m$	$2 \cdot 10^{6}\mu m$	$2,09rad$	$0,116rad$

7 Kosinussatz



$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2$

$0 = b^2 + c^2 - a^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$

$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$

$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha} \qquad \cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta} \qquad \cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$

$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma} \qquad \cos\gamma = \frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}$

7.1 $a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$

7.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Winkel α $[°]$
Länge der Seite c $[m]$
Länge der Seite b $[m]$

Gesucht:
Länge der Seite a $[m]$

(1) $\alpha = 60°$ $c = 7m$ $b = 7m$
(2) $\alpha = 30°$ $c = 1m$ $b = 3m$
(3) $\alpha = 150°$ $c = 12m$ $b = 33m$
(4) $\alpha = 80°$ $c = \frac{1}{2}m$ $b = \frac{3}{4}m$
(5) $\alpha = 30°$ $c = 1m$ $b = 3m$

7.1.2 Lösungen

Aufgabe (1)

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$
$\alpha = 60°$
$c = 7m$
$b = 7m$
$a = \sqrt{(7m)^2 + (7m)^2 - 2 \cdot 7m \cdot 7m \cdot cos60°}$

$a = 7m$

$alpha =$	$c =$	$b =$	$a =$
$60°$	$7m$	$7m$	$7m$
$3,6 \cdot 10^{3}$'	$70dm$	$70dm$	$70dm$
$2,16 \cdot 10^{5}$"	$700cm$	$700cm$	$700cm$
$66\frac{2}{3}gon$	$7 \cdot 10^3mm$	$7 \cdot 10^3mm$	$7 \cdot 10^3mm$
$1,05rad$	$7 \cdot 10^6\mu m$	$7 \cdot 10^6\mu m$	$7 \cdot 10^6\mu m$

Aufgabe (2)

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$
$\alpha = 30°$
$c = 1m$
$b = 3m$
$a = \sqrt{(3m)^2 + (1m)^2 - 2 \cdot 3m \cdot 1m \cdot cos30°}$

$a = 2,19m$

$alpha =$	$c =$	$b =$	$a =$
$30°$	$1m$	$3m$	$2,19m$
$1,8 \cdot 10^{3}$'	$10dm$	$30dm$	$21,9dm$
$1,08 \cdot 10^{5}$"	$100cm$	$300cm$	$219cm$
$33\frac{1}{3}gon$	10^3mm	$3 \cdot 10^3mm$	$2,19 \cdot 10^3mm$
$0,524rad$	$10^6\mu m$	$3 \cdot 10^6\mu m$	$2,19 \cdot 10^6\mu m$

Aufgabe (3)

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$
$\alpha = 150°$
$c = 12m$
$b = 33m$
$a = \sqrt{(33m)^2 + (12m)^2 - 2 \cdot 33m \cdot 12m \cdot cos150°}$

$a = 43,8m$

$alpha =$	$c =$	$b =$	$a =$
$150°$	$12m$	$33m$	$43,8m$
$9 \cdot 10^{3}$'	$120dm$	$330dm$	$438dm$
$5,4 \cdot 10^{5}$"	$1,2 \cdot 10^3cm$	$3,3 \cdot 10^3cm$	$4,38 \cdot 10^3cm$
$166\frac{2}{3}gon$	$1,2 \cdot 10^4mm$	$3,3 \cdot 10^4mm$	$4,38 \cdot 10^4mm$
$2,62rad$	$1,2 \cdot 10^7\mu m$	$3,3 \cdot 10^7\mu m$	$4,38 \cdot 10^7\mu m$

Aufgabe (4)

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$
$\alpha = 80°$
$c = \frac{1}{2}m$
$b = \frac{3}{4}m$
$a = \sqrt{(\frac{3}{4}m)^2 + (\frac{1}{2}m)^2 - 2 \cdot \frac{3}{4}m \cdot \frac{1}{2}m \cdot cos80°}$

$a = 0,826m$

$alpha =$	$c =$	$b =$	$a =$
$80°$	$\frac{1}{2}m$	$\frac{3}{4}m$	$0,826m$
$4,8 \cdot 10^{3}$'	$5dm$	$7\frac{1}{2}dm$	$8,26dm$
$2,88 \cdot 10^{5}$"	$50cm$	$75cm$	$82,6cm$
$88\frac{8}{9}gon$	$500mm$	$750mm$	$826mm$
$1,4rad$	$5 \cdot 10^5\mu m$	$7,5 \cdot 10^5\mu m$	$8,26 \cdot 10^5\mu m$

Aufgabe (5)

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha}$
$\alpha = 30°$
$c = 1m$
$b = 3m$
$a = \sqrt{(3m)^2 + (1m)^2 - 2 \cdot 3m \cdot 1m \cdot cos30°}$

$a = 2,19m$

$alpha =$	$c =$	$b =$	$a =$
$30°$	$1m$	$3m$	$2,19m$
$1,8 \cdot 10^{3}$'	$10dm$	$30dm$	$21,9dm$
$1,08 \cdot 10^{5}$"	$100cm$	$300cm$	$219cm$
$33\frac{1}{3}gon$	10^3mm	$3 \cdot 10^3mm$	$2,19 \cdot 10^3mm$
$0,524rad$	$10^6\mu m$	$3 \cdot 10^6\mu m$	$2,19 \cdot 10^6\mu m$

7.2 $cos\alpha = \frac{b^2+c^2-a^2}{2 \cdot b \cdot c}$

7.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Länge der Seite c $[m]$
Länge der Seite b $[m]$
Länge der Seite a $[m]$

Gesucht:
Winkel α $[^\circ]$

(1) $c = 3m$ $b = 7m$ $a = 9m$
(2) $c = 10m$ $b = 9m$ $a = 5m$
(3) $c = 3m$ $b = 3m$ $a = 3m$
(4) $c = 6m$ $b = 6m$ $a = 5m$
(5) $c = 1\frac{9}{10}m$ $b = 3\frac{3}{5}m$ $a = 5\frac{1}{5}m$
(6) $c = 3m$ $b = 4m$ $a = 5m$

7.2.2 Lösungen

Aufgabe (1)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 3m$
$b = 7m$
$a = 9m$
$cos\alpha = \frac{(7m)^2+(3m)^2-(9m)^2}{2\cdot 7m\cdot 3m}$

$\alpha = 123°$

$c =$	$b =$	$a =$	$alpha =$
$3m$	$7m$	$9m$	$123°$
$30dm$	$70dm$	$90dm$	$7,39 \cdot 10^{3}\,'$
$300cm$	$700cm$	$900cm$	$4,44 \cdot 10^{5}\,''$
$3 \cdot 10^{3}mm$	$7 \cdot 10^{3}mm$	$9 \cdot 10^{3}mm$	$137gon$
$3 \cdot 10^{6}\mu m$	$7 \cdot 10^{6}\mu m$	$9 \cdot 10^{6}\mu m$	$2,15rad$

Aufgabe (2)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 10m$
$b = 9m$
$a = 5m$
$cos\alpha = \frac{(9m)^2+(10m)^2-(5m)^2}{2\cdot 9m\cdot 10m}$

$\alpha = 29,9°$

$c =$	$b =$	$a =$	$alpha =$
$10m$	$9m$	$5m$	$29,9°$
$100dm$	$90dm$	$50dm$	$1,8 \cdot 10^{3}\,'$
$10^{3}cm$	$900cm$	$500cm$	$1,08 \cdot 10^{5}\,''$
$10^{4}mm$	$9 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$33,3gon$
$10^{7}\mu m$	$9 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$0,522rad$

Aufgabe (3)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 3m$
$b = 3m$
$a = 3m$
$cos\alpha = \frac{(3m)^2+(3m)^2-(3m)^2}{2\cdot 3m\cdot 3m}$

$\alpha = 60°$

$c =$	$b =$	$a =$	$alpha =$
$3m$	$3m$	$3m$	$60°$
$30dm$	$30dm$	$30dm$	$3,6 \cdot 10^{3}\,'$
$300cm$	$300cm$	$300cm$	$2,16 \cdot 10^{5}\,''$
$3 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$3 \cdot 10^{3}mm$	$66\frac{2}{3}gon$
$3 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$3 \cdot 10^{6}\mu m$	$1,05rad$

Aufgabe (4)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 6m$
$b = 6m$
$a = 5m$
$cos\alpha = \frac{(6m)^2+(6m)^2-(5m)^2}{2\cdot 6m\cdot 6m}$

$\alpha = 49,2°$

$c =$	$b =$	$a =$	$alpha =$
$6m$	$6m$	$5m$	$49,2°$
$60dm$	$60dm$	$50dm$	$2,95 \cdot 10^{3}\,'$
$600cm$	$600cm$	$500cm$	$1,77 \cdot 10^{5}\,''$
$6 \cdot 10^{3}mm$	$6 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$54,7gon$
$6 \cdot 10^{6}\mu m$	$6 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$0,86rad$

Aufgabe (5)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 1\frac{9}{10}m$
$b = 3\frac{3}{5}m$
$a = 5\frac{1}{5}m$
$cos\alpha = \frac{(3\frac{3}{5}m)^2+(1\frac{9}{10}m)^2-(5\frac{1}{5}m)^2}{2\cdot 3\frac{3}{5}m\cdot 1\frac{9}{10}m}$

$\alpha = 140°$

$c =$	$b =$	$a =$	$alpha =$
$1\frac{9}{10}m$	$3\frac{3}{5}m$	$5\frac{1}{5}m$	$140°$
$19dm$	$36dm$	$52dm$	$8,4 \cdot 10^{3}\,'$
$190cm$	$360cm$	$520cm$	$5,04 \cdot 10^{5}\,''$
$1,9 \cdot 10^{3}mm$	$3,6 \cdot 10^{3}mm$	$5,2 \cdot 10^{3}mm$	$155gon$
$1,9 \cdot 10^{6}\mu m$	$3,6 \cdot 10^{6}\mu m$	$5,2 \cdot 10^{6}\mu m$	$2,44rad$

Aufgabe (6)

$cos\alpha = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$
$c = 3m$
$b = 4m$
$a = 5m$
$cos\alpha = \frac{(4m)^2+(3m)^2-(5m)^2}{2\cdot 4m\cdot 3m}$

$\alpha = 90°$

$c =$	$b =$	$a =$	$alpha =$
$3m$	$4m$	$5m$	$90°$
$30dm$	$40dm$	$50dm$	$5,4 \cdot 10^{3}\,'$
$300cm$	$400cm$	$500cm$	$3,24 \cdot 10^{5}\,''$
$3 \cdot 10^{3}mm$	$4 \cdot 10^{3}mm$	$5 \cdot 10^{3}mm$	$100gon$
$3 \cdot 10^{6}\mu m$	$4 \cdot 10^{6}\mu m$	$5 \cdot 10^{6}\mu m$	$1,57rad$

8 Kongruenzsätze - Berechnungen am Dreieck

Seite - Seite - Seite (SSS)

Seite	Seite	Seite
a	b	c

1. Zwei Winkel mit Kosinus-Satz berechnen

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$

$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$

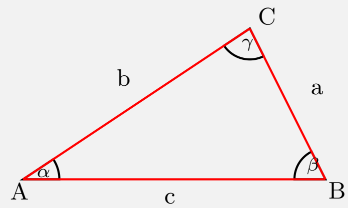
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

entsprechend

$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c} \qquad \cos\gamma = \frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}$

2. Fehlenden Winkel über die Winkelsumme im Dreieck berechnen

$\alpha + \beta + \gamma = 180°$



$a = 2,2 \quad b = 3,6 \quad c = 4$

$\cos\alpha = \frac{3,6^2 + 4^2 - 2,2^2}{2 \cdot 3,6 \cdot 4}$

$\cos\alpha = 0,8$

$\alpha = \arccos(0,8)$

$\alpha = 33,1°$

$\cos\beta = \frac{2,2^2 + 4^2 - 3,6^2}{2 \cdot 2,2 \cdot 4}$

$\cos\beta = 0,4$

$\beta = \arccos(0,4)$

$\beta = 63,4°$

$\gamma = 180° - 33,1° - 63,4°$

$\gamma = 83,5°$

Seite - Winkel - Seite (SWS)

Seite	Winkel	Seite
a	β	c
a	γ	b
b	α	c

1. Gegenüberliegende Seite mit Kosinussatz berechnen

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$

entsprechend

$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta} \qquad c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$

2. Winkel mit Kosinussatz berechnen

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$

$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$

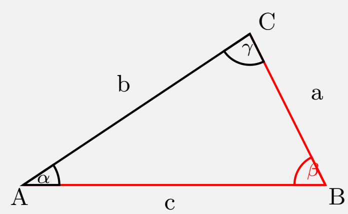
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

entsprechend

$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c} \qquad \cos\gamma = \frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}$

3. Fehlenden Winkel über die Winkelsumme im Dreieck berechnen

$\alpha + \beta + \gamma = 180°$



$a = 2,2 \quad c = 4 \quad \beta = 63,4°$

$b = \sqrt{2,2^2 + 4^2 - 2 \cdot 2,2 \cdot 4 \cdot \cos 63,4°}$

$b = 3,6$

$\cos\alpha = \frac{3,6^2 + 4^2 - 2,2^2}{2 \cdot 3,6 \cdot 4}$

$\cos\alpha = 0,8$

$\alpha = \arccos(0,8)$

$\alpha = 33,1°$

$\gamma = 180° - 33,1° - 63,4°$

$\gamma = 83,5°$

Winkel - Seite - Winkel (WSW,WWS)

Winkel	Seite	Winkel
α	c	β
α	b	γ
β	a	γ

Winkel	Winkel	Seite
α	β	a
α	β	b
α	γ	a
α	γ	c
β	γ	b
β	γ	c

1. Fehlenden Winkel über die Winkelsumme im Dreieck berechnen

$\alpha + \beta + \gamma = 180°$

2. Eine Seite über den Sinussatz

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta$

$b = \frac{a \cdot \sin\beta}{\sin\alpha}$

entsprechend

$b = \frac{c \cdot \sin\beta}{\sin\gamma}$

$c = \frac{a \cdot \sin\gamma}{\sin\alpha} \qquad c = \frac{b \cdot \sin\gamma}{\sin\beta}$

$a = \frac{b \cdot \sin\alpha}{\sin\beta} \qquad a = \frac{c \cdot \sin\alpha}{\sin\gamma}$

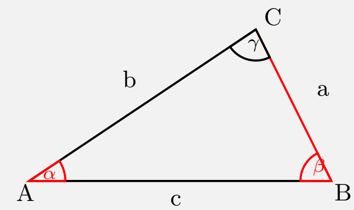
3. Fehlende Seite mit dem Kosinussatz berechnen

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$

$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$

entsprechend

$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta} \qquad c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$



$a = 2,2 \quad \alpha = 33,1° \quad \beta = 63,4°$

$\gamma = 180° - \alpha - \beta$

$\gamma = 180° - 33,1° - 63,4°$

$\gamma = 83,5°$

$b = \frac{2,2 \cdot \sin 63,4}{\sin 33,1}$

$b = 3,6$

$c = \sqrt{2,2^2 + 3,6^2 - 2 \cdot 2,2 \cdot 3,6 \cdot \cos 83,5°}$

$c = 4$

Seite - Seite - Winkel (SsW)

Seite	Seite	Winkel	
a	b	α	a>b
a	b	β	b>a
a	c	α	a>c
a	c	γ	c>a
b	c	β	b>c
b	c	γ	c>b

1. Winkel mit dem Sinussatz berechnen

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$

$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : b$

$\sin\alpha = \frac{a \cdot \sin\beta}{b}$

entsprechend

$\sin\beta = \frac{b \cdot \sin\alpha}{a} \qquad \sin\gamma = \frac{c \cdot \sin\alpha}{a}$

2. Fehlenden Winkel über die Winkelsumme im Dreieck berechnen

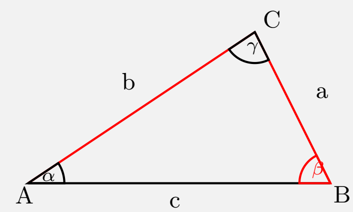
$\alpha + \beta + \gamma = 180°$

3. Fehlende Seite mit dem Kosinussatz berechnen

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta \qquad a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$

entsprechend

$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta} \qquad c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$



$a = 2,2 \quad b = 3,6 \quad \beta = 63,4°$

$\sin\alpha = \frac{2,2 \cdot \sin 63,4°}{3,6}$

$\sin\alpha = 0,5$

$\alpha = \arcsin(0,5)$

$\alpha = 33,1°$

$\gamma = 180° - 33,1° - 63,4°$

$\gamma = 83,5°$

$c = \sqrt{2,2^2 + 3,6^2 - 2 \cdot 2,2 \cdot 3,6 \cdot \cos 83,5°}$

$c = 4$

8.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung
Gegeben:
Seite-Seite-Seite (SSS): $a - b - c$
Seite-Winkel-Seite (SWS):
$a - b - \gamma, a - c - \beta, b - c - \alpha$
Seite-Seite-Winkel(SsW):
$a - b - \alpha, a - b - \beta, a - c - \alpha, a - c - \gamma,$
$b - c - \beta, b - c - \gamma$
Winkel-Winkel-Seite (WWS,WSW):
$c - \beta - \gamma, a - \alpha - \beta, a - \alpha - \gamma,$
$a - \beta - \gamma, b - \alpha - \beta, b - \alpha - \gamma,$
$b - \beta - \gamma, c - \alpha - \beta, c - \alpha - \gamma$
Gesucht:
- alle Winkel und alle Seiten
- Fläche
- Umfang
- Höhen,Seitenhalbierende,Winkelhalbierende
- In- und Umkreisradius
Eingabe:
Nur drei Eingaben können ungleich Null sein.
Ausgabe der Grafik nur im PDF-Format.

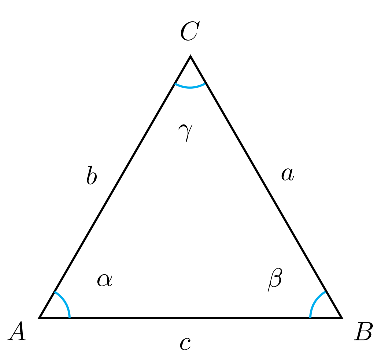
(1) $a = 4$ $b = 4$ $c = 4$
(2) $b = 7$ $c = 5$ $\alpha = 30$
(3) $a = 4$ $b = 4$ $c = 4$
(4) $a = 3$ $b = 4$ $c = 5$
(5) $a = 3$ $b = 5$ $c = 4$
(6) $a = 5$ $b = 4$ $c = 3$
(7) $a = 4$ $b = 3$ $\alpha = 90$
(8) $a = 8$ $c = 5$ $\alpha = 90$
(9) $b = 3$ $c = 5$ $\alpha = 90$
(10) $a = 3$ $b = 4$ $\beta = 90$
(11) $a = 3$ $c = 5$ $\beta = 90$
(12) $b = 8$ $c = 5$ $\beta = 90$
(13) $a = 3$ $b = 4$ $\gamma = 90$
(14) $a = 3$ $c = 5$ $\gamma = 90$
(15) $b = 3$ $c = 5$ $\gamma = 90$
(16) $a = 4$ $\alpha = 90$ $\beta = 70$
(17) $b = 5$ $\alpha = 90$ $\beta = 30$
(18) $c = 5$ $\alpha = 90$ $\gamma = 40$
(19) $a = 3$ $\alpha = 20$ $\beta = 90$
(20) $c = 5$ $\alpha = 30$ $\beta = 90$
(21) $b = 8$ $\beta = 90$ $\gamma = 45$
(22) $a = 3$ $\alpha = 20$ $\gamma = 90$
(23) $c = 5$ $\alpha = 35$ $\gamma = 90$
(24) $b = 3$ $\beta = 65$ $\gamma = 90$
(25) $a = 6$ $\alpha = 90$ $\beta = 30$
(26) $a = 5$ $\alpha = 90$ $\gamma = 30$
(27) $b = 3$ $c = 5$ $\alpha = 90$
(28) $a = 3$ $b = 4$ $\beta = 90$
(29) $a = 3$ $c = 5$ $\beta = 90$
(30) $a = 8$ $b = 4$ $c = 5$
(31) $a = 3$ $b = 7$ $c = 4$
(32) $a = 7$ $b = 4$ $c = 5$
(33) $a = 6$ $b = 2$ $c = 5$
(34) $a = 6$ $b = 5$ $\gamma = 25$
(35) $b = 5$ $c = 10$ $\alpha = 155$
(36) $b = 7$ $c = 5$ $\alpha = 30$
(37) $a = 6$ $c = 5$ $\beta = 40$
(38) $a = 6$ $b = 5$ $\gamma = 120$
(39) $a = 6$ $b = 5$ $\alpha = 50$
(40) $a = 6$ $b = 7$ $\beta = 60$
(41) $a = 6$ $c = 3\frac{1}{2}$ $\alpha = 50$
(42) $a = 2\frac{1}{2}$ $c = 4\frac{1}{2}$ $\beta = 60$
(43) $b = 4$ $c = 3\frac{1}{2}$ $\beta = 40$
(44) $b = 3\frac{1}{2}$ $c = 4\frac{1}{2}$ $\gamma = 70$
(45) $a = 6$ $\alpha = 30$ $\beta = 50$
(46) $a = 6$ $\alpha = 30$ $\gamma = 50$
(47) $b = 7$ $\alpha = 30$ $\beta = 50$
(48) $b = 7$ $\beta = 50$ $\gamma = 80$
(49) $c = 7$ $\alpha = 30$ $\gamma = 70$
(50) $c = 6$ $\beta = 50$ $\gamma = 40$
(51) $a = 2$ $b = 3$ $c = 4$
(52) $a = 2$ $b = 3$ $c = 4$
(53) $a = 2$ $b = 3$ $c = 4$
(54) $a = 3$ $b = 4$ $c = 5$
(55) $a = 3$ $b = 4$ $c = 5$
(56) $a = 3$ $b = 4$ $c = 5$
(57) $a = 3$ $b = 4$ $c = 5$
(58) $a = 3$ $b = 4$ $c = 5$
(59) $a = 3$ $b = 4$ $c = 5$
(60) $a = 3$ $b = 4$ $c = 5$
(61) $a = 3$ $b = 4$ $c = 5$
(62) $a = 3$ $b = 4$ $c = 5$
(63) $a = 3$ $b = 4$ $c = 5$
(64) $b = 4$ $c = 5$ $\alpha = 12$
(65) $b = 4$ $c = 5$ $\alpha = 120$
(66) $b = 4$ $c = 5$ $\alpha = 120$
(67) $b = 4$ $\alpha = 120$ $\gamma = 3$
(68) $b = 4$ $\alpha = 120$ $\gamma = 3$
(69) $b = 4$ $\alpha = 120$ $\gamma = 3$
(70) $b = 4$ $\alpha = 20$ $\beta = 40$
(71) $b = 4$ $\alpha = 20$ $\beta = 40$
(72) $b = 4$ $\alpha = 20$ $\beta = 40$
(73) $b = 4$ $\alpha = 20$ $\beta = 40$
(74) $c = 4$ $\alpha = 30$ $\beta = 40$
(75) $a = 3$ $c = 6$ $\beta = 56$
(76) $a = 3$ $c = 6$ $\beta = 56$
(77) $a = 3$ $c = 6$ $\beta = 56$
(78) $a = 3$ $b = 4$ $c = 5$
(79) $a = 3$ $b = 4$ $c = 5$
(80) $a = 3$ $b = 4$ $c = 5$
(81) $a = 4$ $b = 3$ $\gamma = 45$
(82) $a = 4$ $b = 3$ $\gamma = 45$
(83) $a = 2$ $b = 4$ $\alpha = 45$
(84) $a = 5$ $b = 4$ $\alpha = 45$
(85) $a = 5$ $b = 4$ $\alpha = 45$
(86) $a = 4$ $b = 5$ $c = 6$
(87) $a = 4$ $b = 5$ $c = 6$
(88) $a = 3$ $b = 4$ $c = 5$

8.2 Lösungen

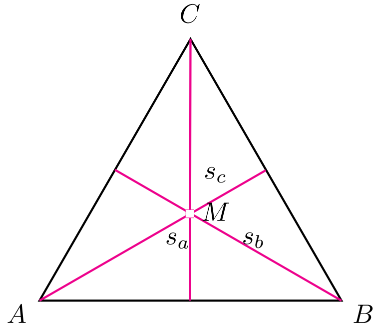
Aufgabe (1)

Seite-Seite-Seite
$a = 4 \quad b = 4 \quad c = 4$
Gleichseitiges Dreieck
$\alpha = 60° \quad \beta = 60° \quad \gamma = 60°$
Höhe: h_c
$h_c = \frac{1}{2} \cdot a \cdot \sqrt{3}$
$h_c = \frac{1}{2} \cdot 4 \cdot \sqrt{3}$
$h_c = 3,46$
$h_a = h_b = h_c = 3,46$
$s_a = s_b = s_c = 3,46$
$wha = whb = whc = 3,46$
Fläche: A
$A = \frac{1}{4} \cdot a^2 \cdot \sqrt{3}$
$A = \frac{1}{3} \cdot 4^2 \cdot \sqrt{3}$
$A = 6,93$

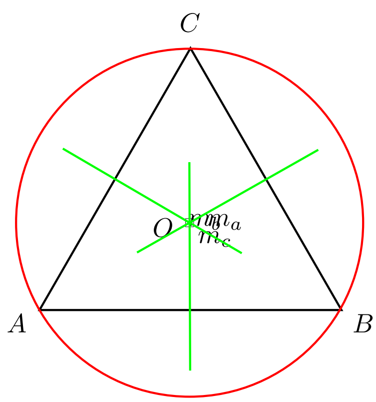
Umfang: $U = a + b + c$
$U = 4 + 4 + 4$
$U = 12$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 60°}$
$r_u = 2,31$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,93}{12}$
$r_i = 1,15$



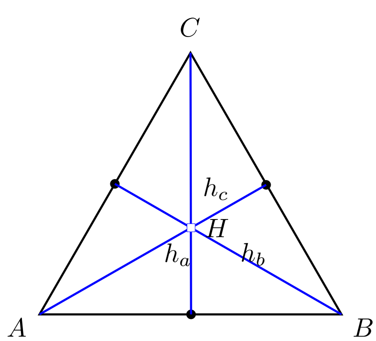
Seitenhalbierende-Schwerpunkt



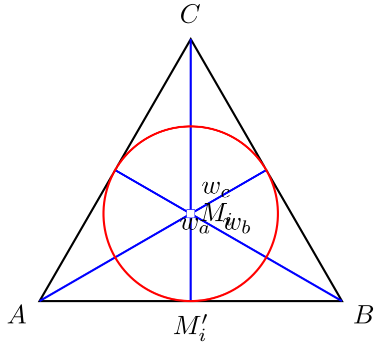
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



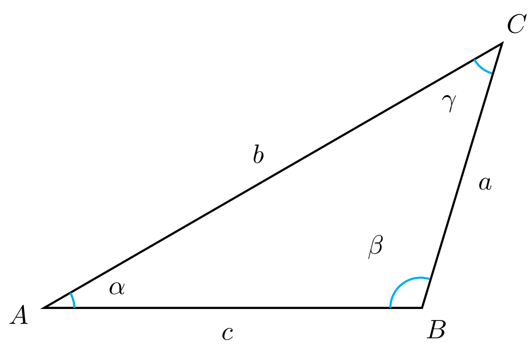
Aufgabe (2)

Seite-Winkel-Seite
$b = 7 \quad c = 5 \quad \alpha = 30°$

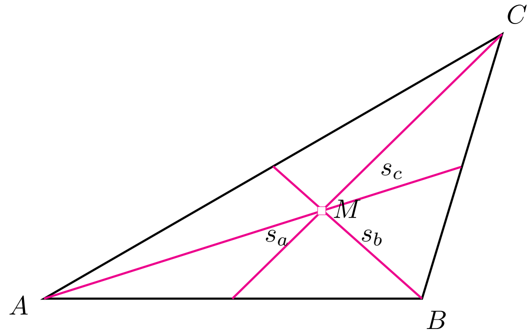
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos 30°}$
$a = 3,66$
Umfang: $U = a + b + c$
$U = 3,66 + 7 + 5$
$U = 15,7$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{3,66^2 + 5^2 - 7^2}{2 \cdot 3,66 \cdot 5}$
$\cos\beta = -0,29$
$\beta = \arccos(-0,29)$
$\beta = 107°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 107°$
$\gamma = 43,1°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 107°$
$h_a = 4,78$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3,66 \cdot 4,78$
$A = 8\frac{3}{4}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3,66 \cdot \sin 43,1°$
$h_b = 2\frac{1}{2}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 30°$
$h_c = 3\frac{1}{2}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 107}{\sin 58,1}$
$wha = 5,63$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3,66 \cdot \sin 43,1}{\sin 83,4}$
$whb = 2,52$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 30}{\sin 58,1}$
$whc = 2,15$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

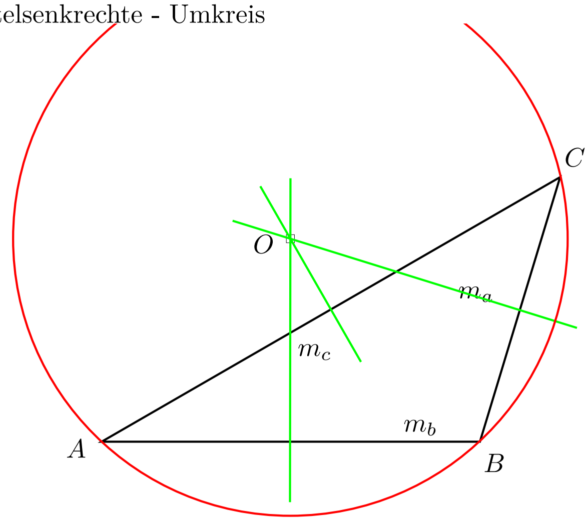
$s_a = \frac{1}{2}\sqrt{2(7^2+5^2)-3,66^2}$
$s_a = 5,8$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(3,66^2+5^2)-7^2}$
$s_b = 2,63$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3,66^2+7^2)-5^2}$
$s_c = 4,35$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3,66}{2 \cdot \sin 30^\circ}$
$r_u = 3,66$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8\frac{3}{4}}{15,7}$
$r_i = 1,12$



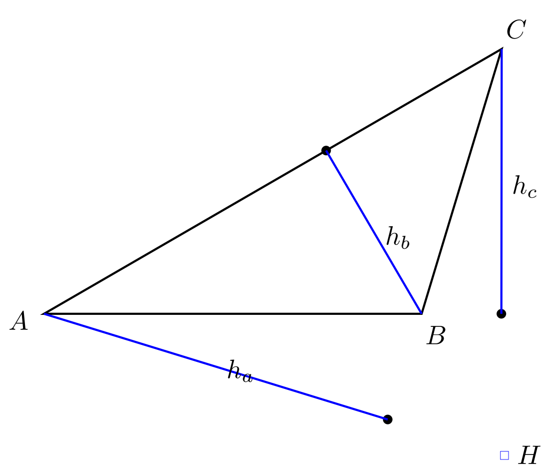
Seitenhalbierende-Schwerpunkt



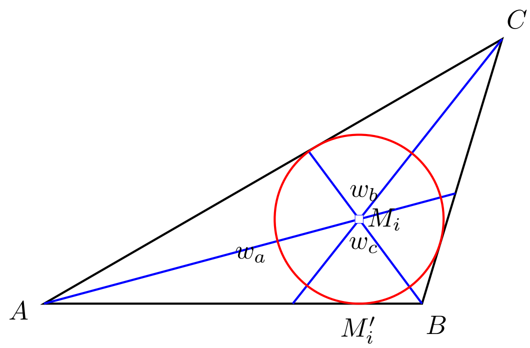
Mittelsenkrechte - Umkreis



Höhen



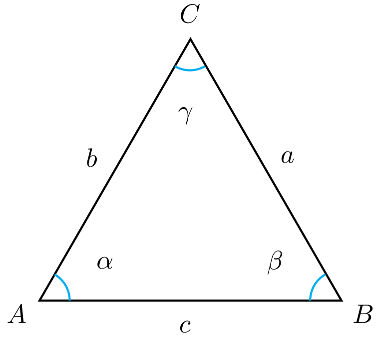
Winkelhalbierende-Inkreis



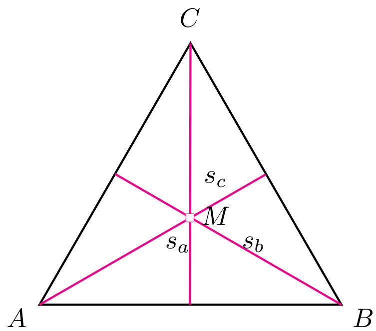
Aufgabe (3)

Seite-Seite-Seite
$a = 4 \quad b = 4 \quad c = 4$
Gleichseitiges Dreieck
$\alpha = 60° \quad \beta = 60° \quad \gamma = 60°$
Höhe: h_c
$h_c = \frac{1}{2} \cdot a \cdot \sqrt{3}$
$h_c = \frac{1}{2} \cdot 4 \cdot \sqrt{3}$
$h_c = 3,46$
$h_a = h_b = h_c = 3,46$
$s_a = s_b = s_c = 3,46$
$wha = whb = whc = 3,46$
Fläche: A
$A = \frac{1}{4} \cdot a^2 \cdot \sqrt{3}$
$A = \frac{1}{3} \cdot 4^2 \cdot \sqrt{3}$
$A = 6,93$

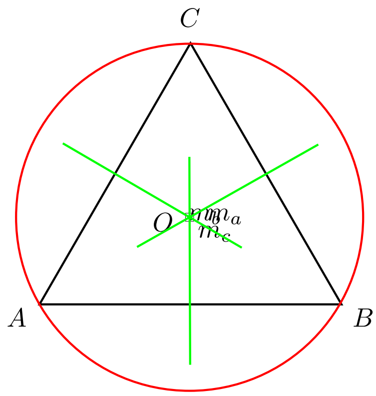
Umfang: $U = a + b + c$
$U = 4 + 4 + 4$
$U = 12$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 60°}$
$r_u = 2,31$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,93}{12}$
$r_i = 1,15$



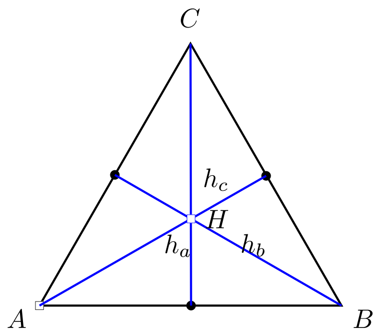
Seitenhalbierende-Schwerpunkt



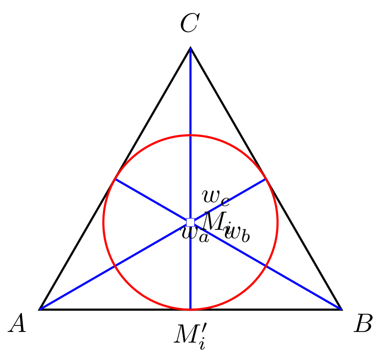
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



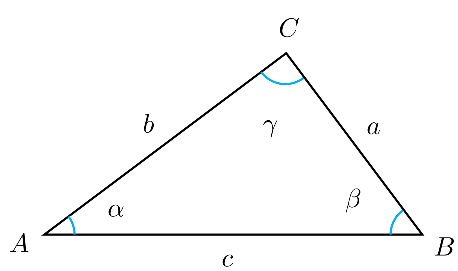
Aufgabe (4)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

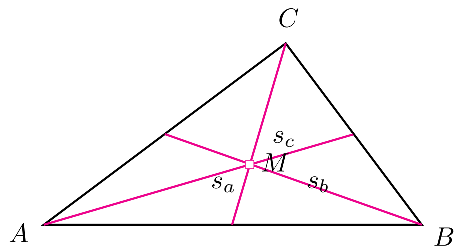
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$

$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ

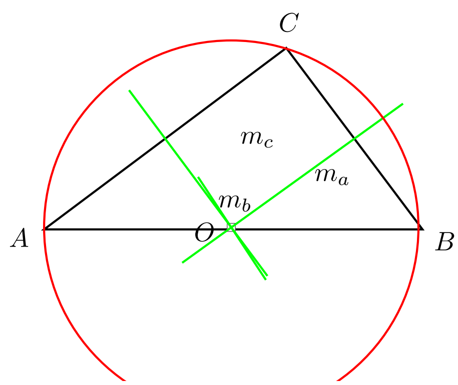
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



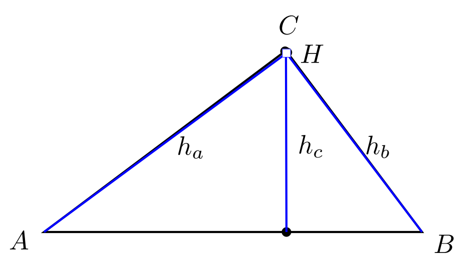
Seitenhalbierende-Schwerpunkt



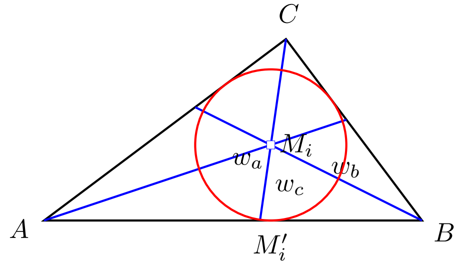
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



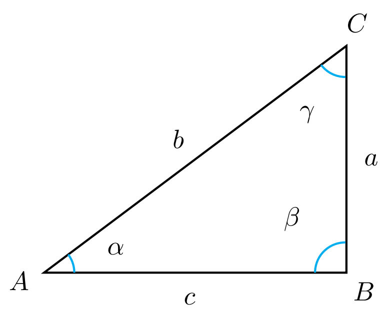
Aufgabe (5)

Seite-Seite-Seite
$a = 3 \quad b = 5 \quad c = 4$
Pythagoras: $b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 4^2}$
$b = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ Hypothenuse: $b = 5$ Kathete: $c = 4 \quad \beta = 90°$

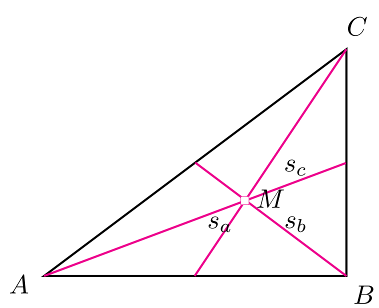
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / -\alpha \qquad / -\beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 36,9° - 90°$
$\gamma = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 5 + 4$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 90°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 53,1°$
$h_b = 2\frac{2}{5}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 36,9°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4 \cdot \sin 90}{\sin 71,6}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 53,1}{\sin 81,9}$
$whb = 2,42$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 36,9}{\sin 71,6}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 4^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_b = 2\frac{1}{2}$

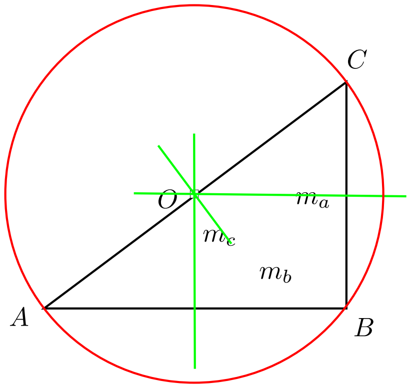
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2+5^2)-4^2}$
$s_c = 3,28$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



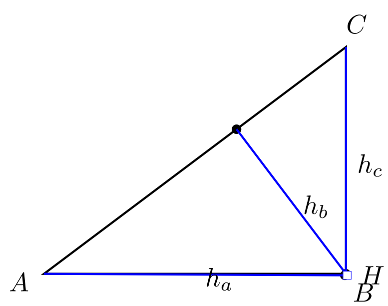
Seitenhalbierende-Schwerpunkt



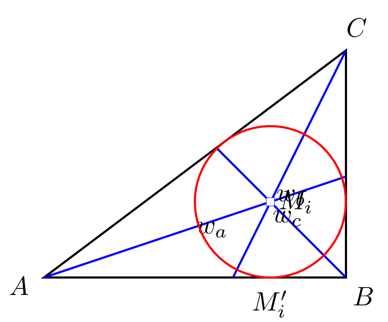
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

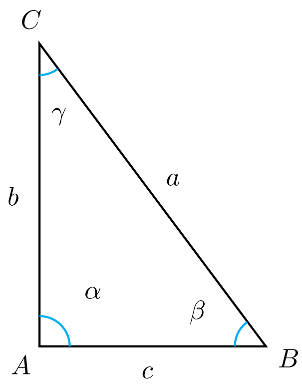


Aufgabe (6)

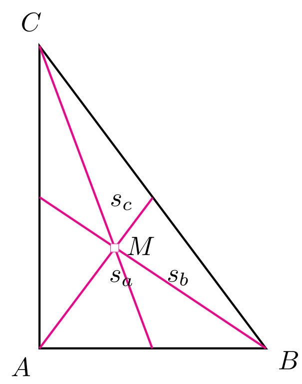
Seite-Seite-Seite
$a = 5 \quad b = 4 \quad c = 3$
Pythagoras: $a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{4^2 + 3^2}$
$a = 5$ Rechtwinkliges Dreieck
Hypothenuse: $a = 5$ Kathete: $b = 4$ Kathete: $c = 3$ $\alpha = 90°$

Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{4}{5}$
$\beta = 53,1$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 53,1°$
$\gamma = 36,9°$
Umfang: $U = a + b + c$
$U = 5 + 4 + 3$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3 \cdot \sin 53,1°$
$h_a = 2\frac{2}{5}$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 2\frac{2}{5}$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 36,9°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 90°$
$h_c = 4$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

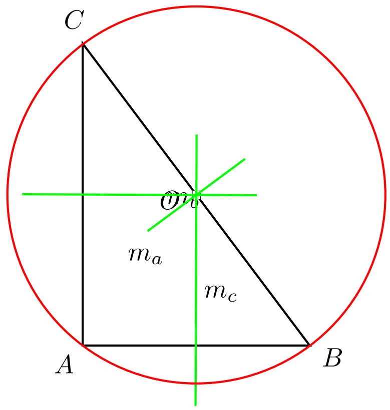
$wha = \frac{3 \cdot \sin 53,1}{\sin 81,9}$
$wha = 2,42$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 36,9}{\sin 117}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 90}{\sin 81,9}$
$whc = 5,05$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 5^2}$
$s_a = 2\frac{1}{2}$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2 + 3^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2 + 4^2) - 3^2}$
$s_c = 4,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 90^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



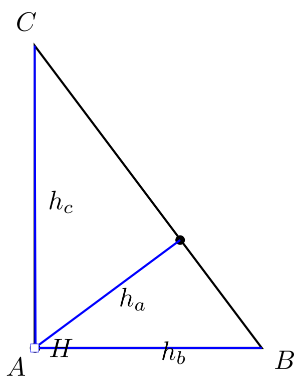
Seitenhalbierende-Schwerpunkt



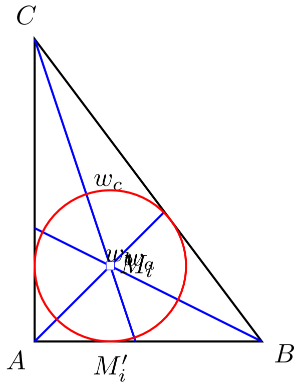
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



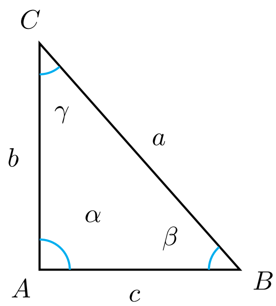
Aufgabe (7)

Seite-Seite-Winkel
$a = 4 \quad b = 3 \quad \alpha = 90°$

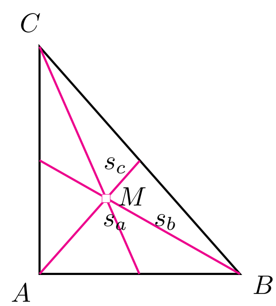
Pythagoras: $a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,65$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{4}$
$\beta = 48,6$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 48,6°$
$\gamma = 41,4°$
Umfang: $U = a + b + c$
$U = 4 + 3 + 2,65$
$U = 9,65$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,65 \cdot \sin 48,6°$
$h_a = 1,98$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 1,98$

$A = 3,97$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 41,4°$
$h_b = 2,65$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,65 \cdot \sin 48,6}{\sin 86,4}$
$wha = 1,99$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 41,4}{\sin 114}$
$whb = 2,9$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 86,4}$
$whc = 4,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 2,65^2) - 4^2}$
$s_a = 2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 2,65^2) - 3^2}$

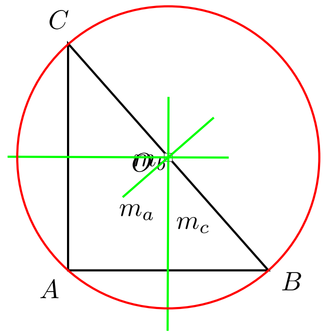
$s_b = 3,04$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2+3^2)-2,65^2}$
$s_c = 3,2$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 90°}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,97}{9,65}$
$r_i = 0,823$



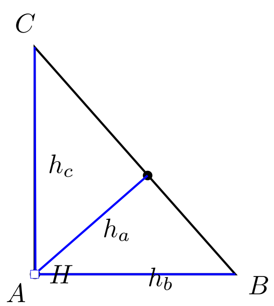
Seitenhalbierende-Schwerpunkt



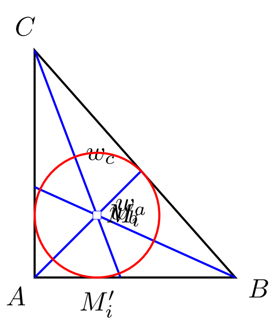
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

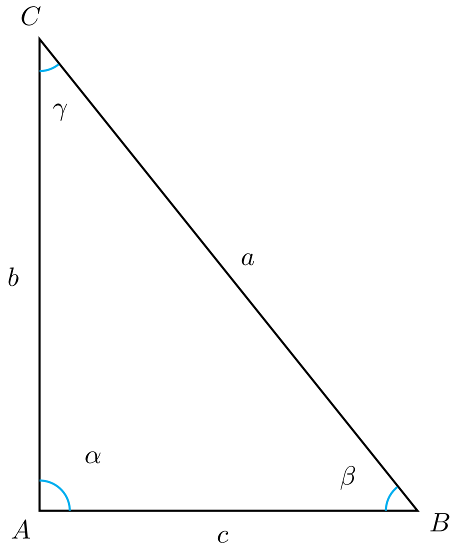


Aufgabe (8)

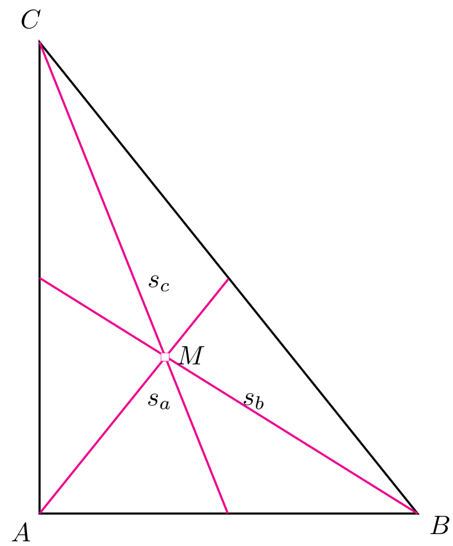
Seite-Seite-Winkel
$a = 8 \quad c = 5 \quad \alpha = 90°$

Pythagoras: $a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{8^2 - 5^2}$
$b = 6,24$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{6,24}{8}$
$\beta = 51,3$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 51,3°$
$\gamma = 38,7°$
Umfang: $U = a + b + c$
$U = 8 + 6,24 + 5$
$U = 19,2$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 51,3°$
$h_a = 3,9$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 8 \cdot 3,9$
$A = 15,6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 8 \cdot \sin 38,7°$
$h_b = 5$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 6,24 \cdot \sin 90°$
$h_c = 6,24$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

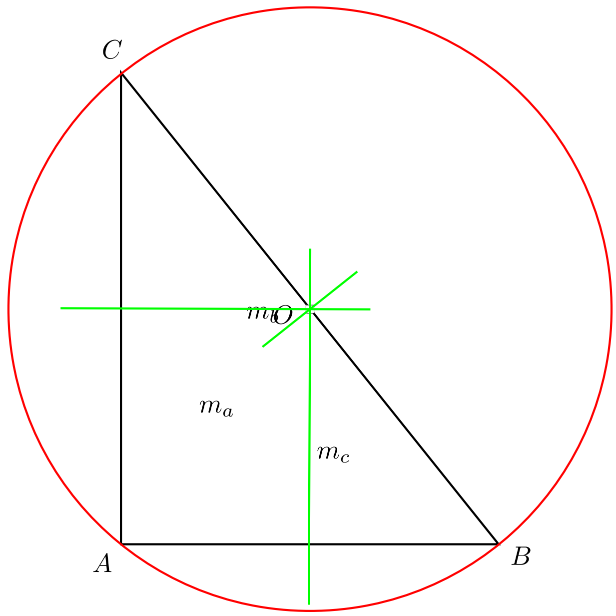
$wha = \frac{5 \cdot \sin 51,3}{\sin 83,7}$
$wha = 3,93$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{8 \cdot \sin 38,7}{\sin 116}$
$whb = 5,55$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{6,24 \cdot \sin 90}{\sin 83,7}$
$whc = 8,05$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(6,24^2 + 5^2) - 8^2}$
$s_a = 4$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 6,24^2}$
$s_b = 5,89$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(8^2 + 6,24^2) - 5^2}$
$s_c = 6,46$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{8}{2 \cdot \sin 90°}$
$r_u = 4$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15,6}{19,2}$
$r_i = 1,62$



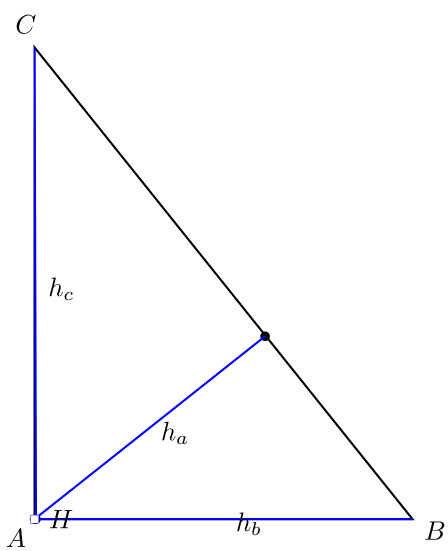
Seitenhalbierende-Schwerpunkt



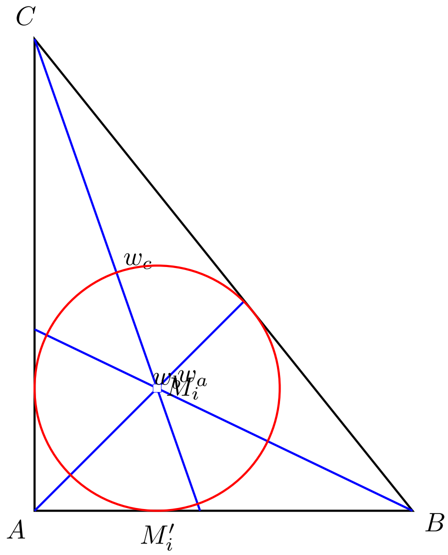
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



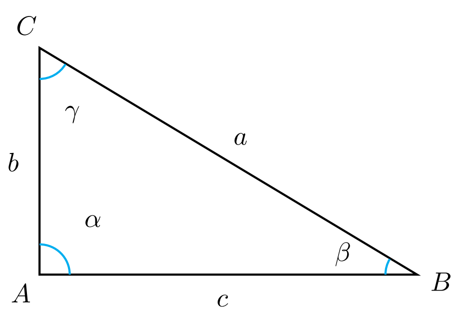
Aufgabe (9)

Seite-Winkel-Seite
$b = 3 \quad c = 5 \quad \alpha = 90°$

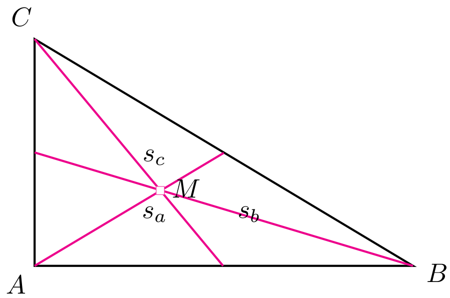
Pythagoras: $a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{3^2 + 5^2}$
$a = 5,83$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{5,83}$
$\beta = 31$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 31°$
$\gamma = 59°$
Umfang: $U = a + b + c$
$U = 5,83 + 3 + 5$
$U = 13,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 31°$

$h_a = 2,57$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,83 \cdot 2,57$
$A = 7\frac{1}{2}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,83 \cdot \sin 59°$
$h_b = 5$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 31}{\sin 104}$
$wha = 2,65$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,83 \cdot \sin 59}{\sin 105}$
$whb = 5,19$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 104}$
$whc = 6,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

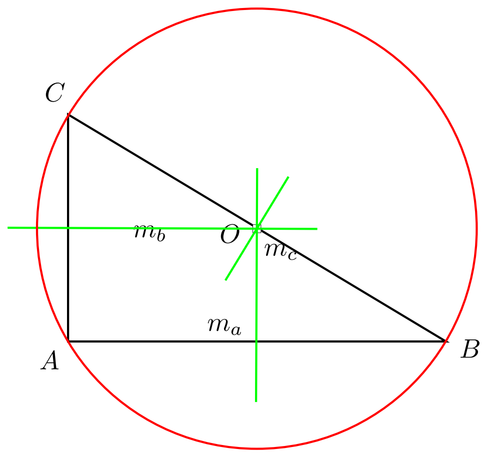
$s_a = \frac{1}{2}\sqrt{2(3^2+5^2)-5,83^2}$
$s_a = 2,92$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,83^2+5^2)-3^2}$
$s_b = 5,22$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,83^2+3^2)-5^2}$
$s_c = 4,39$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,83}{2 \cdot \sin 90°}$
$r_u = 2,92$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,8}$
$r_i = 1,08$



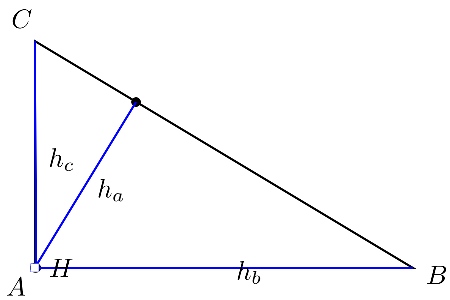
Seitenhalbierende-Schwerpunkt



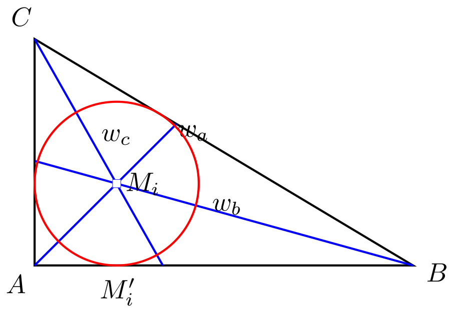
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



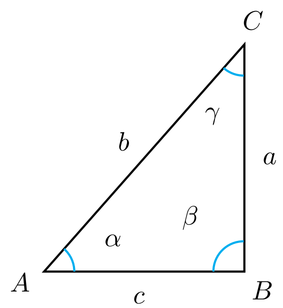
Aufgabe (10)

Seite-Seite-Winkel
$a = 3 \quad b = 4 \quad \beta = 90°$

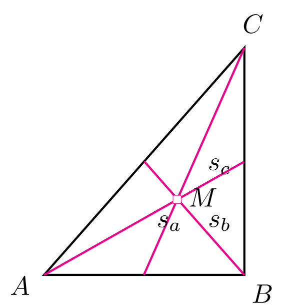
Pythagoras: $b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,65$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{4}$
$\alpha = 48,6$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 48,6° - 90°$
$\gamma = 41,4°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 2,65$
$U = 9,65$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,65 \cdot \sin 90°$
$h_a = 2,65$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 2,65$
$A = 3,97$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 41,4°$
$h_b = 1,98$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 48,6°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,65 \cdot \sin 90}{\sin 65,7}$
$wha = 2,9$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 41,4}{\sin 93,6}$
$whb = 1,99$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 48,6}{\sin 65,7}$
$whc = 2,47$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 2,65^2) - 3^2}$
$s_a = 3,04$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 2,65^2) - 4^2}$
$s_b = 2$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

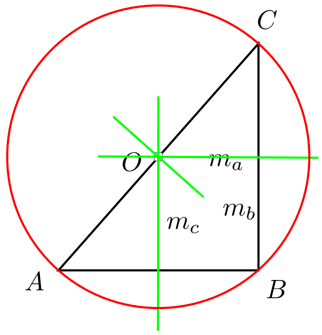
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-2,65^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 48,6°}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,97}{9,65}$
$r_i = 0,823$



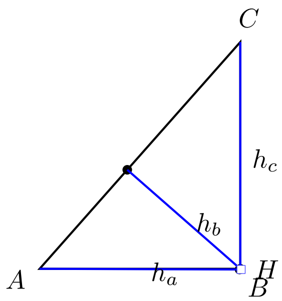
Seitenhalbierende-Schwerpunkt



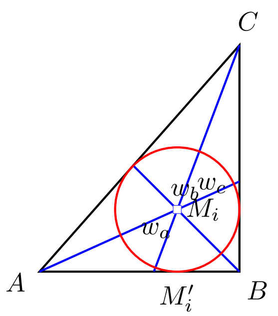
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

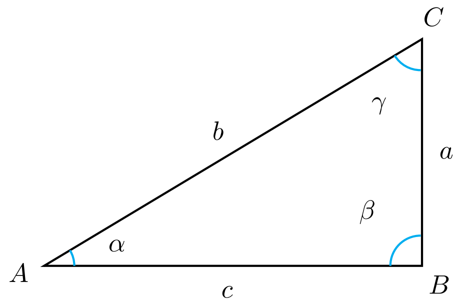


Aufgabe (11)

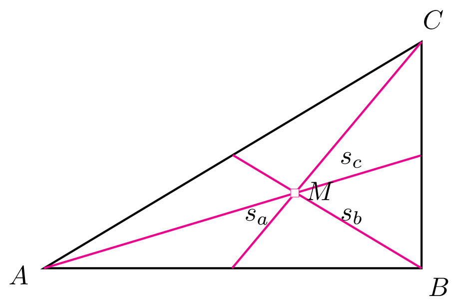
Seite-Winkel-Seite
$a = 3 \quad c = 5 \quad \beta = 90°$

Pythagoras: $b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 5^2}$
$b = 5,83$
Sinus: $\quad \sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5,83}$
$\alpha = 31$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 31° - 90°$
$\gamma = 59°$
Umfang: $U = a + b + c$
$U = 3 + 5,83 + 5$
$U = 13,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 5$
$A = 7\frac{1}{2}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 59°$
$h_b = 2,57$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,83 \cdot \sin 31°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

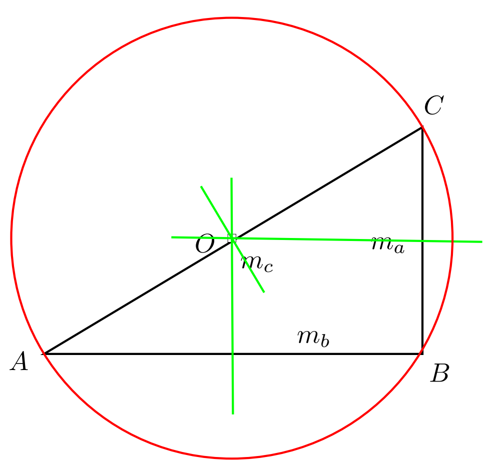
$wha = \frac{5 \cdot \sin 90}{\sin 74,5}$
$wha = 5,19$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 59}{\sin 76}$
$whb = 2,65$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,83 \cdot \sin 31}{\sin 74,5}$
$whc = 1,6$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,83^2 + 5^2) - 3^2}$
$s_a = 5,22$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,83^2}$
$s_b = 2,92$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 5,83^2) - 5^2}$
$s_c = 3,61$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 31^\circ}$
$r_u = 2,92$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,8}$
$r_i = 1,08$



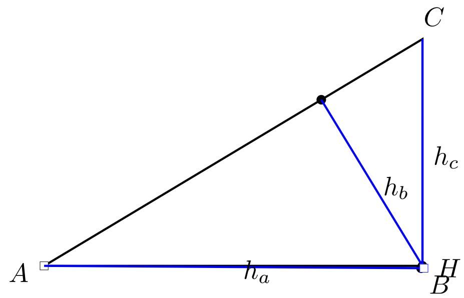
Seitenhalbierende-Schwerpunkt



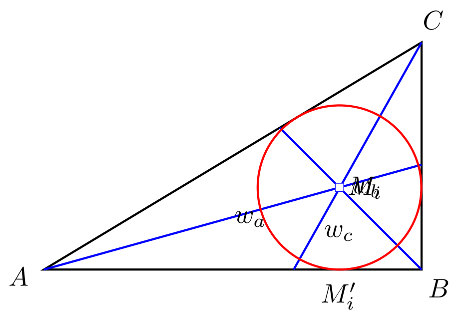
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (12)

Seite-Seite-Winkel
$b = 8 \quad c = 5 \quad \beta = 90°$

Pythagoras: $b^2 = a^2 + c^2 \quad / - c^2$
$a^2 = b^2 - c^2$
$a = \sqrt{b^2 - c^2}$
$a = \sqrt{8^2 - 5^2}$
$a = 6,24$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{6,24}{8}$
$\alpha = 51,3$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 51,3° - 90°$

$\gamma = 38,7°$
Umfang: $U = a + b + c$
$U = 6,24 + 8 + 5$
$U = 19,2$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6,24 \cdot 5$
$A = 15,6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6,24 \cdot \sin 38,7°$
$h_b = 3,9$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8 \cdot \sin 51,3°$
$h_c = 6,24$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 64,3}$
$wha = 5,55$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6,24 \cdot \sin 38,7}{\sin 96,3}$
$whb = 3,93$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{8 \cdot \sin 51,3}{\sin 64,3}$

$whc = 5,41$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 6,24^2}$

$s_a = 5,89$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(6,24^2 + 5^2) - 8^2}$

$s_b = 4$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(6,24^2 + 8^2) - 5^2}$

$s_c = 5,96$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

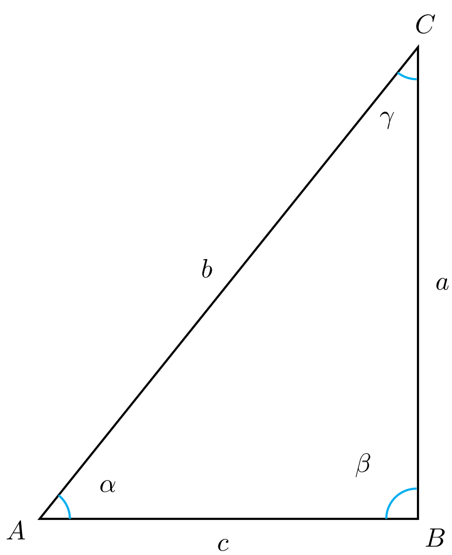
$r_u = \frac{6,24}{2 \cdot \sin 51,3^\circ}$

$r_u = 4$

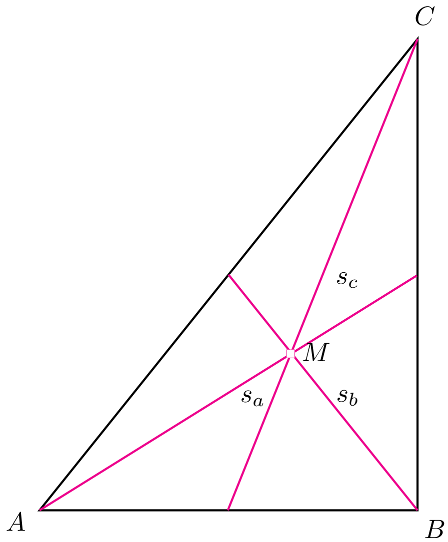
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 15,6}{19,2}$

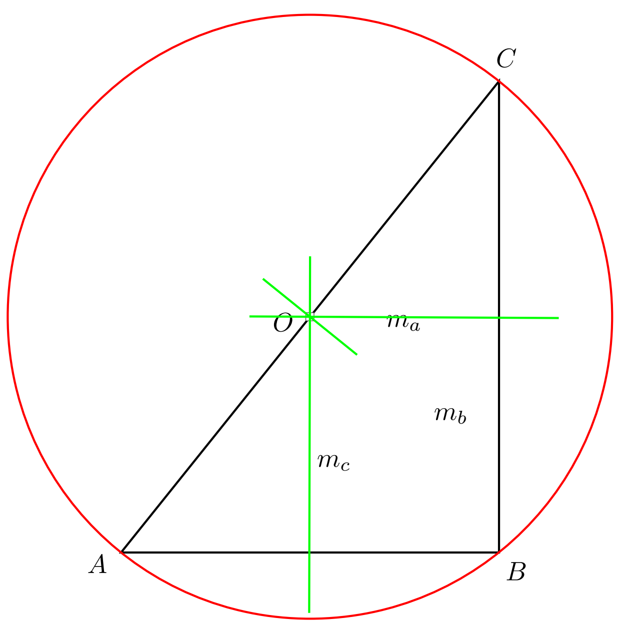
$r_i = 1,62$



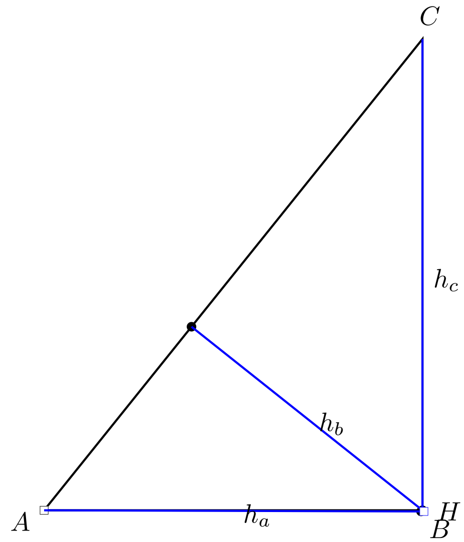
Seitenhalbierende-Schwerpunkt



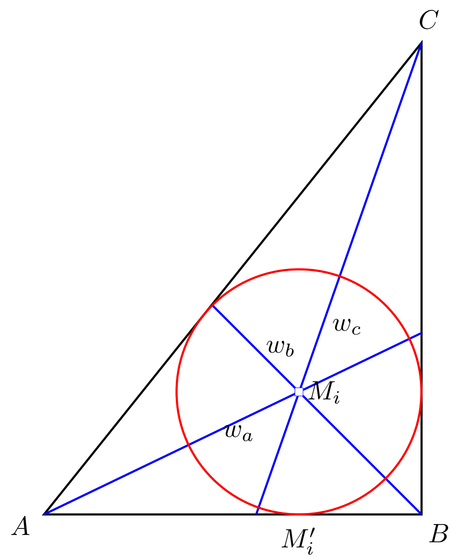
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (13)

Seite-Winkel-Seite

$a = 3 \quad b = 4 \quad \gamma = 90°$

Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$

$whb = 3,35$

Winkelhalbierende: γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$

$whc = 1,9$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$

$s_a = 4,27$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$

$s_b = 3,61$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$

$s_c = 2,92$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

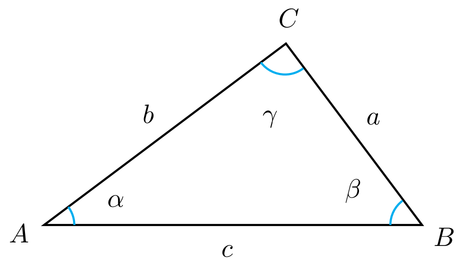
$r_u = \frac{3}{2 \cdot \sin 36,9°}$

$r_u = 2\frac{1}{2}$

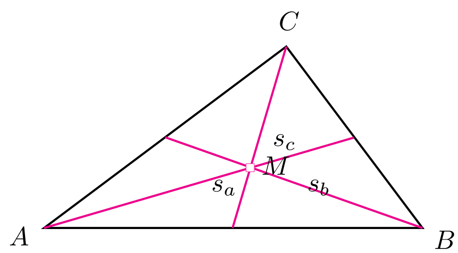
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 6}{12}$

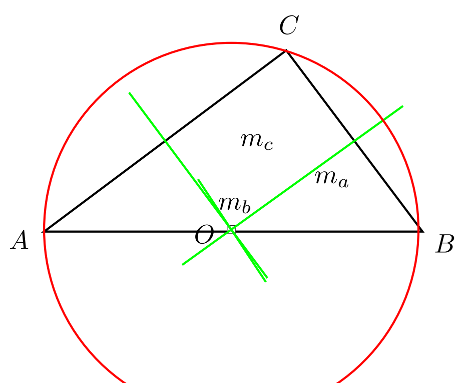
$r_i = 1$



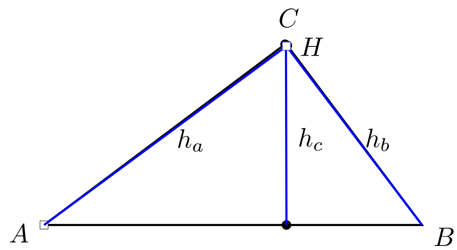
Seitenhalbierende-Schwerpunkt



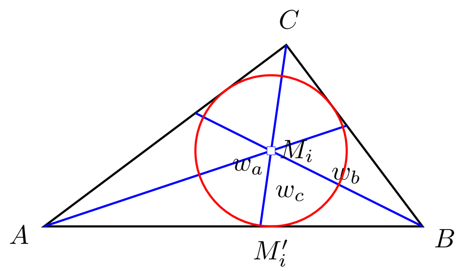
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



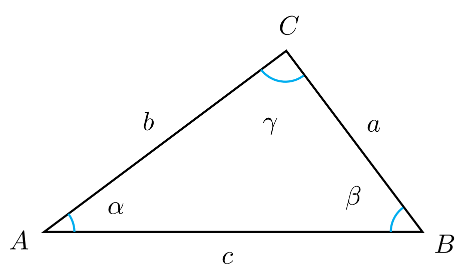
Aufgabe (14)

Seite-Seite-Winkel
$a = 3 \quad c = 5 \quad \gamma = 90°$

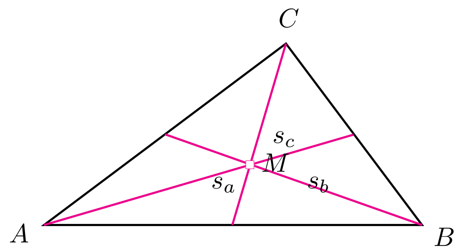
Pythagoras: $c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{5^2 - 3^2}$
$b = 4$
Sinus: $\sin\alpha = \dfrac{a}{c}$
$\sin\alpha = \dfrac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$

$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

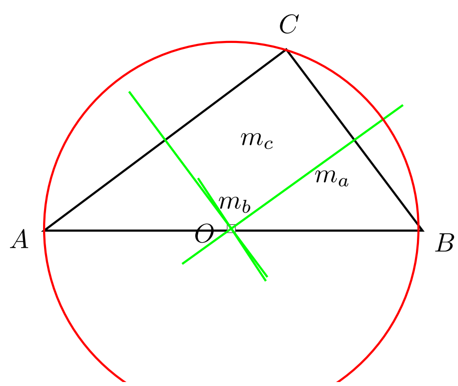
$whc = \frac{b \cdot \sin \alpha}{\sin \delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2+5^2)-3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2+5^2)-4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin \alpha}$
$r_u = \frac{a}{2 \cdot \sin \alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



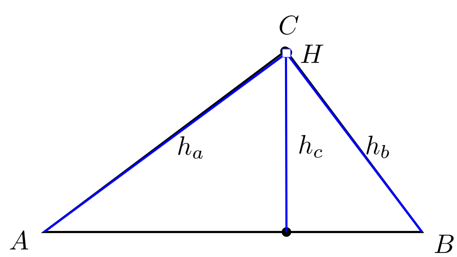
Seitenhalbierende-Schwerpunkt



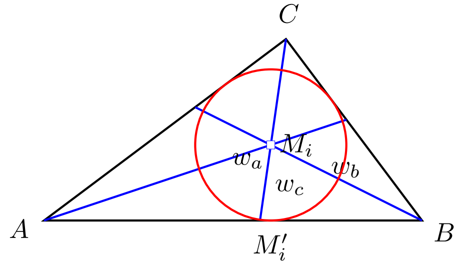
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



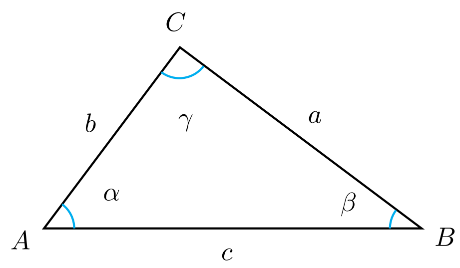
Aufgabe (15)

Seite-Seite-Winkel
$b = 3 \quad c = 5 \quad \gamma = 90°$

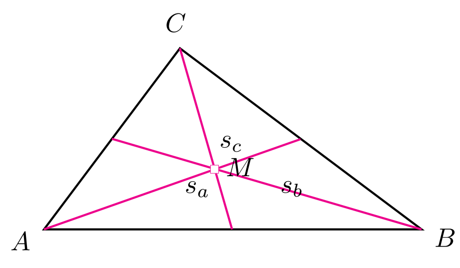
Pythagoras: $c^2 = a^2 + b^2 \quad / - b^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2 - b^2}$
$a = \sqrt{5^2 - 3^2}$
$a = 4$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{4}{5}$
$\alpha = 53,1$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 53,1° - 90°$
$\beta = 36,9°$
Umfang: $U = a + b + c$
$U = 4 + 3 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 36,9°$
$h_a = 3$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 3$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 90°$
$h_b = 4$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 53,1°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 36,9}{\sin 117}$
$wha = 3,35$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 90}{\sin 71,6}$
$whb = 4,22$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 53,1}{\sin 117}$
$whc = 3,58$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_a = 3,61$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_b = 4,27$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

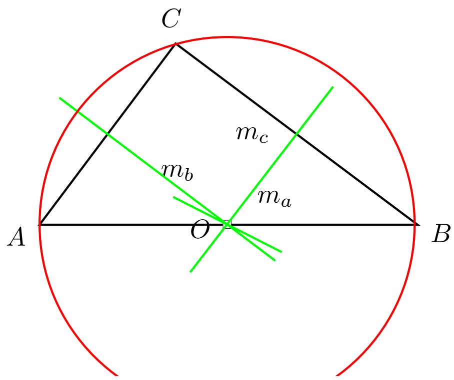
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 5^2}$
$s_c = 3,2$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 53,1°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



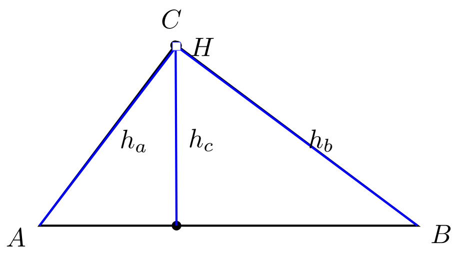
Seitenhalbierende-Schwerpunkt



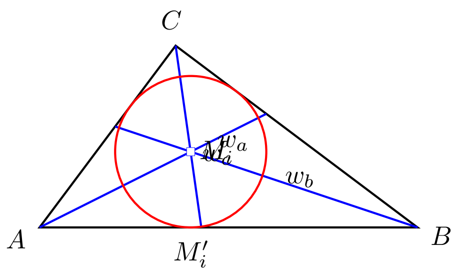
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

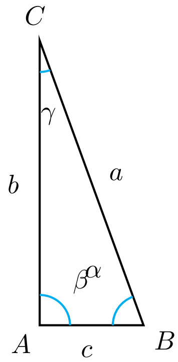


Aufgabe (16)

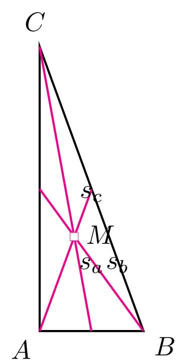
Winkel-Winkel-Seite
$a = 4 \quad \alpha = 90° \quad \beta = 70°$

Winkelsumme: $\alpha+\beta+\gamma=180°$
$\alpha+\beta+\gamma=180 \qquad /-\alpha \qquad /-\beta$
$\gamma=180°-\alpha-\beta$
$\gamma=180°-90°-70°$
$\gamma=20°$
Sinus: $\quad \sin\beta=\dfrac{b}{a}$
$\sin\beta=\dfrac{b}{a} \quad /\cdot a$
$b=a\cdot\sin\beta$
$b=4\cdot\sin 70$
$b=3,76$
Pythagoras: $a^2=b^2+c^2 \quad /-b^2$
$c^2=a^2-b^2$
$c=\sqrt{a^2-b^2}$
$c=\sqrt{4^2-3,76^2}$
$c=1,37$
Umfang: $U=a+b+c$
$U=4+3,76+1,37$
$U=9,13$
Höhe: h_a
$\sin\beta=\dfrac{h_a}{c}$
$\sin\beta=\dfrac{h_a}{c} \quad /\cdot c$
$h_a=c\cdot\sin\beta$
$h_a=1,37\cdot\sin 70°$
$h_a=1,29$
Flaeche: $\quad A=\dfrac{1}{2}\cdot a\cdot h_a$
$A=\dfrac{1}{2}\cdot 4\cdot 1,29$
$A=2,57$
Höhe: h_b
$\sin\gamma=\dfrac{h_b}{a}$
$\sin\gamma=\dfrac{h_b}{a} \quad /\cdot a$
$h_b=a\cdot\sin\gamma$
$h_b=4\cdot\sin 20°$
$h_b=1,37$
Höhe: h_c
$\sin\alpha=\dfrac{h_c}{b}$
$\sin\alpha=\dfrac{h_c}{b} \quad /\cdot b$
$h_c=b\cdot\sin\alpha$
$h_c=3,76\cdot\sin 90°$
$h_c=3,76$
Winkelhalbierende: α
$\delta=180-\beta-\dfrac{\alpha}{2}$
Sinus-Satz: $\dfrac{wha}{\sin\beta}=\dfrac{c}{\sin\delta}$
$\dfrac{wha}{\sin\beta}=\dfrac{c}{\sin\delta} \qquad /\cdot\sin\beta$
$wha=\dfrac{c\cdot\sin\beta}{\sin\delta}$
$wha=\dfrac{1,37\cdot\sin 70}{\sin 65}$
$wha=1,42$

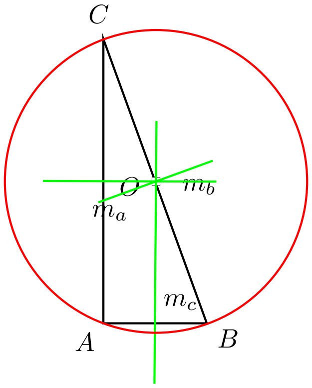
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 20}{\sin 125}$
$whb = 1,67$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3,76 \cdot \sin 90}{\sin 65}$
$whc = 4,41$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,76^2 + 1,37^2) - 4^2}$
$s_a = 2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 1,37^2) - 3,76^2}$
$s_b = 2,32$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3,76^2) - 1,37^2}$
$s_c = 3,4$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 90°}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,57}{9,13}$
$r_i = 0,563$



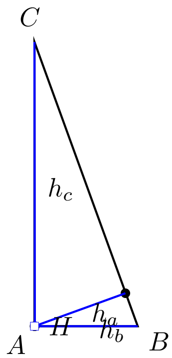
Seitenhalbierende-Schwerpunkt



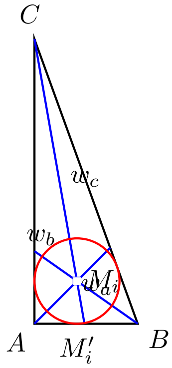
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



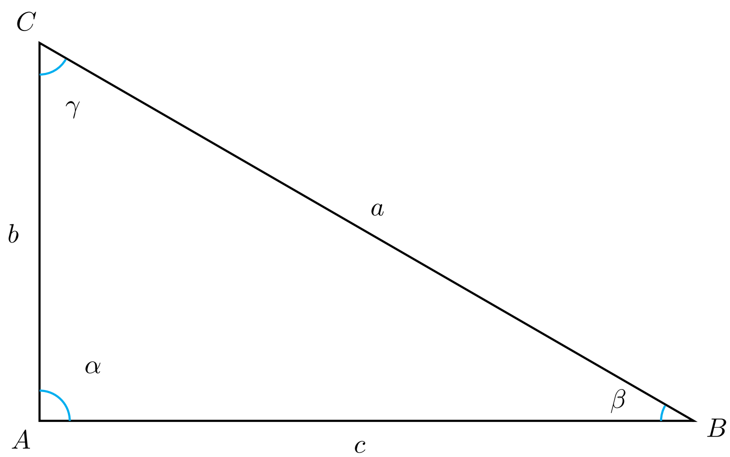
Aufgabe (17)

inkel-Winkel-Seite
$b = 5 \quad \alpha = 90° \quad \beta = 30°$

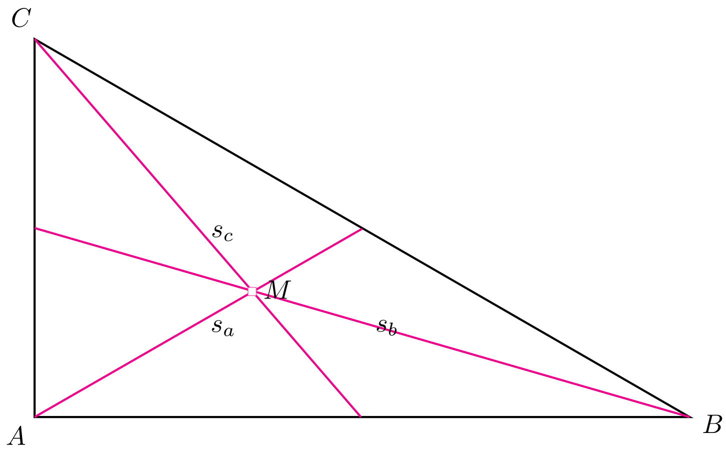
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30°$
$\gamma = 60°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$a \cdot \sin\beta = b \quad / : sin\beta$
$a = \frac{b}{\sin\beta}$
$a = \frac{5}{\sin 30}$
$a = 10$
Pythagoras: $a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{10^2 - 5^2}$
$c = 8,66$
Umfang: $U = a + b + c$
$U = 10 + 5 + 8,66$
$U = 23,7$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,66 \cdot \sin 30°$
$h_a = 4,33$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 10 \cdot 4,33$
$A = 21,7$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 10 \cdot \sin 60°$
$h_b = 8,66$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 90°$
$h_c = 5$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{8,66 \cdot \sin 30}{\sin 105}$
$wha = 4,48$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{10 \cdot \sin 60}{\sin 105}$
$whb = 8,97$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 90}{\sin 105}$
$whc = 10,4$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 8,66^2) - 10^2}$
$s_a = 5$

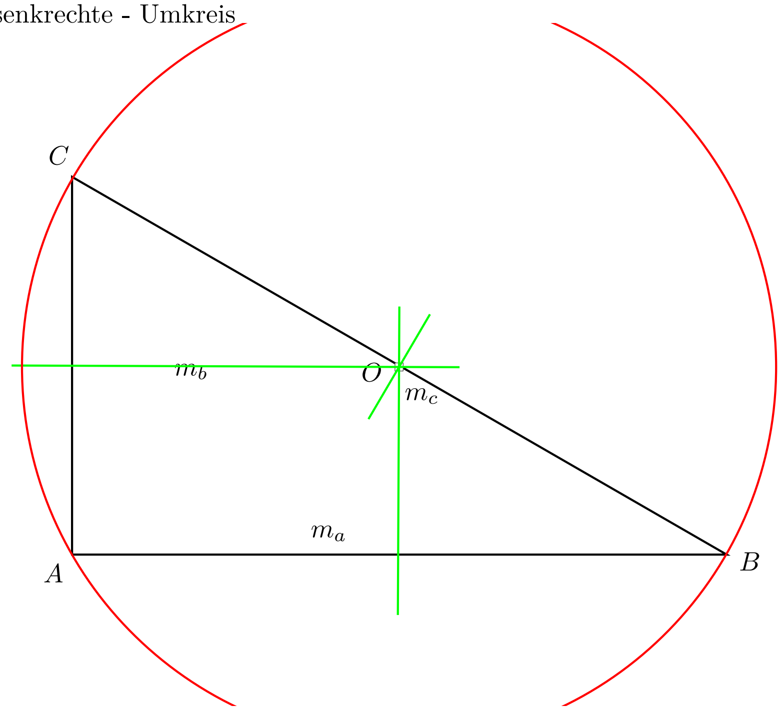
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(10^2+8,66^2)-5^2}$
$s_b = 9,01$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(10^2+5^2)-8,66^2}$
$s_c = 7\frac{1}{2}$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2\cdot\sin\alpha}$
$r_u = \frac{10}{2\cdot\sin 90°}$
$r_u = 5$
Inkreisradius: $r_i = \frac{2\cdot A}{U}$
$r_i = \frac{2\cdot 21,7}{23,7}$
$r_i = 1,83$



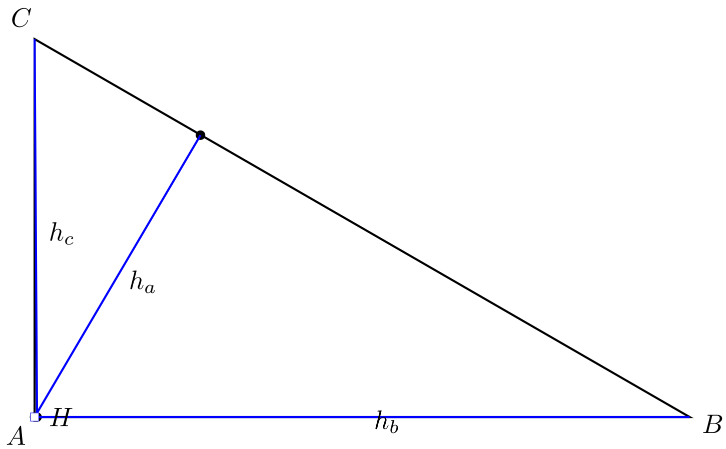
Seitenhalbierende-Schwerpunkt



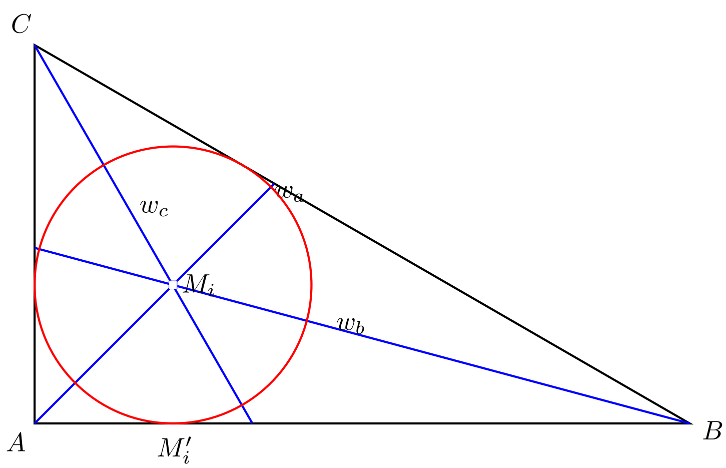
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



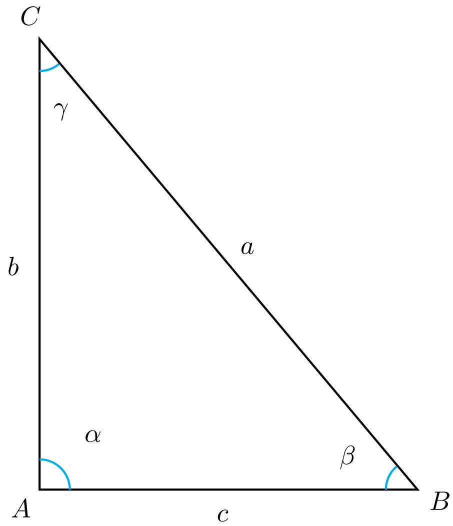
Aufgabe (18)

Winkel-Winkel-Seite
$c = 5 \quad \gamma = 40° \quad \alpha = 90°$

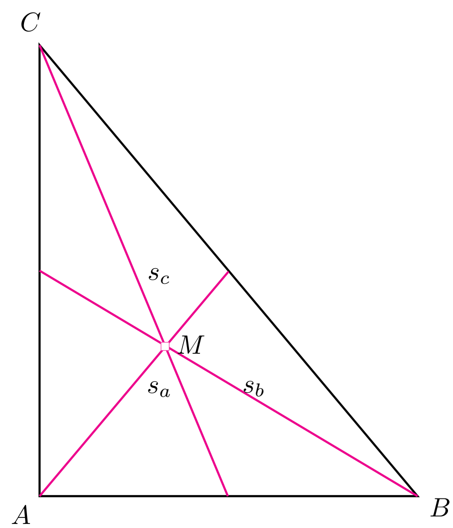
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 90° - 40°$

$\beta = 50°$
Kosinus: $\cos\beta = \frac{c}{a}$
$\cos\beta = \frac{c}{a} \quad / \cdot a$
$a \cdot \cos\beta = c \quad / : cos\beta$
$a = \frac{c}{\cos\beta}$
$a = \frac{5}{\cos 50}$
$a = 7,78$
Pythagoras: $a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{7,78^2 - 5^2}$
$b = 5,96$
Umfang: $U = a + b + c$
$U = 7,78 + 5,96 + 5$
$U = 18,7$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 50°$
$h_a = 3,83$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7,78 \cdot 3,83$
$A = 14,9$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7,78 \cdot \sin 40°$
$h_b = 5$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,96 \cdot \sin 90°$
$h_c = 5,96$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 50}{\sin 85}$
$wha = 3,84$
Winkelhalbierende: β

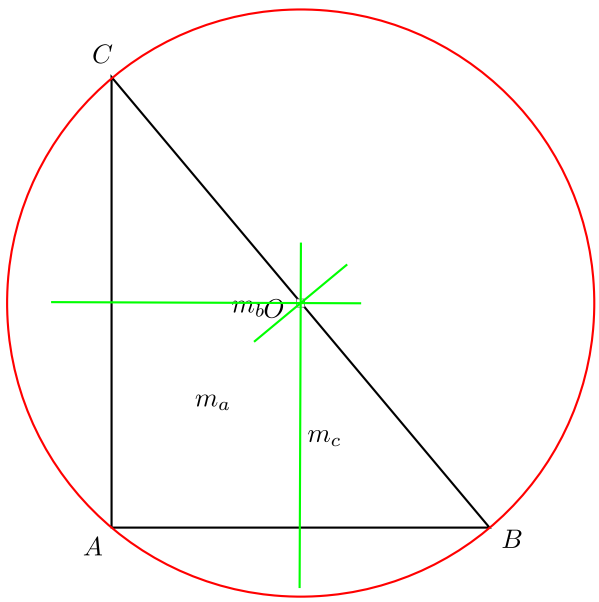
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7,78 \cdot \sin 40}{\sin 115}$
$whb = 5,52$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,96 \cdot \sin 90}{\sin 85}$
$whc = 7,81$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,96^2 + 5^2) - 7,78^2}$
$s_a = 3,89$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(7,78^2 + 5^2) - 5,96^2}$
$s_b = 5,82$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(7,78^2 + 5,96^2) - 5^2}$
$s_c = 6,26$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7,78}{2 \cdot \sin 90^\circ}$
$r_u = 3,89$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 14,9}{18,7}$
$r_i = 1,59$



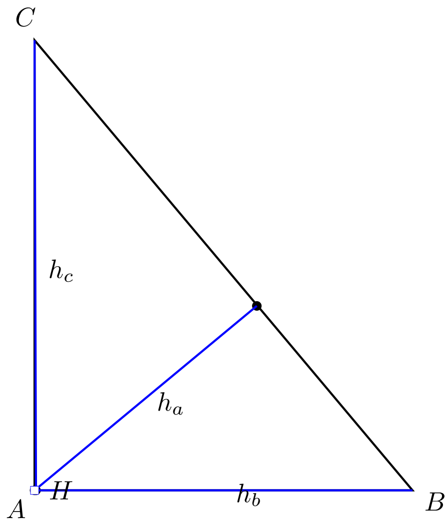
Seitenhalbierende-Schwerpunkt



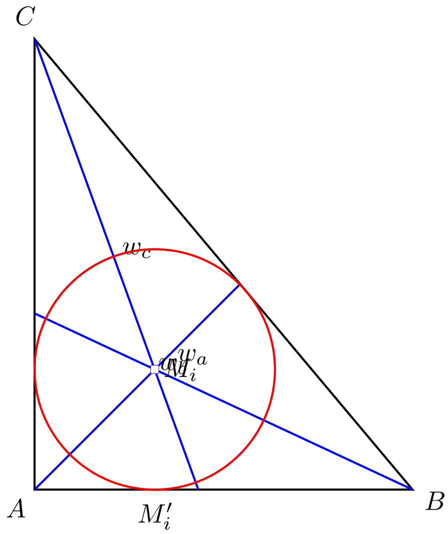
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



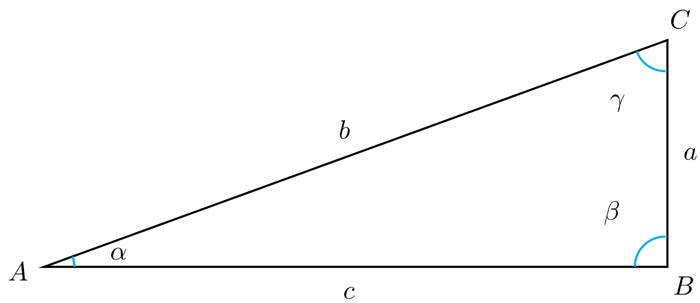
Aufgabe (19)

Winkel-Winkel-Seite
$a = 3 \quad \alpha = 20° \quad \beta = 90°$

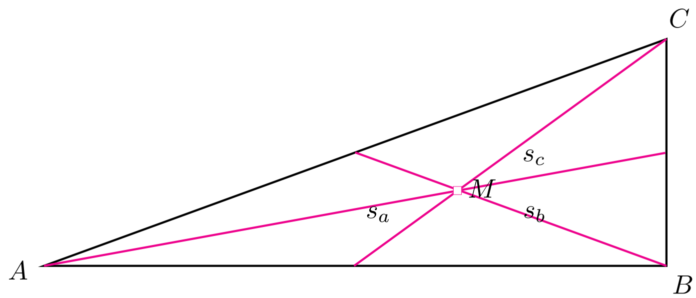
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 20° - 90°$
$\gamma = 70°$
Kosinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{a}{b} \quad / \cdot b$
$b \cdot \sin\alpha = a \quad / : sin\alpha$
$b = \frac{a}{\sin\alpha}$
$b = \frac{3}{\sin 20}$
$b = 8,77$
Pythagoras: $b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{8,77^2 - 3^2}$
$c = 8,24$
Umfang: $U = a + b + c$
$U = 3 + 8,77 + 8,24$
$U = 20$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$

$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,24 \cdot \sin 90°$
$h_a = 8,24$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 8,24$
$A = 12,4$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 70°$
$h_b = 2,82$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8,77 \cdot \sin 20°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{8,24 \cdot \sin 90}{\sin 80}$
$wha = 8,37$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 70}{\sin 65}$
$whb = 3,11$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8,77 \cdot \sin 20}{\sin 80}$
$whc = 1,04$

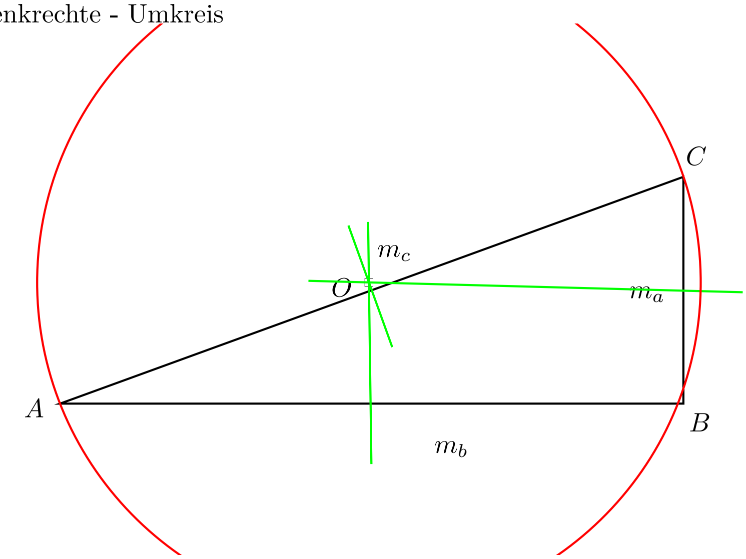
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(8,77^2 + 8,24^2) - 3^2}$
$s_a = 8,38$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 8,24^2) - 8,77^2}$
$s_b = 4,39$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 8,77^2) - 8,24^2}$
$s_c = 4,87$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 20°}$
$r_u = 4,39$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,4}{20}$
$r_i = 1,24$



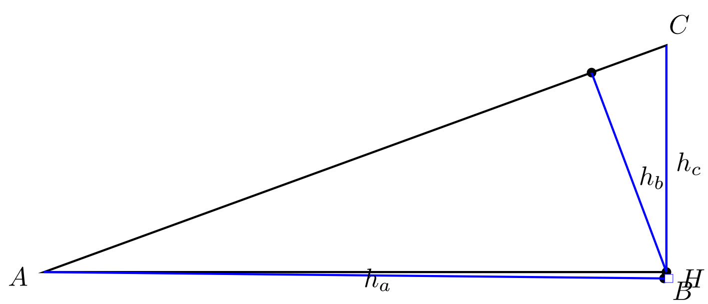
Seitenhalbierende-Schwerpunkt



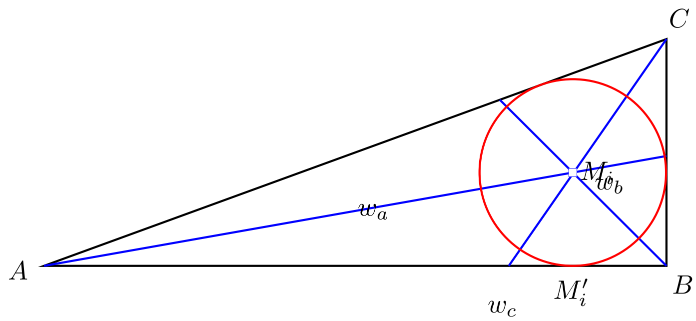
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



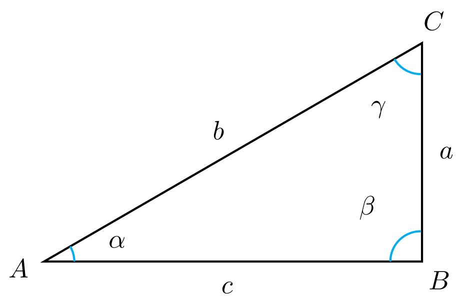
Aufgabe (20)

Winkel-Seite-Winkel
$c = 5 \quad \alpha = 30° \quad \beta = 90°$

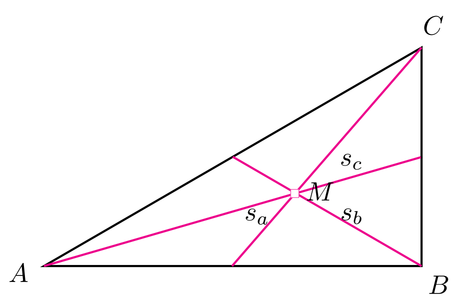
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 90°$
$\gamma = 60°$
Sinus: $\cos\alpha = \frac{c}{b}$
$\cos\alpha = \frac{c}{b} \quad / \cdot b$
$b \cdot \cos\alpha = c \quad / : cos\alpha$
$b = \frac{c}{\cos\alpha}$
$b = \frac{5}{\cos 30}$
$b = 5,77$
Pythagoras: $b^2 = a^2 + c^2 \quad / - c^2$
$a^2 = b^2 - c^2$
$a = \sqrt{b^2 - c^2}$
$a = \sqrt{5,77^2 - 5^2}$
$a = 2,89$
Umfang: $U = a + b + c$
$U = 2,89 + 5,77 + 5$
$U = 13,7$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$

$A = \frac{1}{2} \cdot 2,89 \cdot 5$
$A = 7,22$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,89 \cdot \sin 60^\circ$
$h_b = 2\frac{1}{2}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,77 \cdot \sin 30^\circ$
$h_c = 2,89$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 75}$
$wha = 5,18$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,89 \cdot \sin 60}{\sin 75}$
$whb = 2,59$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,77 \cdot \sin 30}{\sin 75}$
$whc = 1,49$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,77^2 + 5^2) - 2,89^2}$
$s_a = 5,2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

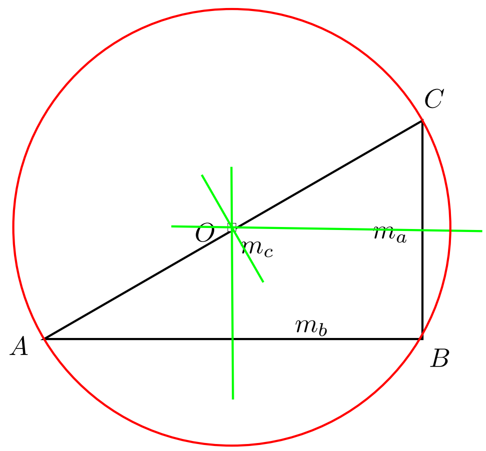
$s_b = \frac{1}{2}\sqrt{2(2,89^2 + 5^2) - 5,77^2}$
$s_b = 2,89$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,89^2 + 5,77^2) - 5^2}$
$s_c = 3,54$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,89}{2 \cdot \sin 30°}$
$r_u = 2,89$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,22}{13,7}$
$r_i = 1,06$



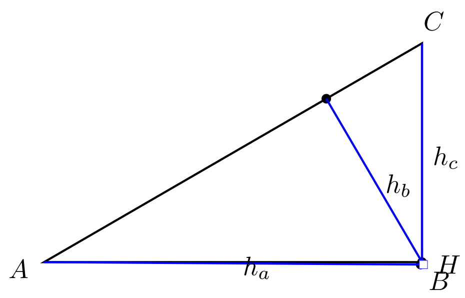
Seitenhalbierende-Schwerpunkt



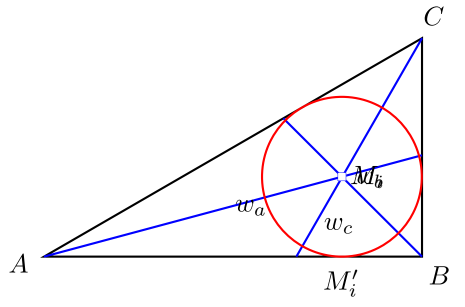
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

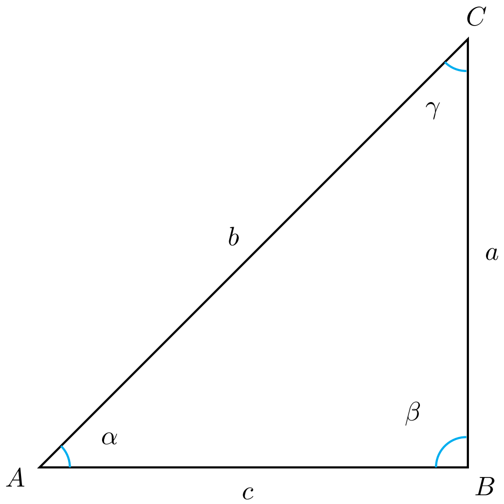


Aufgabe (21)

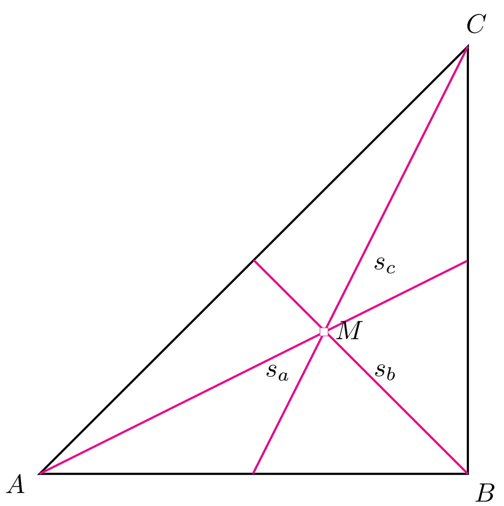
Winkel-Winkel-Seite
$b = 8 \quad \gamma = 45° \quad \beta = 90°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 90° - 45°$
$\alpha = 45°$
Sinus: $\quad \sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{a}{b} \quad / \cdot b \; a = b \cdot \sin\alpha$
$a = 8 \cdot \sin 45$
$a = 5,66$
Pythagoras: $b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{8^2 - 5,66^2}$
$c = 5,66$
Umfang: $U = a + b + c$
$U = 5,66 + 8 + 5,66$
$U = 19,3$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,66 \cdot \sin 90°$
$h_a = 5,66$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,66 \cdot 5,66$
$A = 16$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,66 \cdot \sin 45°$
$h_b = 4$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8 \cdot \sin 45°$
$h_c = 5,66$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

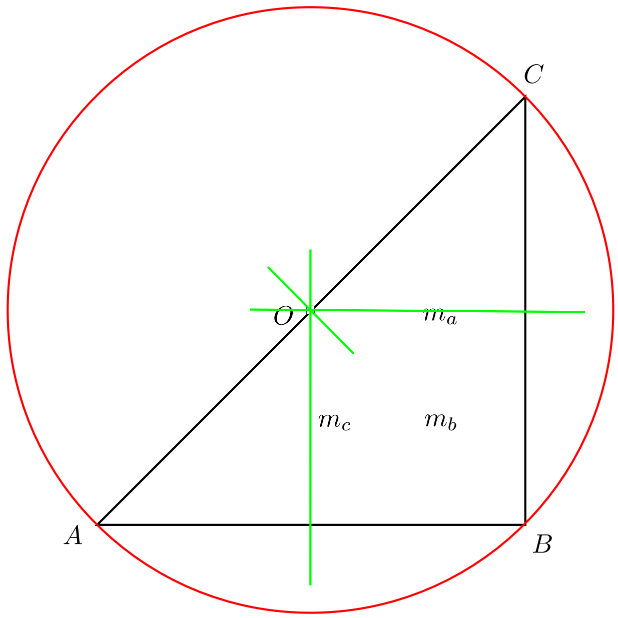
$wha = \frac{5,66 \cdot \sin 90}{\sin 67\frac{1}{2}}$
$wha = 6,12$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,66 \cdot \sin 45}{\sin 90}$
$whb = 4$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8 \cdot \sin 45}{\sin 67\frac{1}{2}}$
$whc = 4,33$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(8^2 + 5,66^2) - 5,66^2}$
$s_a = 6,32$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,66^2 + 5,66^2) - 8^2}$
$s_b = 4$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,66^2 + 8^2) - 5,66^2}$
$s_c = 5,66$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,66}{2 \cdot \sin 45°}$
$r_u = 4$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 16}{19,3}$
$r_i = 1,66$



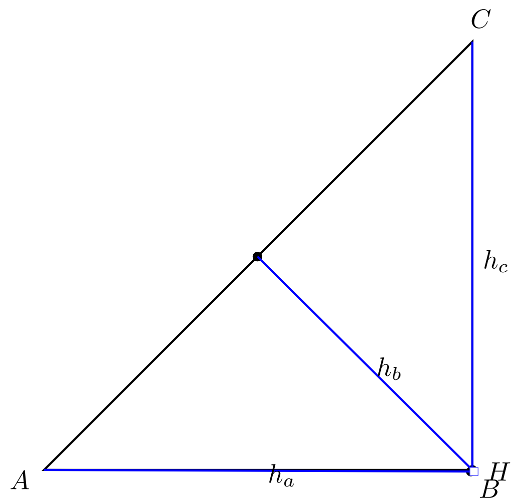
Seitenhalbierende-Schwerpunkt



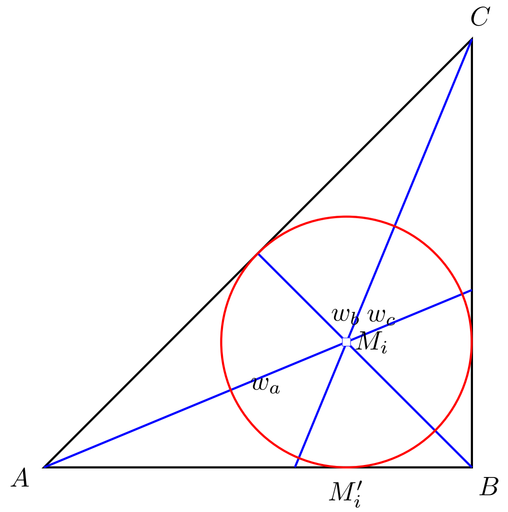
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



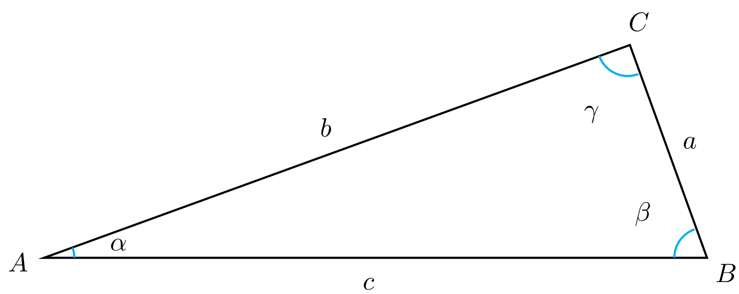
Aufgabe (22)

Winkel-Winkel-Seite
$a = 3 \quad \alpha = 20° \quad \gamma = 90°$

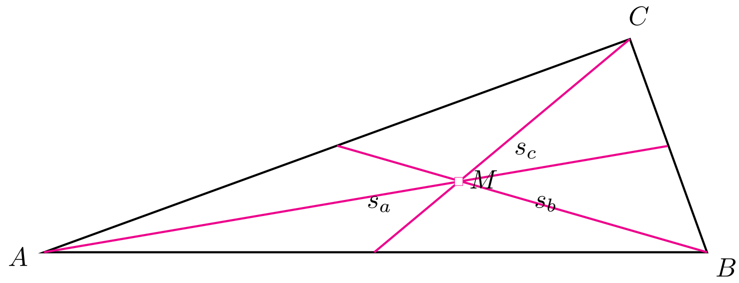
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 20° - 90°$
$\beta = 70°$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{a}{c} \quad / \cdot c$
$c \cdot \sin\alpha = a \quad / : sin\alpha$
$c = \frac{a}{\sin\alpha}$
$c = \frac{3}{\sin 20}$
$c = 8,77$
Pythagoras: $c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{8,77^2 - 3^2}$
$b = 8,24$
Umfang: $U = a + b + c$
$U = 3 + 8,24 + 8,77$
$U = 20$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$

$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,77 \cdot \sin 70°$
$h_a = 8,24$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 8,24$
$A = 12,4$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8,24 \cdot \sin 20°$
$h_c = 2,82$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{8,77 \cdot \sin 70}{\sin 100}$
$wha = 8,37$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 55}$
$whb = 3,66$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8,24 \cdot \sin 20}{\sin 100}$
$whc = 1,04$

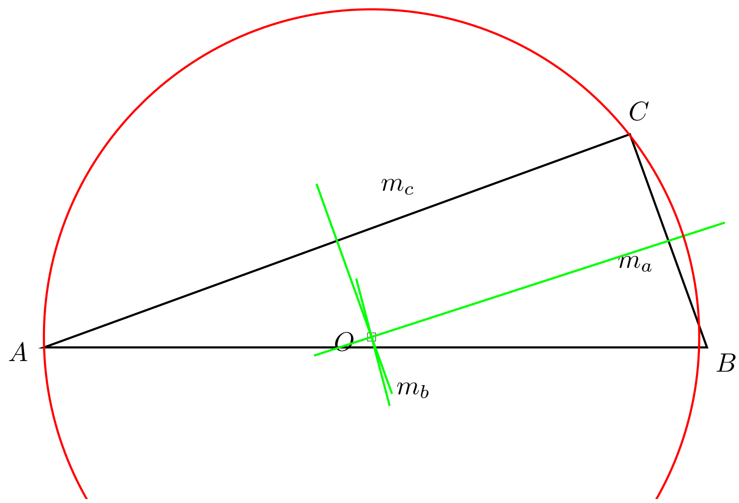
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(8,24^2+8,77^2)-3^2}$
$s_a = 8,38$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2+8,77^2)-8,24^2}$
$s_b = 5,1$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2+8,24^2)-8,77^2}$
$s_c = 4,64$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 20°}$
$r_u = 4,39$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,4}{20}$
$r_i = 1,24$



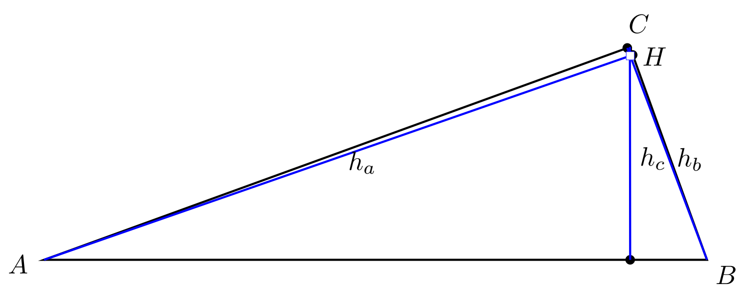
Seitenhalbierende-Schwerpunkt



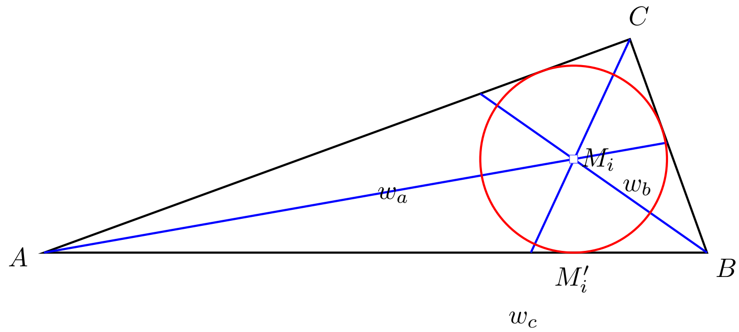
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (23)

Winkel-Winkel-Seite
$c = 5 \quad \gamma = 90° \quad \alpha = 35°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 35° - 90°$
$\beta = 55°$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{a}{c} \quad / \cdot c \; a = c \cdot \sin\alpha$
$a = 5 \cdot \sin 35$
$a = 2,87$
Pythagoras: $c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{5^2 - 2,87^2}$
$b = 4,1$
Umfang: $U = a + b + c$
$U = 2,87 + 4,1 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 55°$
$h_a = 4,1$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,87 \cdot 4,1$
$A = 5,87$
Höhe: h_b

$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,87 \cdot \sin 90^\circ$
$h_b = 2,87$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,1 \cdot \sin 35^\circ$
$h_c = 2,35$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 55}{\sin 107\frac{1}{2}}$
$wha = 4,29$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,87 \cdot \sin 90}{\sin 62\frac{1}{2}}$
$whb = 3,23$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,1 \cdot \sin 35}{\sin 107\frac{1}{2}}$
$whc = 1,72$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,1^2 + 5^2) - 2,87^2}$
$s_a = 4,34$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,87^2 + 5^2) - 4,1^2}$
$s_b = 3,52$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$

$s_c = \frac{1}{2}\sqrt{2(2,87^2+4,1^2)-5^2}$

$s_c = 2,88$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

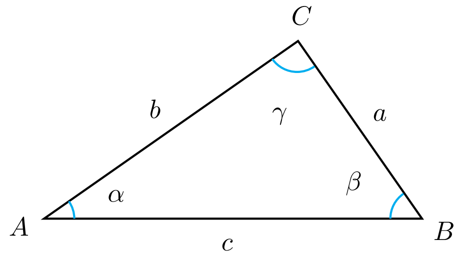
$r_u = \frac{2,87}{2 \cdot \sin 35°}$

$r_u = 2\frac{1}{2}$

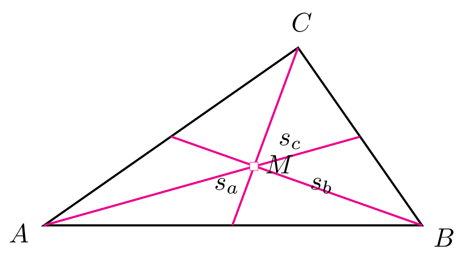
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 5,87}{12}$

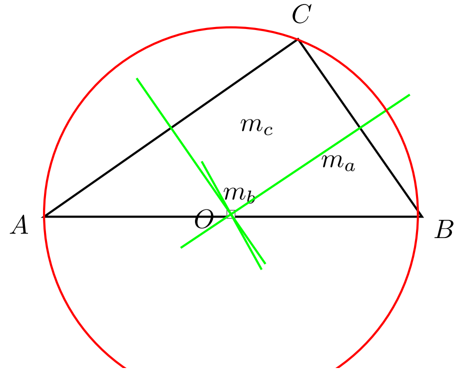
$r_i = 0,982$



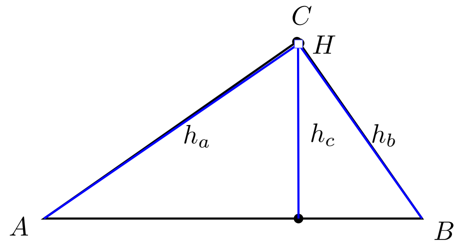
Seitenhalbierende-Schwerpunkt



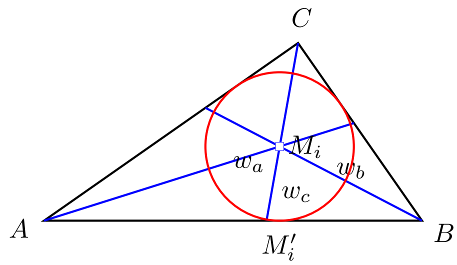
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

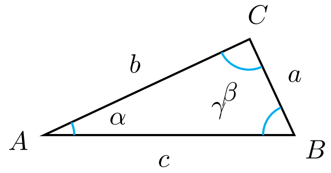


Aufgabe (24)

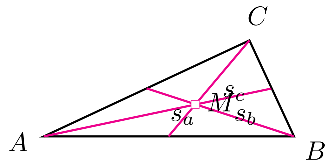
Winkel-Winkel-Seite
$b = 3 \quad \gamma = 90° \quad \beta = 65°$

Winkelsumme:$\alpha+\beta+\gamma=180°$
$\alpha+\beta+\gamma=180 \qquad /-\alpha \qquad /-\gamma$
$\alpha=180°-\beta-\gamma$
$\alpha=180°-65°-90°$
$\alpha=25°$
Kosinus: $\quad \cos\alpha=\frac{b}{c}$
$\cos\alpha=\frac{b}{c} \quad /\cdot c$
$c\cdot\cos\alpha=b \quad /:cos\alpha$
$c=\frac{b}{\cos\alpha}$
$c=\frac{3}{\cos 25}$
$c=3,31$
Pythagoras: $c^2=a^2+b^2 \quad /-b^2$
$a^2=c^2-b^2$
$a=\sqrt{c^2-b^2}$
$a=\sqrt{3,31^2-3^2}$
$a=1,4$
Umfang: $U=a+b+c$
$U=1,4+3+3,31$
$U=7,71$
Höhe: h_a
$\sin\beta=\frac{h_a}{c}$
$\sin\beta=\frac{h_a}{c} \quad /\cdot c$
$h_a=c\cdot\sin\beta$
$h_a=3,31\cdot\sin 65°$
$h_a=3$
Flaeche: $\quad A=\frac{1}{2}\cdot a\cdot h_a$
$A=\frac{1}{2}\cdot 1,4\cdot 3$
$A=2,1$
Höhe: h_b
$\sin\gamma=\frac{h_b}{a}$
$\sin\gamma=\frac{h_b}{a} \quad /\cdot a$
$h_b=a\cdot\sin\gamma$
$h_b=1,4\cdot\sin 90°$
$h_b=1,4$
Höhe: h_c
$\sin\alpha=\frac{h_c}{b}$
$\sin\alpha=\frac{h_c}{b} \quad /\cdot b$
$h_c=b\cdot\sin\alpha$
$h_c=3\cdot\sin 25°$
$h_c=1,27$
Winkelhalbierende: α
$\delta=180-\beta-\frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta}=\frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta}=\frac{c}{\sin\delta} \qquad /\cdot\sin\beta$
$wha=\frac{c\cdot\sin\beta}{\sin\delta}$

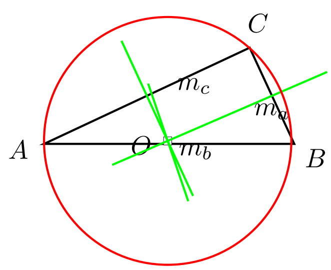
$wha = \frac{3,31 \cdot \sin 65}{\sin 102\frac{1}{2}}$
$wha = 3,07$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{1,4 \cdot \sin 90}{\sin 57\frac{1}{2}}$
$whb = 1,66$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 25}{\sin 102\frac{1}{2}}$
$whc = 0,606$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 3,31^2) - 1,4^2}$
$s_a = 3,08$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(1,4^2 + 3,31^2) - 3^2}$
$s_b = 2,05$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(1,4^2 + 3^2) - 3,31^2}$
$s_c = 1,8$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{1,4}{2 \cdot \sin 25°}$
$r_u = 1,66$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,1}{7,71}$
$r_i = 0,544$



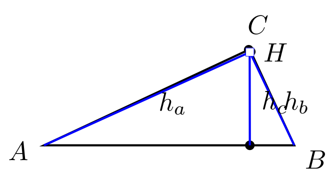
Seitenhalbierende-Schwerpunkt



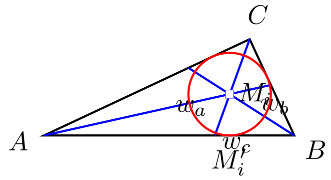
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



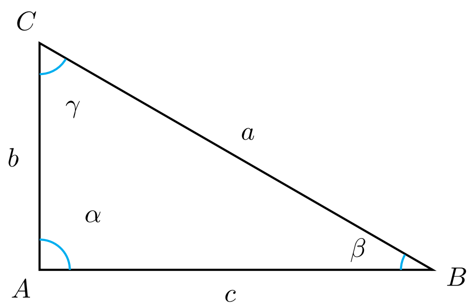
Aufgabe (25)

Winkel-Winkel-Seite
$a = 6 \quad \alpha = 90° \quad \beta = 30°$

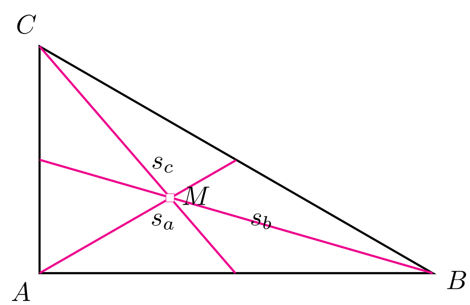
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30°$
$\gamma = 60°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$b = a \cdot \sin\beta$
$b = 6 \cdot \sin 30$
$b = 3$
Pythagoras: $a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6^2 - 3^2}$
$c = 5,2$
Umfang: $U = a + b + c$
$U = 6 + 3 + 5,2$
$U = 14,2$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,2 \cdot \sin 30°$
$h_a = 2,6$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 2,6$
$A = 7,79$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$

$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 60°$
$h_b = 5,2$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,2 \cdot \sin 30}{\sin 105}$
$wha = 2,69$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 60}{\sin 105}$
$whb = 5,38$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 105}$
$whc = 6,21$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5,2^2) - 6^2}$
$s_a = 3$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5,2^2) - 3^2}$
$s_b = 5,41$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 3^2) - 5,2^2}$

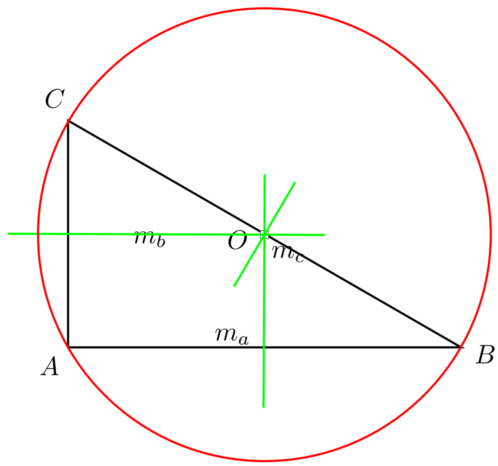
$s_c = 4\frac{1}{2}$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 90°}$
$r_u = 3$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,79}{14,2}$
$r_i = 1,1$



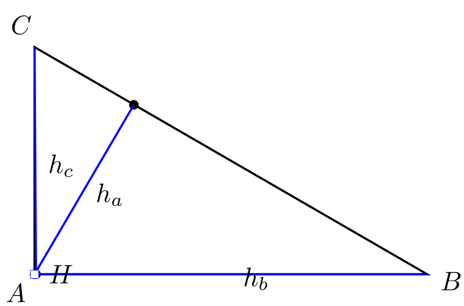
Seitenhalbierende-Schwerpunkt



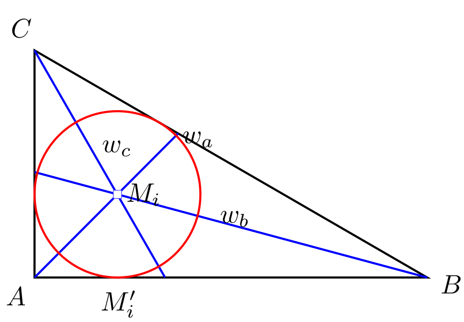
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

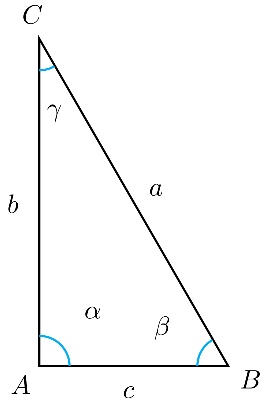


Aufgabe (26)

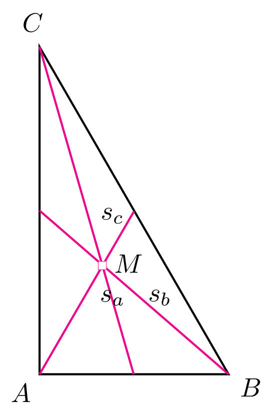
Winkel-Winkel-Seite
$a = 5 \quad \alpha = 90° \quad \gamma = 30°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 90° - 30°$
$\beta = 60°$
Sinus: $\quad \sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$b = a \cdot \sin\beta$
$b = 5 \cdot \sin 60$
$b = 4,33$
Pythagoras: $a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{5^2 - 4,33^2}$
$c = 2\frac{1}{2}$
Umfang: $U = a + b + c$
$U = 5 + 4,33 + 2\frac{1}{2}$
$U = 11,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2\frac{1}{2} \cdot \sin 60°$
$h_a = 2,17$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 2,17$
$A = 5,41$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 30°$
$h_b = 2\frac{1}{2}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,33 \cdot \sin 90°$
$h_c = 4,33$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

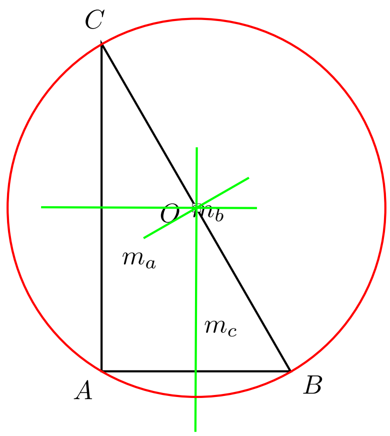
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2\frac{1}{2} \cdot \sin 60}{\sin 75}$
$wha = 2,24$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 30}{\sin 120}$
$whb = 2,89$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,33 \cdot \sin 90}{\sin 75}$
$whc = 5,18$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,33^2 + 2\frac{1}{2}^2) - 5^2}$
$s_a = 2\frac{1}{2}$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2 + 2\frac{1}{2}^2) - 4,33^2}$
$s_b = 3,31$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2 + 4,33^2) - 2\frac{1}{2}^2}$
$s_c = 4,15$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 90°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 5,41}{11,8}$
$r_i = 0,915$



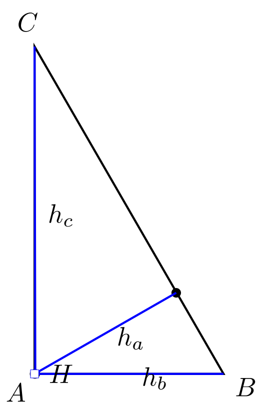
Seitenhalbierende-Schwerpunkt



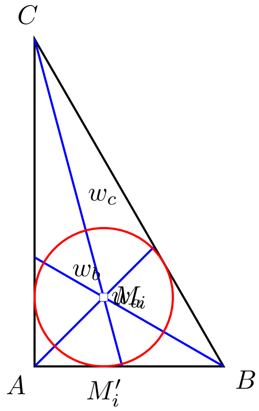
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



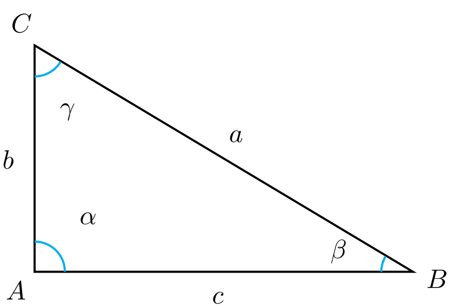
Aufgabe (27)

Seite-Winkel-Seite
$b = 3 \quad c = 5 \quad \alpha = 90°$

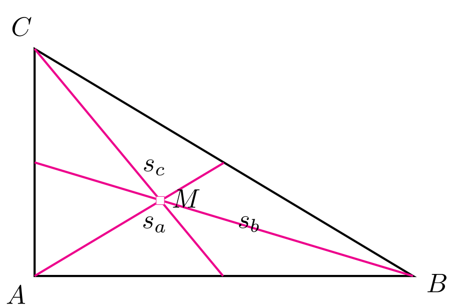
Pythagoras: $a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{3^2 + 5^2}$
$a = 5,83$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{5,83}$
$\beta = 31$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 31°$
$\gamma = 59°$
Umfang: $U = a + b + c$
$U = 5,83 + 3 + 5$
$U = 13,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 31°$
$h_a = 2,57$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,83 \cdot 2,57$

$A = 7\frac{1}{2}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,83 \cdot \sin 59°$
$h_b = 5$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 31}{\sin 104}$
$wha = 2,65$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,83 \cdot \sin 59}{\sin 105}$
$whb = 5,19$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 104}$
$whc = 6,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,83^2}$
$s_a = 2,92$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

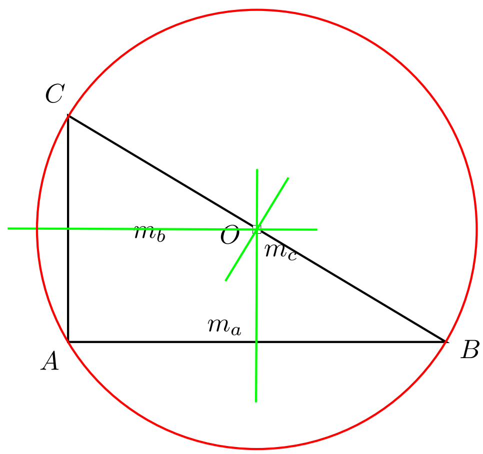
$s_b = \frac{1}{2}\sqrt{2(5,83^2+5^2)-3^2}$
$s_b = 5,22$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,83^2+3^2)-5^2}$
$s_c = 4,39$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2\cdot\sin\alpha}$
$r_u = \frac{5,83}{2\cdot\sin 90^\circ}$
$r_u = 2,92$
Inkreisradius: $r_i = \frac{2\cdot A}{U}$
$r_i = \frac{2\cdot 7\frac{1}{2}}{13,8}$
$r_i = 1,08$



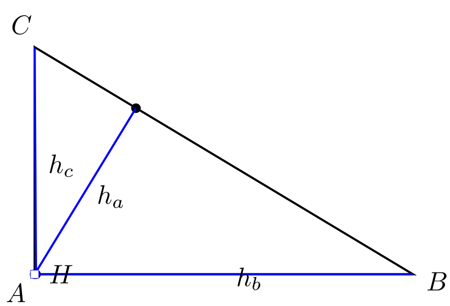
Seitenhalbierende-Schwerpunkt



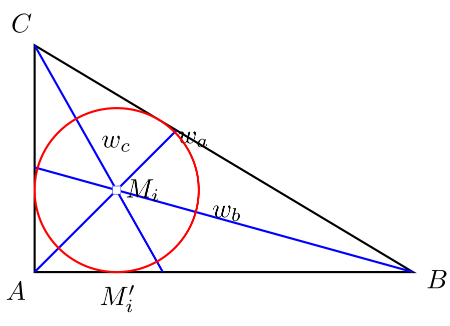
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

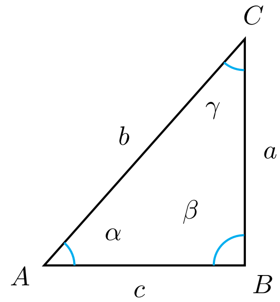


Aufgabe (28)

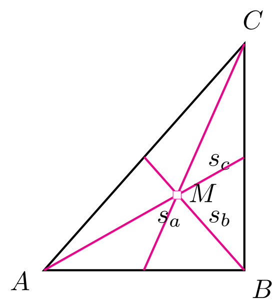
Seite-Seite-Winkel
$a = 3 \quad b = 4 \quad \beta = 90°$

Pythagoras: $b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,65$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{4}$
$\alpha = 48,6$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 48,6° - 90°$
$\gamma = 41,4°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 2,65$
$U = 9,65$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,65 \cdot \sin 90°$
$h_a = 2,65$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 2,65$
$A = 3,97$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 41,4°$
$h_b = 1,98$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 48,6°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

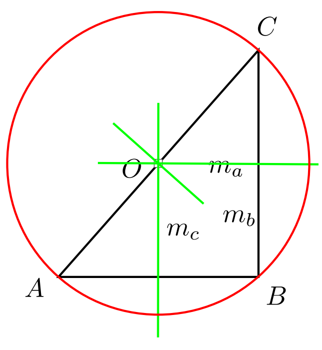
$wha = \frac{2,65 \cdot \sin 90}{\sin 65,7}$
$wha = 2,9$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 41,4}{\sin 93,6}$
$whb = 1,99$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 48,6}{\sin 65,7}$
$whc = 2,47$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 2,65^2) - 3^2}$
$s_a = 3,04$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 2,65^2) - 4^2}$
$s_b = 2$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2,65^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 48,6°}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,97}{9,65}$
$r_i = 0,823$



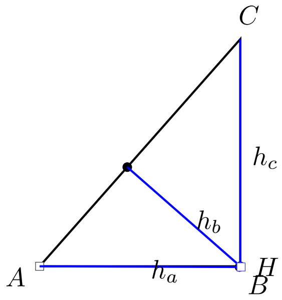
Seitenhalbierende-Schwerpunkt



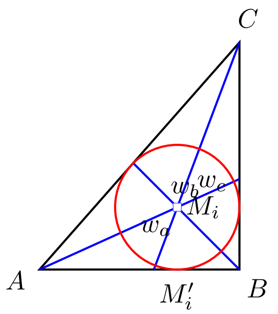
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (29)

Seite-Winkel-Seite
$a = 3 \quad c = 5 \quad \beta = 90°$

Pythagoras: $b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 5^2}$
$b = 5,83$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5,83}$
$\alpha = 31$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 31° - 90°$
$\gamma = 59°$

Umfang: $U = a + b + c$
$U = 3 + 5,83 + 5$
$U = 13,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 5$
$A = 7\frac{1}{2}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 59°$
$h_b = 2,57$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,83 \cdot \sin 31°$
$h_c = 3$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 74,5}$
$wha = 5,19$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 59}{\sin 76}$
$whb = 2,65$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{5,83 \cdot \sin 31}{\sin 74,5}$

$whc = 1,6$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(5,83^2 + 5^2) - 3^2}$

$s_a = 5,22$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,83^2}$

$s_b = 2,92$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(3^2 + 5,83^2) - 5^2}$

$s_c = 3,61$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

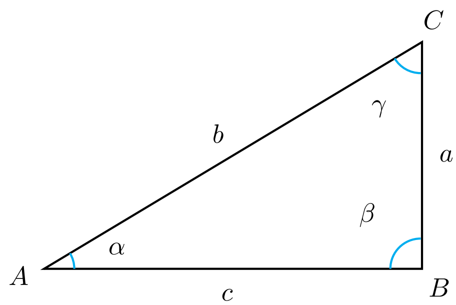
$r_u = \frac{3}{2 \cdot \sin 31^\circ}$

$r_u = 2,92$

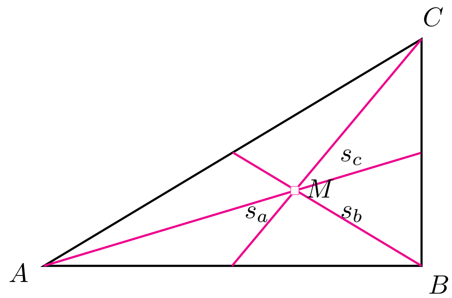
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,8}$

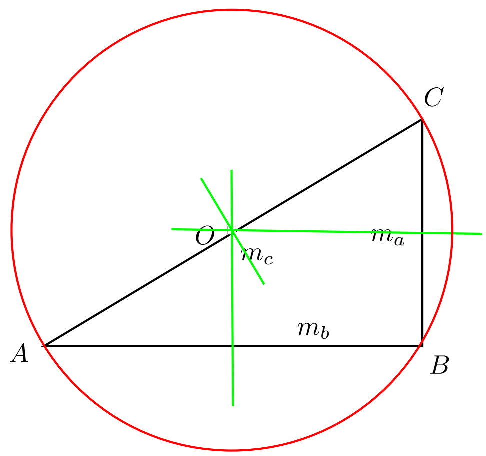
$r_i = 1,08$



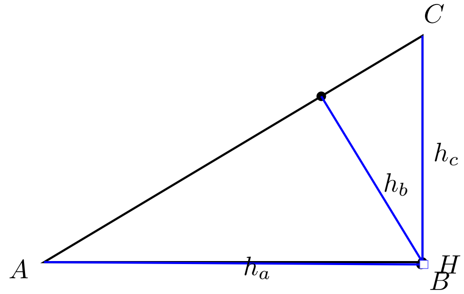
Seitenhalbierende-Schwerpunkt



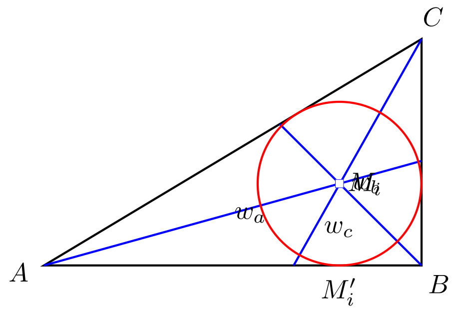
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



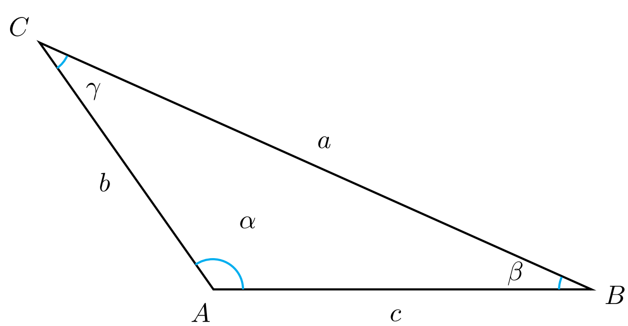
Aufgabe (30)

Seite-Seite-Seite
$a = 8 \quad b = 4 \quad c = 5$

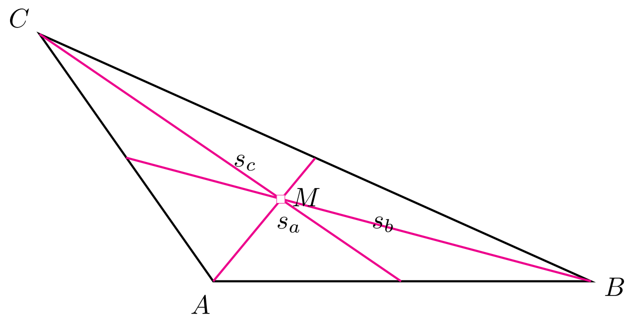
Umfang: $U = a + b + c$
$U = 8 + 4 + 5$
$U = 17$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{4^2 + 5^2 - 8^2}{2 \cdot 4 \cdot 5}$
$\cos\alpha = -\frac{23}{40}$
$\alpha = \arccos(-\frac{23}{40})$
$\alpha = 125°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{8^2 + 5^2 - 4^2}{2 \cdot 8 \cdot 5}$
$\cos\beta = \frac{73}{80}$
$\beta = \arccos(\frac{73}{80})$
$\beta = 24,1°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 125° - 24,1°$
$\gamma = 30,8°$

Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 24,1°$
$h_a = 2,05$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 8 \cdot 2,05$
$A = 8,18$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 8 \cdot \sin 30,8°$
$h_b = 4,09$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 125°$
$h_c = 3,27$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 24,1}{\sin 93,3}$
$wha = 2,05$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{8 \cdot \sin 30,8}{\sin 137}$
$whb = 6,02$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

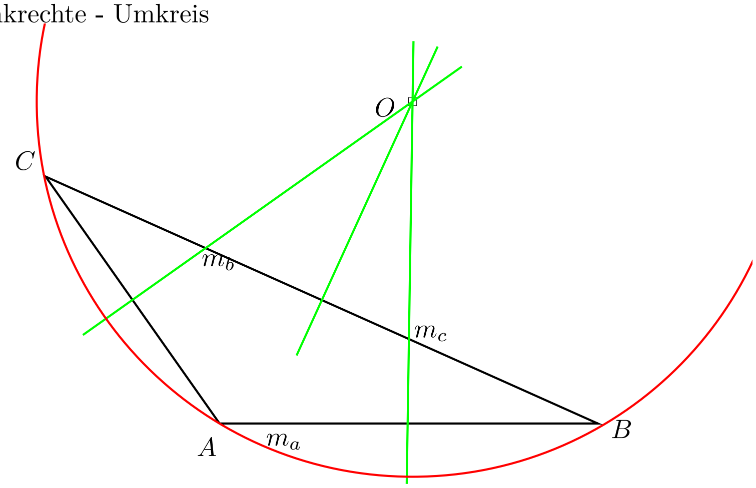
$whc = \frac{4 \cdot \sin 125}{\sin 93,3}$
$whc = 6,56$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 8^2}$
$s_a = 2,12$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 4^2}$
$s_b = 6,36$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(8^2 + 4^2) - 5^2}$
$s_c = 6$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{8}{2 \cdot \sin 125°}$
$r_u = 4,89$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8,18}{17}$
$r_i = 0,963$



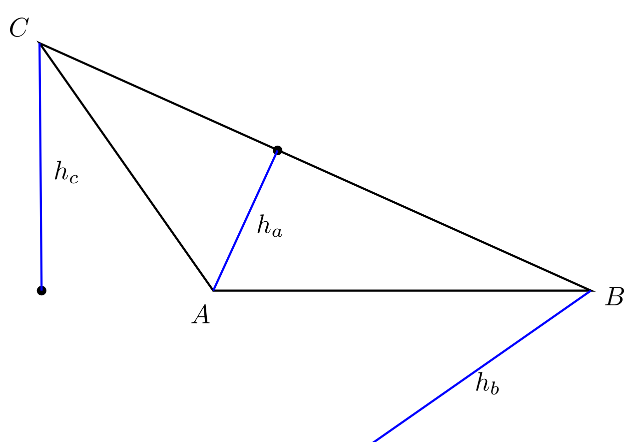
Seitenhalbierende-Schwerpunkt



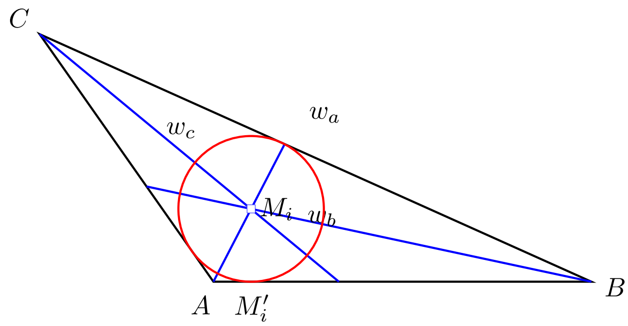
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (31)

Seite-Seite-Seite
$a = 3 \quad b = 7 \quad c = 4$
$b > a + c$ Berechnung nicht möglich: Dreiecksungleichung nicht erfüllt
$b > a + c$
Zeichnung nicht möglich

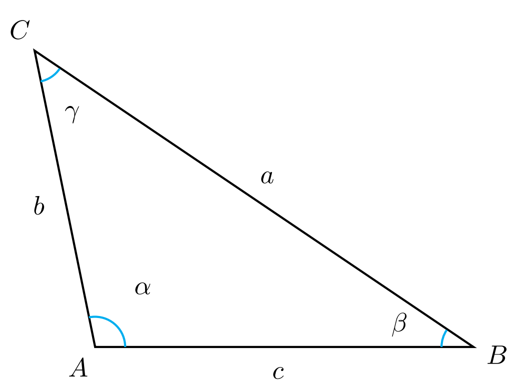
Aufgabe (32)

Seite-Seite-Seite
$a = 7 \quad b = 4 \quad c = 5$

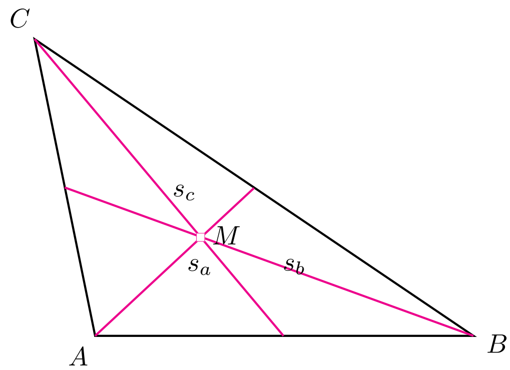
Umfang: $U = a + b + c$
$U = 7 + 4 + 5$
$U = 16$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{4^2 + 5^2 - 7^2}{2 \cdot 4 \cdot 5}$
$\cos\alpha = -\dfrac{1}{5}$
$\alpha = \arccos(-\dfrac{1}{5})$
$\alpha = 102°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \dfrac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \dfrac{7^2 + 5^2 - 4^2}{2 \cdot 7 \cdot 5}$

$\cos\beta = \frac{29}{35}$
$\beta = \arccos(\frac{29}{35})$
$\beta = 34°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 102° - 34°$
$\gamma = 44,4°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 34°$
$h_a = 2,8$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7 \cdot 2,8$
$A = 9,8$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7 \cdot \sin 44,4°$
$h_b = 4,9$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 102°$
$h_c = 3,92$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 34}{\sin 95,2}$
$wha = 2,81$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7 \cdot \sin 44,4}{\sin 119}$

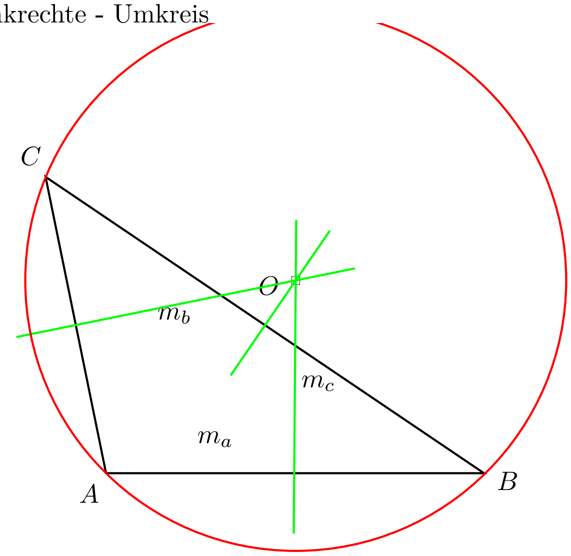
$whb = 5,58$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 102}{\sin 95,2}$
$whc = 6,89$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 7^2}$
$s_a = 2,87$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(7^2 + 5^2) - 4^2}$
$s_b = 5,74$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(7^2 + 4^2) - 5^2}$
$s_c = 5,34$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7}{2 \cdot \sin 102°}$
$r_u = 3,57$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,8}{16}$
$r_i = 1,22$



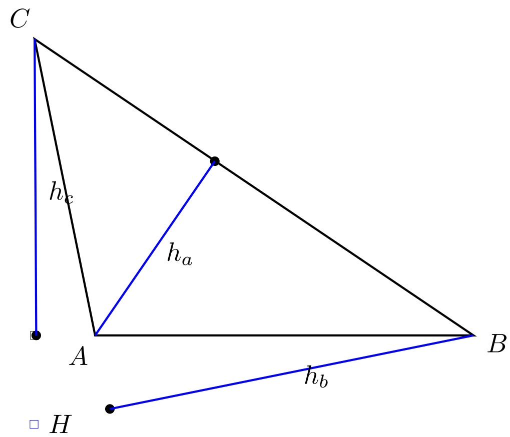
Seitenhalbierende-Schwerpunkt



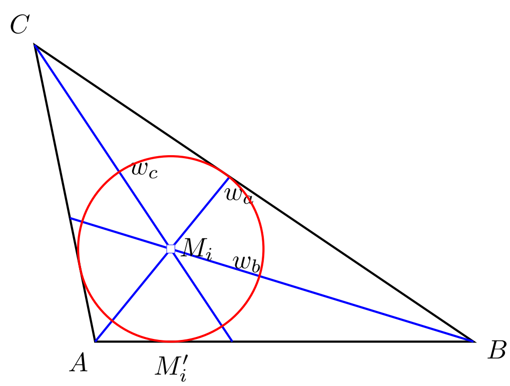
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



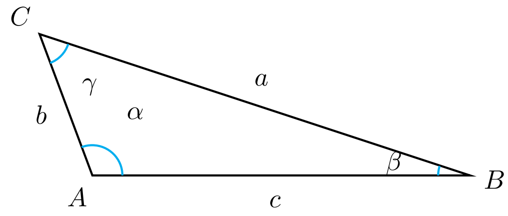
Aufgabe (33)

Seite-Seite-Seite
$a = 6 \quad b = 2 \quad c = 5$

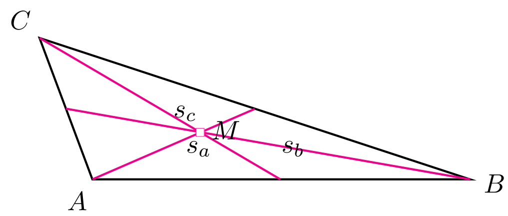
Umfang: $U = a + b + c$
$U = 6 + 2 + 5$
$U = 13$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

$\cos\alpha = \dfrac{2^2+5^2-6^2}{2\cdot 2\cdot 5}$
$\cos\alpha = -\dfrac{7}{20}$
$\alpha = \arccos(-\dfrac{7}{20})$
$\alpha = 110^\circ$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2\cdot b\cdot c\cdot\cos\beta$
$b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos\beta \qquad / - b^2 \qquad / + 2\cdot a\cdot c\cdot\cos\beta$
$2\cdot a\cdot c\cdot\cos\beta = a^2 + c^2 - b^2 \qquad / : (2\cdot a\cdot c)$
$\cos\beta = \dfrac{a^2+c^2-b^2}{2\cdot a\cdot c}$
$\cos\beta = \dfrac{6^2+5^2-2^2}{2\cdot 6\cdot 5}$
$\cos\beta = \dfrac{19}{20}$
$\beta = \arccos(\dfrac{19}{20})$
$\beta = 18,2^\circ$
Winkelsumme: $\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180^\circ - \alpha - \beta$
$\gamma = 180^\circ - 110^\circ - 18,2^\circ$
$\gamma = 51,3^\circ$
Höhe: h_a
$\sin\beta = \dfrac{h_a}{c}$
$\sin\beta = \dfrac{h_a}{c} \quad / \cdot c$
$h_a = c\cdot\sin\beta$
$h_a = 5\cdot\sin 18,2^\circ$
$h_a = 1,56$
Flaeche: $A = \dfrac{1}{2}\cdot a\cdot h_a$
$A = \dfrac{1}{2}\cdot 6\cdot 1,56$
$A = 4,68$
Höhe: h_b
$\sin\gamma = \dfrac{h_b}{a}$
$\sin\gamma = \dfrac{h_b}{a} \quad / \cdot a$
$h_b = a\cdot\sin\gamma$
$h_b = 6\cdot\sin 51,3^\circ$
$h_b = 4,68$
Höhe: h_c
$\sin\alpha = \dfrac{h_c}{b}$
$\sin\alpha = \dfrac{h_c}{b} \quad / \cdot b$
$h_c = b\cdot\sin\alpha$
$h_c = 2\cdot\sin 110^\circ$
$h_c = 1,87$
Winkelhalbierende: α
$\delta = 180 - \beta - \dfrac{\alpha}{2}$
Sinus-Satz: $\dfrac{wha}{\sin\beta} = \dfrac{c}{\sin\delta}$
$\dfrac{wha}{\sin\beta} = \dfrac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \dfrac{c\cdot\sin\beta}{\sin\delta}$

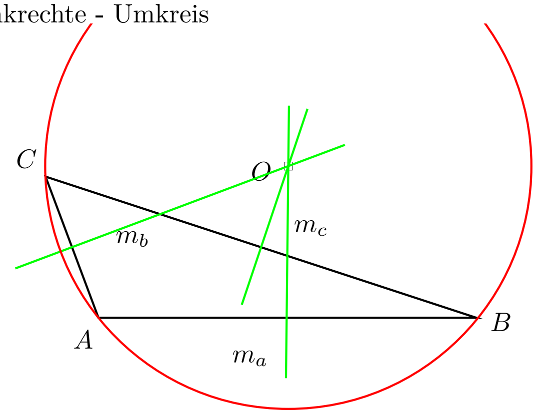
$wha = \frac{5 \cdot \sin 18,2}{\sin 107}$
$wha = 1,63$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 51,3}{\sin 120}$
$whb = 5,39$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{2 \cdot \sin 110}{\sin 107}$
$whc = 5,86$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(2^2 + 5^2) - 6^2}$
$s_a = 2,35$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 2^2}$
$s_b = 5,43$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 2^2) - 5^2}$
$s_c = 4,36$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 110°}$
$r_u = 3,2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 4,68}{13}$
$r_i = 0,721$



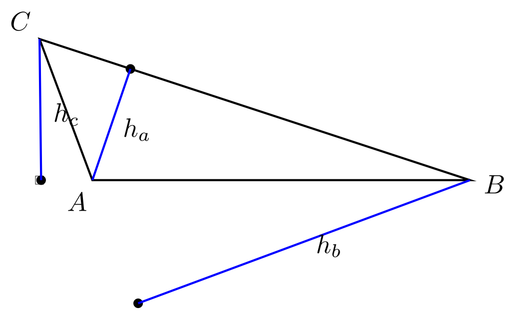
Seitenhalbierende-Schwerpunkt



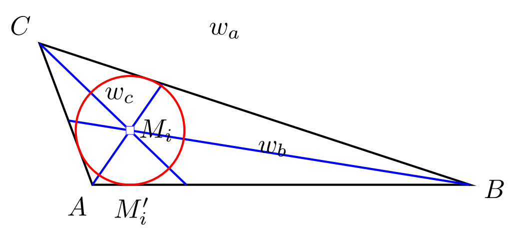
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



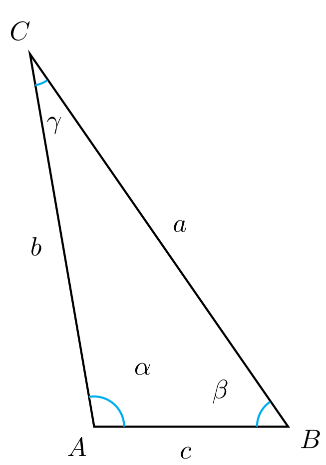
Aufgabe (34)

Seite-Winkel-Seite
$a = 6 \quad b = 5 \quad \gamma = 25°$

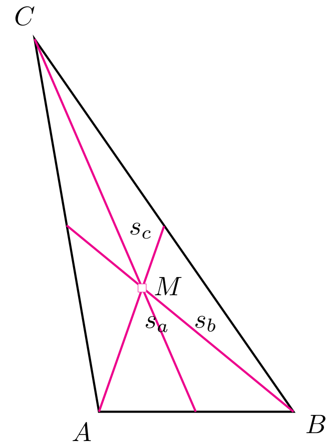
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 25°}$
$c = 2,57$
Umfang: $U = a + b + c$
$U = 6 + 5 + 2,57$
$U = 13,6$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{5^2 + 2,57^2 - 6^2}{2 \cdot 5 \cdot 2,57}$
$\cos\alpha = -0,17$
$\alpha = \arccos(-0,17)$
$\alpha = 99,8°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$

$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 99,8° - 25°$
$\beta = 55,2°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,57 \cdot \sin 55,2°$
$h_a = 2,11$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 2,11$
$A = 6,34$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 25°$
$h_b = 2,54$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 99,8°$
$h_c = 4,93$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,57 \cdot \sin 55,2}{\sin 74,9}$
$wha = 2,19$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 25}{\sin 127}$
$whb = 3,19$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

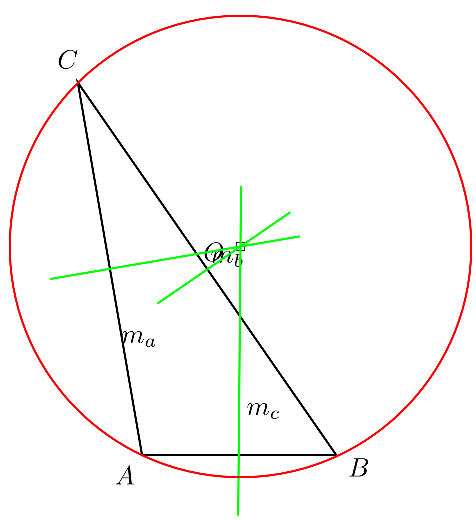
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 99,8}{\sin 74,9}$
$whc = 6,12$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 2,57^2) - 6^2}$
$s_a = 2,61$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 2,57^2) - 5^2}$
$s_b = 3,88$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 2,57^2}$
$s_c = 4,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 99,8°}$
$r_u = 3,04$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,34}{13,6}$
$r_i = 0,934$



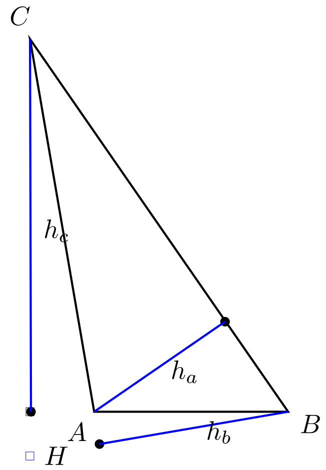
Seitenhalbierende-Schwerpunkt



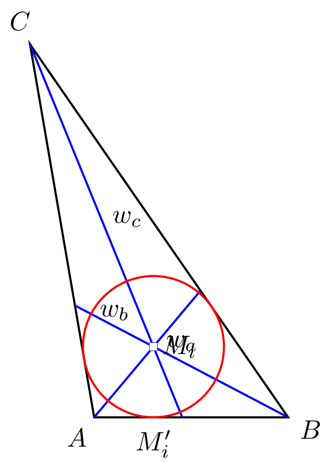
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (35)

Seite-Winkel-Seite
$b = 5 \quad c = 10 \quad \alpha = 155°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{5^2 + 10^2 - 2 \cdot 5 \cdot 10 \cdot \cos 155°}$

$a = 14,7$
Umfang: $U = a + b + c$
$U = 14,7 + 5 + 10$
$U = 29,7$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{14,7^2 + 10^2 - 5^2}{2 \cdot 14,7 \cdot 10}$
$\cos\beta = 0,99$
$\beta = \arccos(0,99)$
$\beta = 8,27°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 155° - 8,27°$
$\gamma = 16,7°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 10 \cdot \sin 8,27°$
$h_a = 1,44$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 14,7 \cdot 1,44$
$A = 10,6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 14,7 \cdot \sin 16,7°$
$h_b = 4,23$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 155°$
$h_c = 2,11$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{10 \cdot \sin 8,27}{\sin 94,2}$
$wha = 1,44$

Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{14,7 \cdot \sin 16,7}{\sin 159}$
$whb = 11,9$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 155}{\sin 94,2}$
$whc = 6,22$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 10^2) - 14,7^2}$
$s_a = 2,93$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(14,7^2 + 10^2) - 5^2}$
$s_b = 12,3$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(14,7^2 + 5^2) - 10^2}$
$s_c = 10,7$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{14,7}{2 \cdot \sin 155°}$
$r_u = 17,4$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 10,6}{29,7}$
$r_i = 0,712$

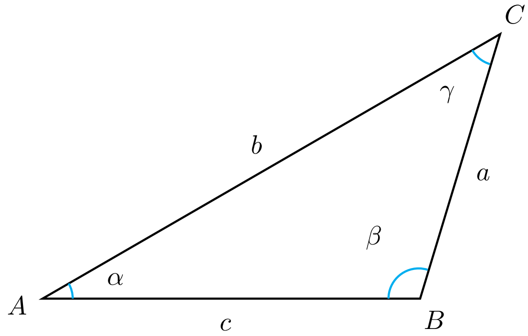
Werte zu groß - Zeichnung nicht möglich

Aufgabe (36)

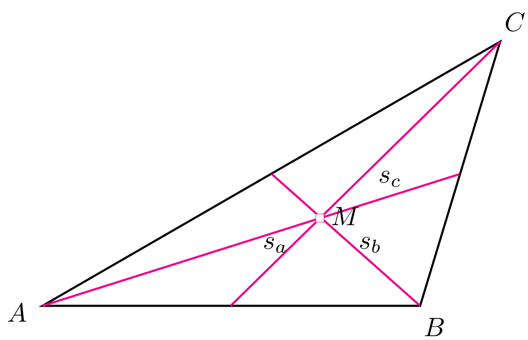
Seite-Winkel-Seite
$b = 7 \quad c = 5 \quad \alpha = 30°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos 30°}$
$a = 3,66$
Umfang: $U = a + b + c$
$U = 3,66 + 7 + 5$
$U = 15,7$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{3,66^2 + 5^2 - 7^2}{2 \cdot 3,66 \cdot 5}$
$\cos\beta = -0,29$
$\beta = \arccos(-0,29)$
$\beta = 107°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 107°$
$\gamma = 43,1°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 107°$
$h_a = 4,78$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3,66 \cdot 4,78$
$A = 8\frac{3}{4}$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3,66 \cdot \sin 43,1°$
$h_b = 2\frac{1}{2}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 30°$
$h_c = 3\frac{1}{2}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

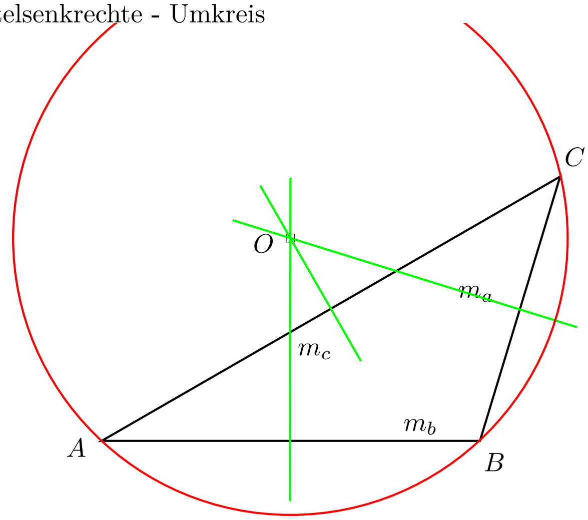
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 107}{\sin 58,1}$
$wha = 5,63$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3,66 \cdot \sin 43,1}{\sin 83,4}$
$whb = 2,52$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 30}{\sin 58,1}$
$whc = 2,15$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 5^2) - 3,66^2}$
$s_a = 5,8$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3,66^2 + 5^2) - 7^2}$
$s_b = 2,63$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3,66^2 + 7^2) - 5^2}$
$s_c = 4,35$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3,66}{2 \cdot \sin 30°}$
$r_u = 3,66$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8\frac{3}{4}}{15,7}$
$r_i = 1,12$



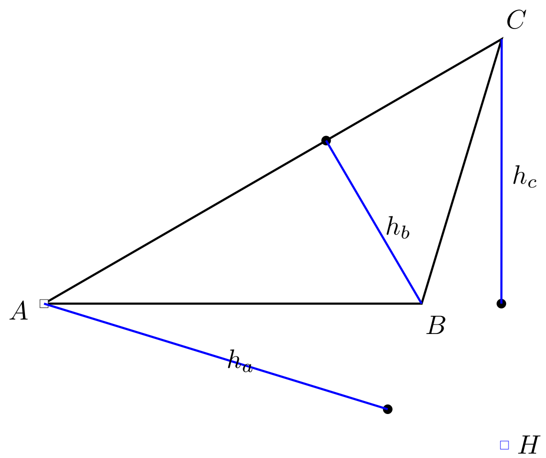
Seitenhalbierende-Schwerpunkt



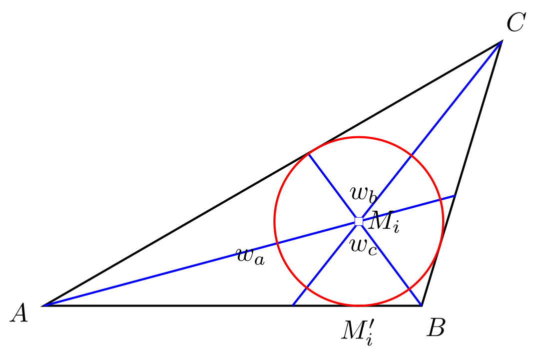
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



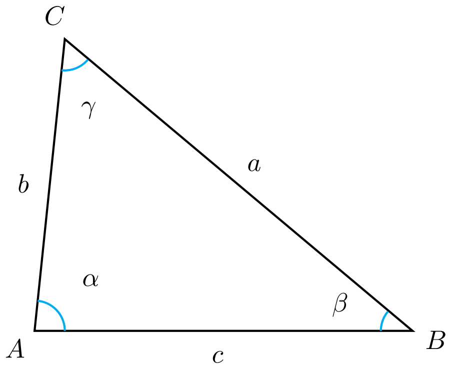
Aufgabe (37)

Seite-Winkel-Seite
$a = 6 \quad c = 5 \quad \beta = 40°$

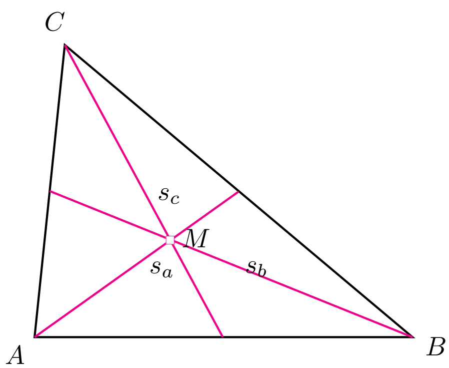
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 40°}$
$b = 3,88$
Umfang: $U = a + b + c$
$U = 6 + 3,88 + 5$
$U = 14,9$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$

$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3,88^2 + 5^2 - 6^2}{2 \cdot 3,88 \cdot 5}$
$\cos\alpha = 0,104$
$\alpha = \arccos(0,104)$
$\alpha = 84°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 84° - 40°$
$\gamma = 56°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 40°$
$h_a = 3,21$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 3,21$
$A = 9,64$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 56°$
$h_b = 4,97$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3,88 \cdot \sin 84°$
$h_c = 3,86$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 40}{\sin 98}$
$wha = 3,25$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

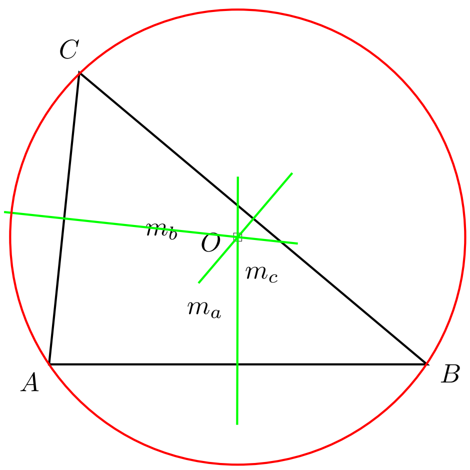
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 56}{\sin 104}$
$whb = 5,13$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3,88 \cdot \sin 84}{\sin 98}$
$whc = 6,03$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,88^2 + 5^2) - 6^2}$
$s_a = 3,32$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 3,88^2}$
$s_b = 5,17$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 3,88^2) - 5^2}$
$s_c = 4,66$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 84^\circ}$
$r_u = 3\frac{1}{61}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,64}{14,9}$
$r_i = 1,3$



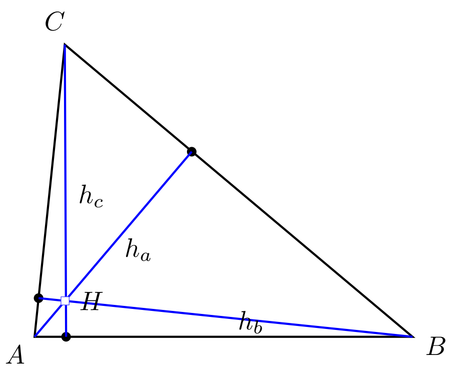
Seitenhalbierende-Schwerpunkt



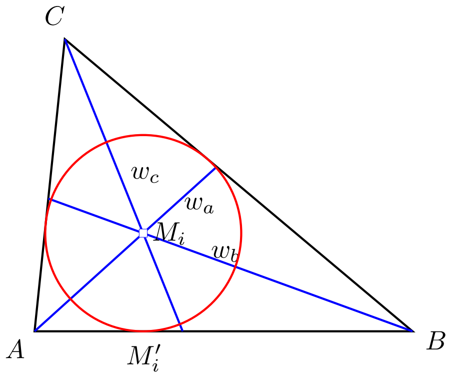
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (38)

Seite-Winkel-Seite
$a = 6 \quad b = 5 \quad \gamma = 120°$

Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 120°}$
$c = 9,54$
Umfang: $U = a + b + c$
$U = 6 + 5 + 9,54$
$U = 20,5$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{5^2 + 9,54^2 - 6^2}{2 \cdot 5 \cdot 9,54}$
$\cos\alpha = 0,839$
$\alpha = \arccos(0,839)$
$\alpha = 33°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 33° - 120°$
$\beta = 27°$
Höhe: h_a
$\sin\beta = \dfrac{h_a}{c}$
$\sin\beta = \dfrac{h_a}{c} \qquad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 9,54 \cdot \sin 27°$

$h_a = 4,33$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 4,33$
$A = 13$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 120°$
$h_b = 5,2$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 33°$
$h_c = 2,72$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9,54 \cdot \sin 27}{\sin 137}$
$wha = 6,29$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 120}{\sin 46,5}$
$whb = 7,16$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 33}{\sin 137}$
$whc = 4,75$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 9,54^2) - 6^2}$

$s_a = 7$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$

$s_b = \frac{1}{2}\sqrt{2(6^2+9,54^2)-5^2}$

$s_b = 7,57$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$

$s_c = \frac{1}{2}\sqrt{2(6^2+5^2)-9,54^2}$

$s_c = 4,92$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2\cdot\sin\alpha}$

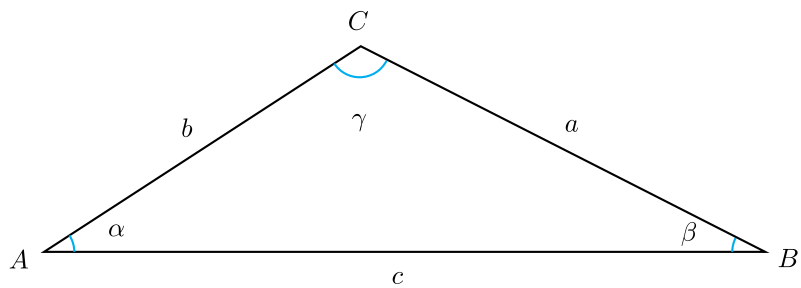
$r_u = \frac{6}{2\cdot\sin 33°}$

$r_u = 5,51$

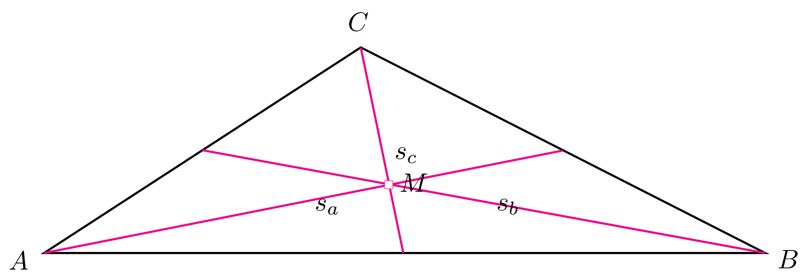
Inkreisradius: $r_i = \frac{2\cdot A}{U}$

$r_i = \frac{2\cdot 13}{20,5}$

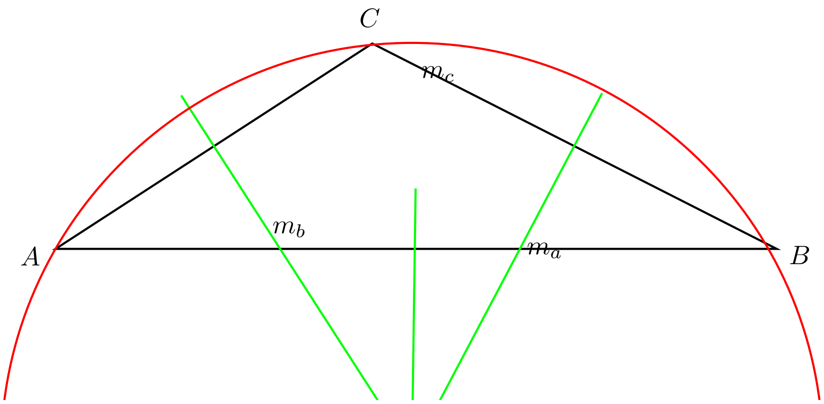
$r_i = 1,26$



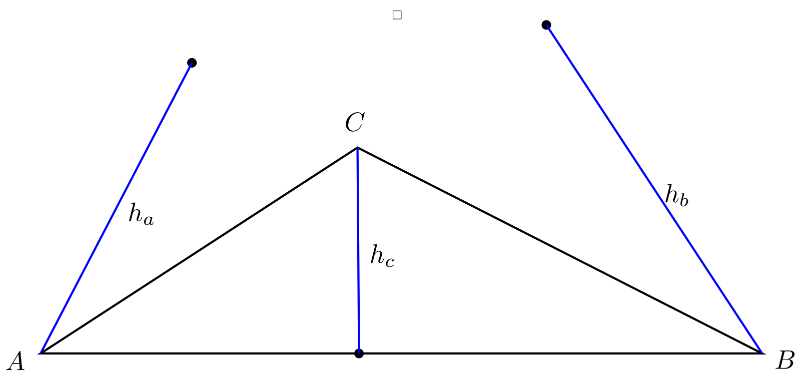
Seitenhalbierende-Schwerpunkt



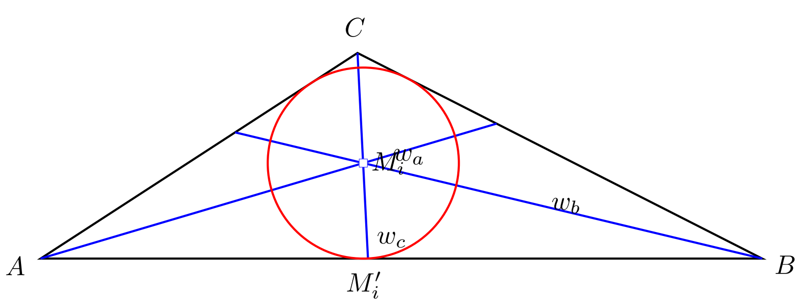
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



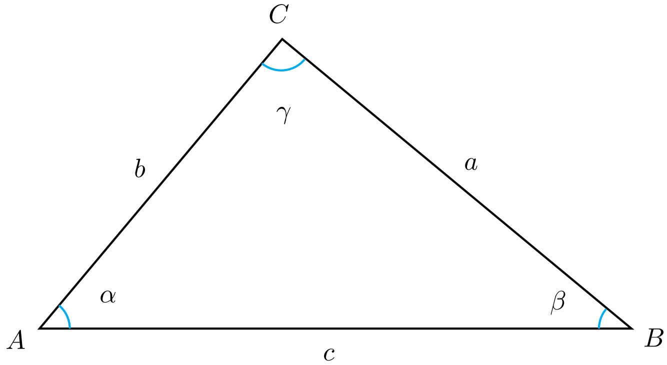
Aufgabe (39)

Seite-Seite-Winkel

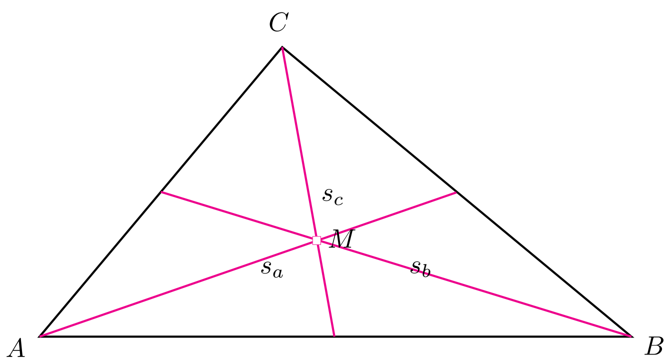
$a = 6 \quad b = 5 \quad \alpha = 50°$

Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{5 \cdot \sin 50°}{6}$
$\sin\beta = 0,638$
$\beta = \arcsin(0,638)$
$\beta = 39,7°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 50° - 39,7°$
$\gamma = 90,3°$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 90,3°}$
$c = 7,83$
Umfang: $U = a + b + c$
$U = 6 + 5 + 7,83$
$U = 18,8$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 7,83 \cdot \sin 39,7°$
$h_a = 5$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 5$
$A = 15$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 90,3°$
$h_b = 6$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 50°$
$h_c = 3,83$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

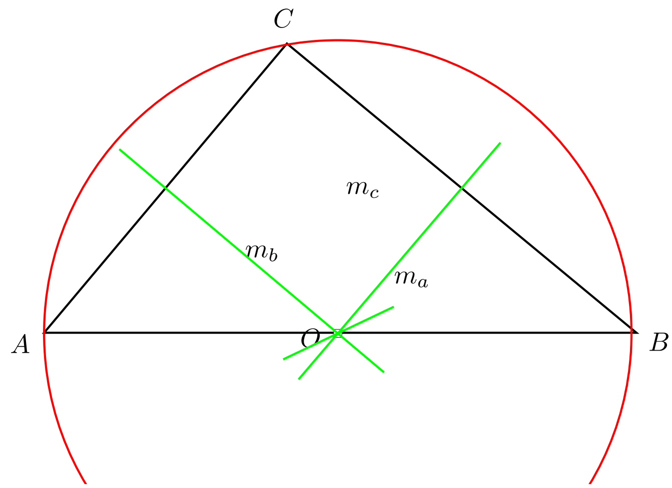
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{7,83 \cdot \sin 39,7}{\sin 115}$
$wha = 5,53$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 90,3}{\sin 69,8}$
$whb = 6,39$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 50}{\sin 115}$
$whc = 5,09$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2+7,83^2)-6^2}$
$s_a = 5,85$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2+7,83^2)-5^2}$
$s_b = 6,51$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2+5^2)-7,83^2}$
$s_c = 4,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 50°}$
$r_u = 3,92$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15}{18,8}$
$r_i = 1,59$



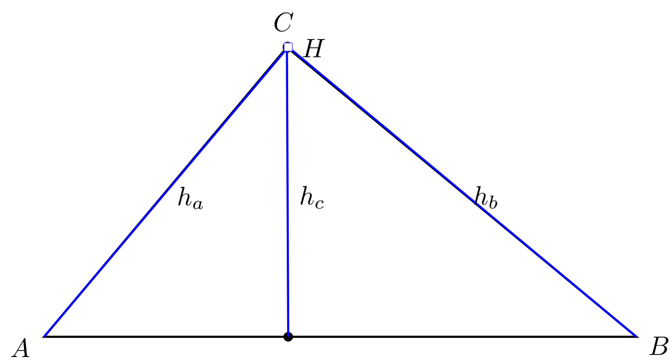
Seitenhalbierende-Schwerpunkt



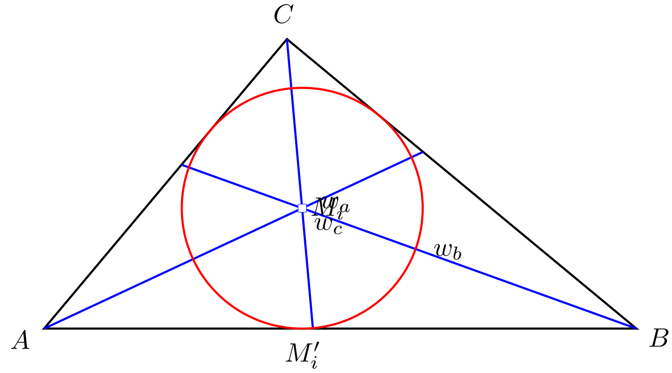
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



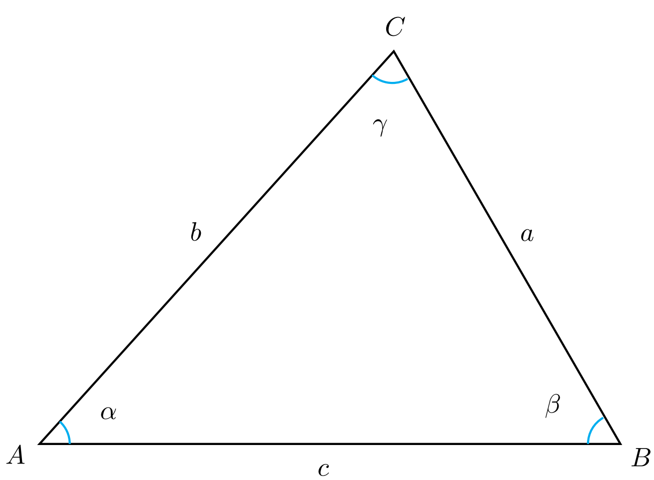
Aufgabe (40)

Seite-Seite-Winkel
$a = 6 \quad b = 7 \quad \beta = 60°$

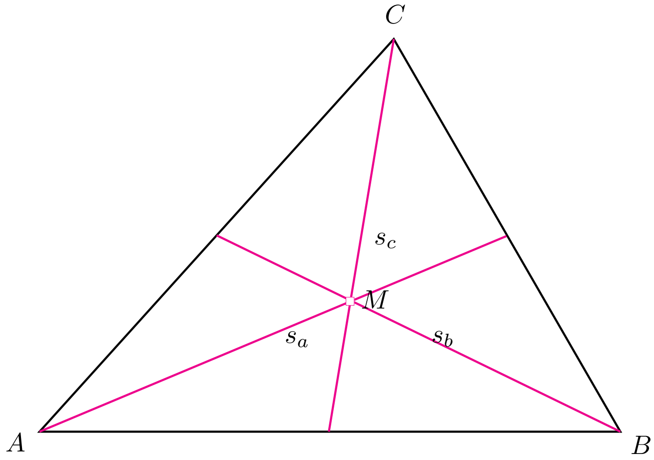
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : b$
$\sin\alpha = \frac{a \cdot \sin\beta}{b}$
$\sin\alpha = \frac{6 \cdot \sin 60°}{7}$
$\sin\alpha = 0,742$
$\alpha = \arcsin(0,742)$
$\alpha = 47,9°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / -\alpha \qquad / -\beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 47,9° - 60°$
$\gamma = 72,1°$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 7^2 - 2 \cdot 6 \cdot 7 \cdot \cos 72,1°}$
$c = 7,69$
Umfang: $U = a + b + c$
$U = 6 + 7 + 7,69$
$U = 20,7$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$

$h_a = c \cdot \sin\beta$
$h_a = 7,69 \cdot \sin 60°$
$h_a = 6,66$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 6,66$
$A = 20$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 72,1°$
$h_b = 5,71$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 47,9°$
$h_c = 5,2$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{7,69 \cdot \sin 60}{\sin 96}$
$wha = 6,7$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 72,1}{\sin 77,9}$
$whb = 5,84$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 47,9}{\sin 96}$
$whc = 4,48$
Seitenhalbierende:

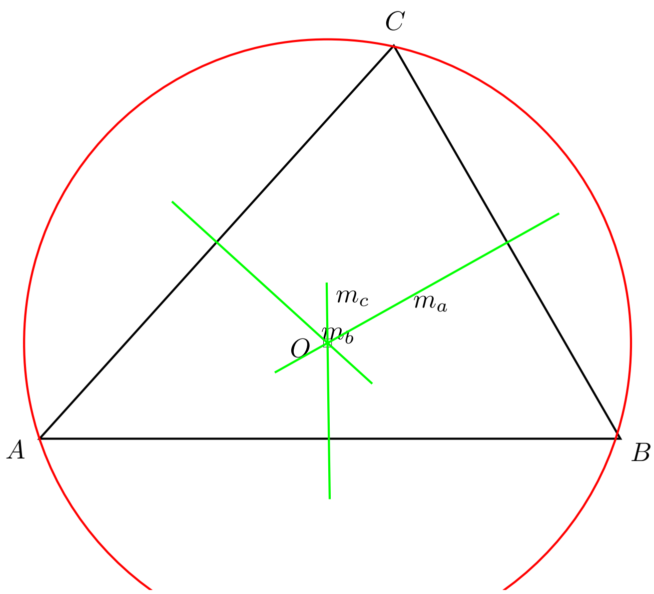
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 7,69^2) - 6^2}$
$s_a = 6,71$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 7,69^2) - 7^2}$
$s_b = 5,94$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 7^2) - 7,69^2}$
$s_c = 5\frac{1}{2}$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 47,9°}$
$r_u = 4,04$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 20}{20,7}$
$r_i = 1,93$



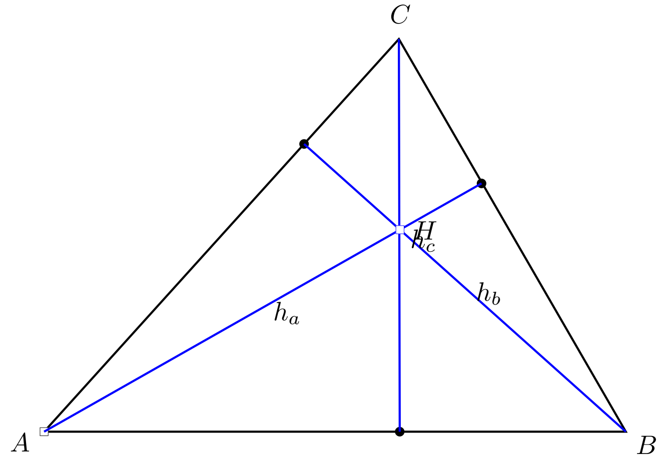
Seitenhalbierende-Schwerpunkt



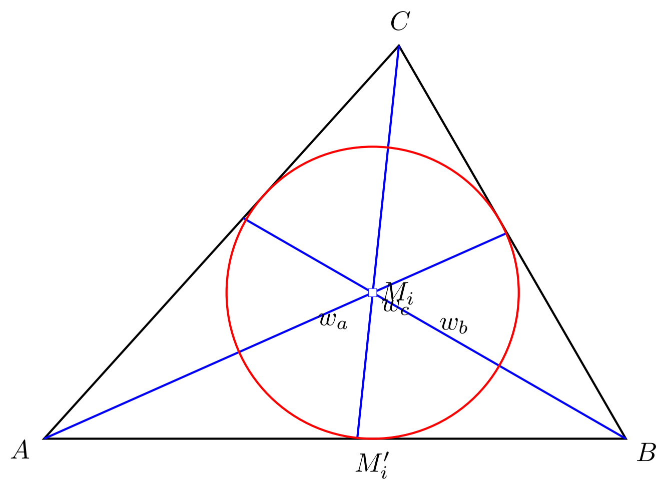
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (41)

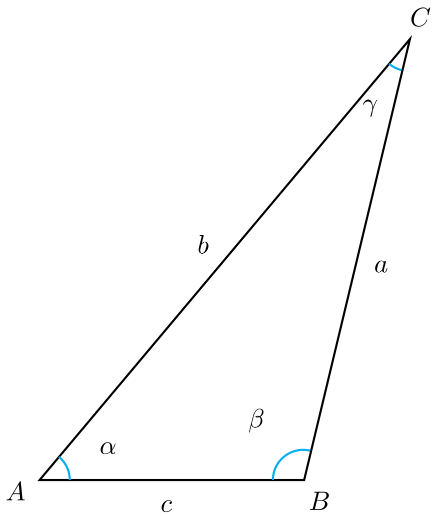
Seite-Seite-Winkel
$a = 6 \quad c = 3\frac{1}{2} \quad \alpha = 50°$

Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

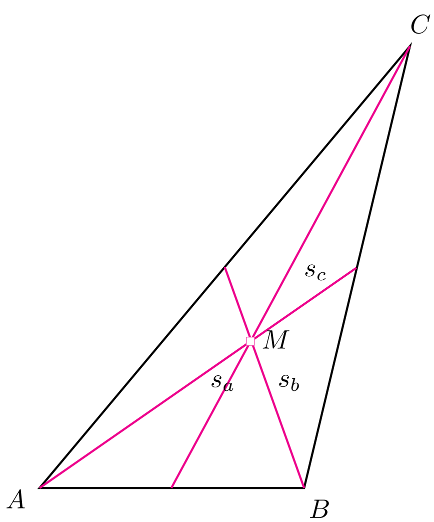
$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\gamma \qquad / \cdot \sin\alpha$

$a \cdot \sin\gamma = c \cdot \sin\alpha \qquad / : a$
$\sin\gamma = \frac{c \cdot \sin\alpha}{a}$
$\sin\gamma = \frac{3\frac{1}{2} \cdot \sin 50^\circ}{6}$
$\sin\gamma = 0,447$
$\gamma = \arcsin(0,447)$
$\gamma = 26,5^\circ$
Winkelsumme: $\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180^\circ - \alpha - \gamma$
$\beta = 180^\circ - 50^\circ - 26,5^\circ$
$\beta = 103^\circ$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{6^2 + 3\frac{1}{2}^2 - 2 \cdot 6 \cdot 3\frac{1}{2} \cdot \cos 103^\circ}$
$b = 7,62$
Umfang: $U = a + b + c$
$U = 6 + 7,62 + 3\frac{1}{2}$
$U = 17,1$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3\frac{1}{2} \cdot \sin 103^\circ$
$h_a = 3,4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 3,4$
$A = 10,2$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 26,5^\circ$
$h_b = 2,68$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,62 \cdot \sin 50^\circ$
$h_c = 5,84$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

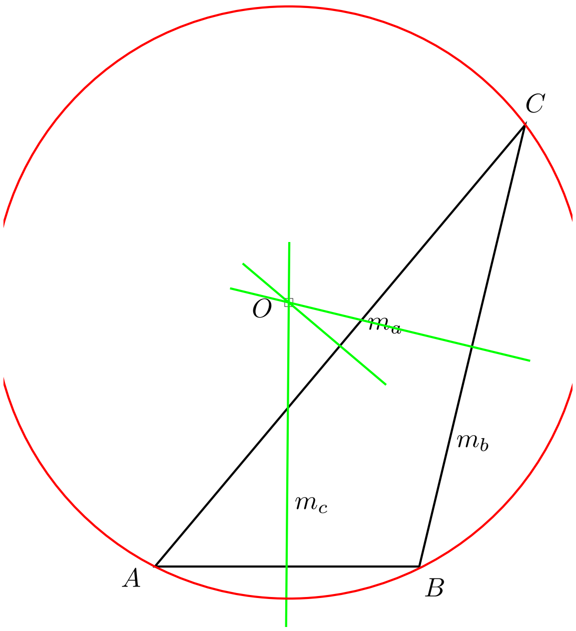
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{3\frac{1}{2} \cdot \sin 103}{\sin 51,5}$
$wha = 4,35$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 26,5}{\sin 102}$
$whb = 2,74$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,62 \cdot \sin 50}{\sin 51,5}$
$whc = 5,87$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,62^2 + 3\frac{1}{2}^2) - 6^2}$
$s_a = 5,11$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 3\frac{1}{2}^2) - 7,62^2}$
$s_b = 3,1$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 7,62^2) - 3\frac{1}{2}^2}$
$s_c = 5,7$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 50°}$
$r_u = 3,92$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 10,2}{17,1}$
$r_i = 1,19$



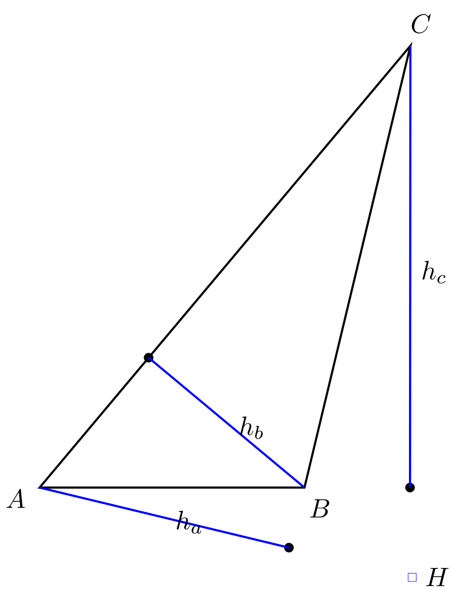
Seitenhalbierende-Schwerpunkt



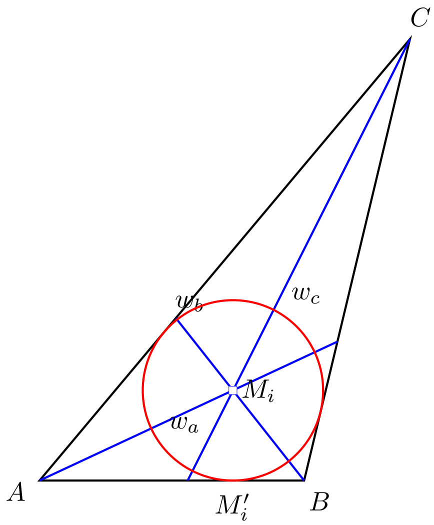
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



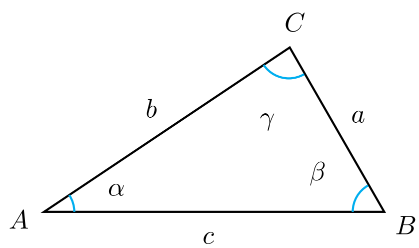
Aufgabe (42)

Seite-Winkel-Seite
$a = 2\frac{1}{2} \quad c = 4\frac{1}{2} \quad \beta = 60°$

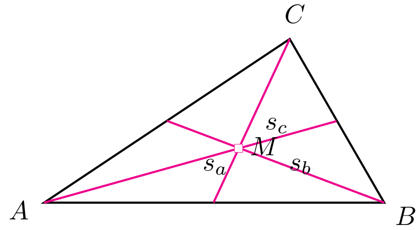
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{2\frac{1}{2}^2 + 4\frac{1}{2}^2 - 2 \cdot 2\frac{1}{2} \cdot 4\frac{1}{2} \cdot \cos 60°}$
$b = 3,91$
Umfang: $U = a + b + c$
$U = 2\frac{1}{2} + 3,91 + 4\frac{1}{2}$
$U = 10,9$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3,91^2 + 4\frac{1}{2}^2 - 2\frac{1}{2}^2}{2 \cdot 3,91 \cdot 4\frac{1}{2}}$
$\cos\alpha = 0,832$
$\alpha = \arccos(0,832)$
$\alpha = 33,7°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 33,7° - 60°$
$\gamma = 86,3°$
Höhe: h_a

$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4\frac{1}{2} \cdot \sin 60°$
$h_a = 3,9$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2\frac{1}{2} \cdot 3,9$
$A = 4,87$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2\frac{1}{2} \cdot \sin 86,3°$
$h_b = 2,49$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3,91 \cdot \sin 33,7°$
$h_c = 2,17$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4\frac{1}{2} \cdot \sin 60}{\sin 103}$
$wha = 4$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2\frac{1}{2} \cdot \sin 86,3}{\sin 63,7}$
$whb = 2,78$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

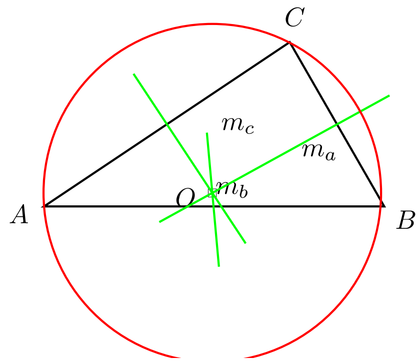
$whc = \frac{3,91 \cdot \sin 33,7}{\sin 103}$
$whc = 1,42$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,91^2+4\frac{1}{2}^2)-2\frac{1}{2}^2}$
$s_a = 4,02$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(2\frac{1}{2}^2+4\frac{1}{2}^2)-3,91^2}$
$s_b = 3,07$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(2\frac{1}{2}^2+3,91^2)-4\frac{1}{2}^2}$
$s_c = 2,63$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2\cdot\sin\alpha}$
$r_u = \frac{2\frac{1}{2}}{2\cdot\sin 33,7°}$
$r_u = 2,25$
Inkreisradius: $r_i = \frac{2\cdot A}{U}$
$r_i = \frac{2\cdot 4,87}{10,9}$
$r_i = 0,893$



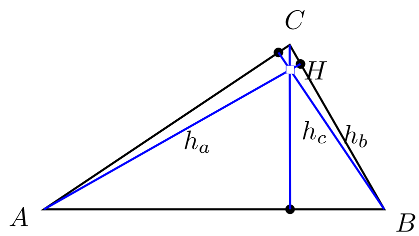
Seitenhalbierende-Schwerpunkt



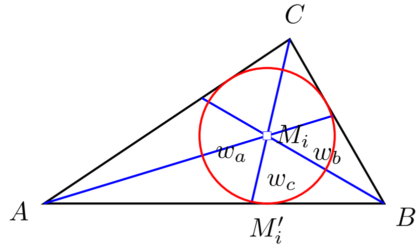
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



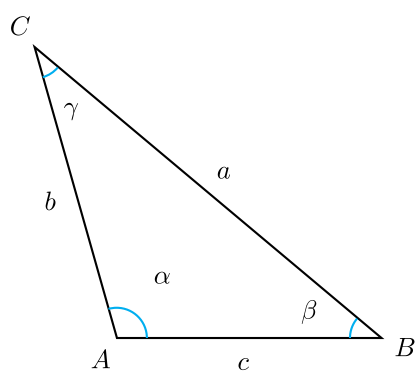
Aufgabe (43)

Seite-Seite-Winkel
$b = 4 \quad c = 3\frac{1}{2} \quad \beta = 40°$

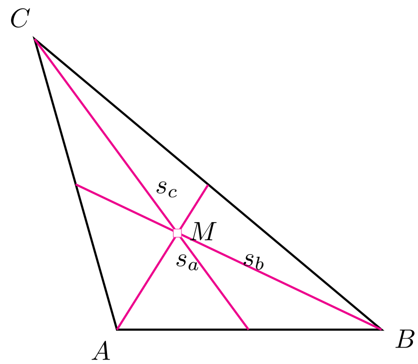
Sinus-Satz: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\beta \qquad / \cdot \sin\gamma$
$b \cdot \sin\gamma = c \cdot \sin\beta \qquad / : b$
$\sin\gamma = \frac{c \cdot \sin\beta}{b}$
$\sin\gamma = \frac{3\frac{1}{2} \cdot \sin 40°}{4}$
$\sin\gamma = 0,562$
$\gamma = \arcsin(0,562)$
$\gamma = 34,2°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 40° - 34,2°$
$\alpha = 106°$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{4^2 + 3\frac{1}{2}^2 - 2 \cdot 4 \cdot 3\frac{1}{2} \cdot \cos 106°}$
$a = 5,99$
Umfang: $U = a + b + c$
$U = 5,99 + 4 + 3\frac{1}{2}$
$U = 13,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \qquad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3\frac{1}{2} \cdot \sin 40°$

$h_a = 2,25$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,99 \cdot 2,25$
$A = 6,74$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,99 \cdot \sin 34,2°$
$h_b = 3,37$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 106°$
$h_c = 3,85$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{3\frac{1}{2} \cdot \sin 40}{\sin 87,1}$
$wha = 2,25$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,99 \cdot \sin 34,2}{\sin 126}$
$whb = 4,15$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 106}{\sin 87,1}$
$whc = 5,77$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

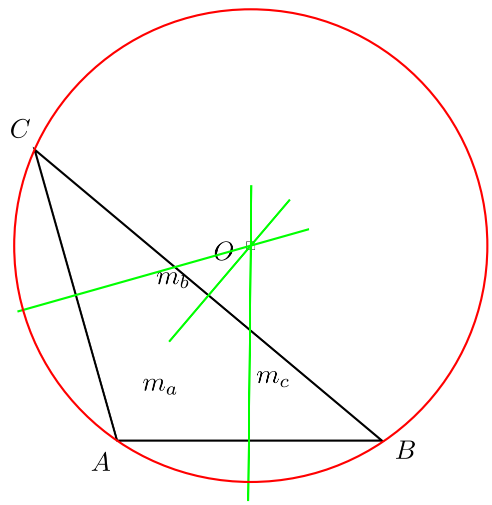
$s_a = \frac{1}{2}\sqrt{2(4^2 + 3\frac{1}{2}^2) - 5,99^2}$
$s_a = 2,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,99^2 + 3\frac{1}{2}^2) - 4^2}$
$s_b = 4,48$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,99^2 + 4^2) - 3\frac{1}{2}^2}$
$s_c = 4,68$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,99}{2 \cdot \sin 106°}$
$r_u = 3,11$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,74}{13,5}$
$r_i = 0,999$



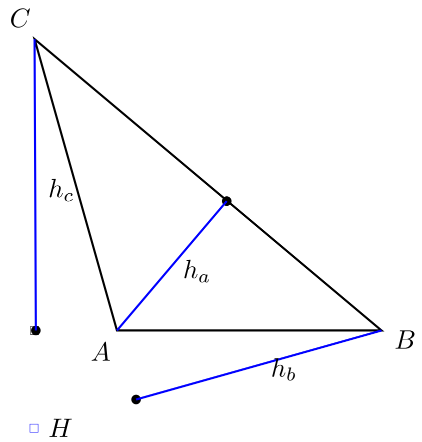
Seitenhalbierende-Schwerpunkt



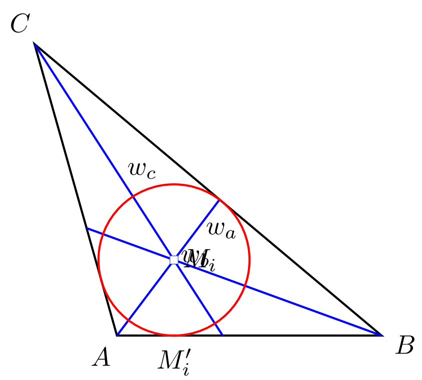
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



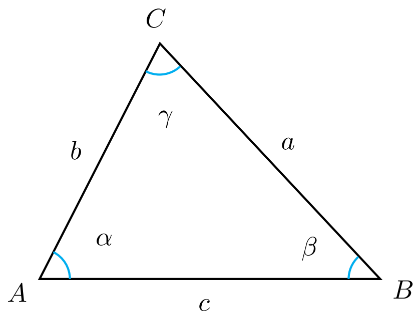
Aufgabe (44)

Seite-Seite-Winkel
$b = 3\frac{1}{2} \quad c = 4\frac{1}{2} \quad \gamma = 70°$

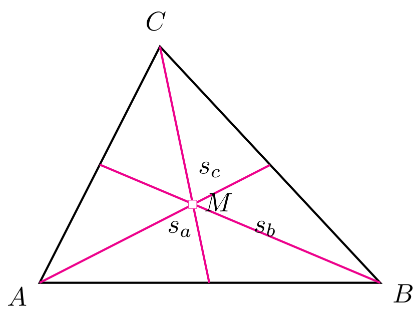
Sinus-Satz: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\beta \qquad / \cdot \sin\gamma$
$b \cdot \sin\gamma = c \cdot \sin\beta \qquad / : c$
$\sin\beta = \frac{b \cdot \sin\gamma}{c}$
$\sin\beta = \frac{3\frac{1}{2} \cdot \sin 70}{4\frac{1}{2}}$
$\sin\beta = 0,731$

$\beta = \arcsin(0,731)$
$\beta = 47°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 47° - 70°$
$\alpha = 63°$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{3\frac{1}{2}^2 + 4\frac{1}{2}^2 - 2 \cdot 3\frac{1}{2} \cdot 4\frac{1}{2} \cdot \cos 63°}$
$a = 4,27$
Umfang: $U = a + b + c$
$U = 4,27 + 3\frac{1}{2} + 4\frac{1}{2}$
$U = 12,3$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4\frac{1}{2} \cdot \sin 47°$
$h_a = 3,29$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,27 \cdot 3,29$
$A = 7,02$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,27 \cdot \sin 70°$
$h_b = 4,01$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3\frac{1}{2} \cdot \sin 63°$
$h_c = 3,12$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4\frac{1}{2} \cdot \sin 47}{\sin 102}$
$wha = 3,36$
Winkelhalbierende: β

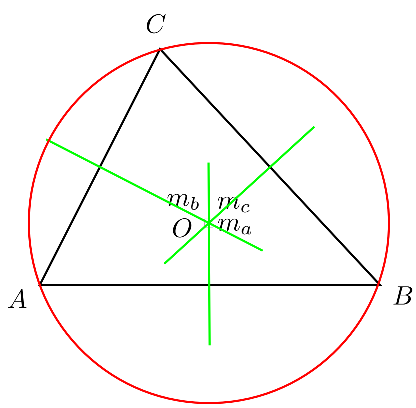
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4,27 \cdot \sin 70}{\sin 86,5}$
$whb = 4,02$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3\frac{1}{2} \cdot \sin 63}{\sin 102}$
$whc = 3,88$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3\frac{1}{2}^2 + 4\frac{1}{2}^2) - 4,27^2}$
$s_a = 3,42$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4,27^2 + 4\frac{1}{2}^2) - 3\frac{1}{2}^2}$
$s_b = 4,02$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4,27^2 + 3\frac{1}{2}^2) - 4\frac{1}{2}^2}$
$s_c = 3,49$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4,27}{2 \cdot \sin 63°}$
$r_u = 2,39$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,02}{12,3}$
$r_i = 1,14$



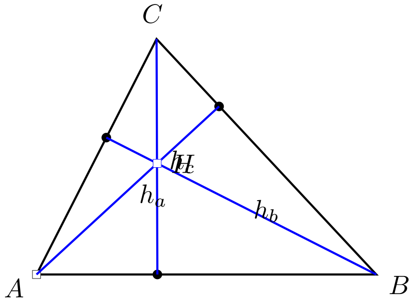
Seitenhalbierende-Schwerpunkt



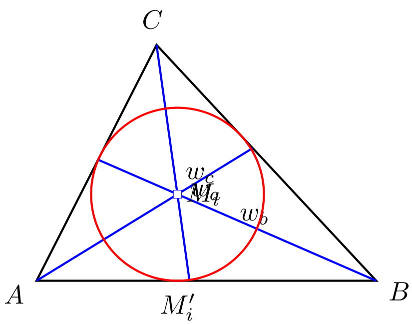
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



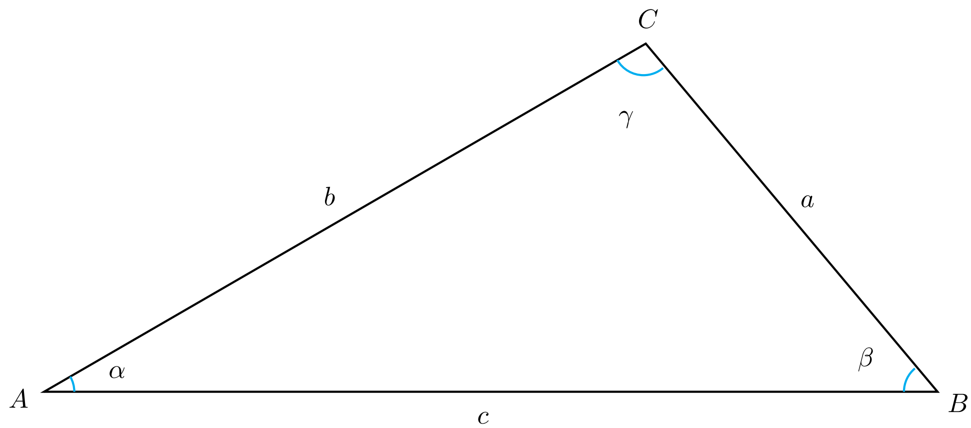
Aufgabe (45)

Winkel-Winkel-Seite
$a = 6 \quad \alpha = 30° \quad \beta = 50°$

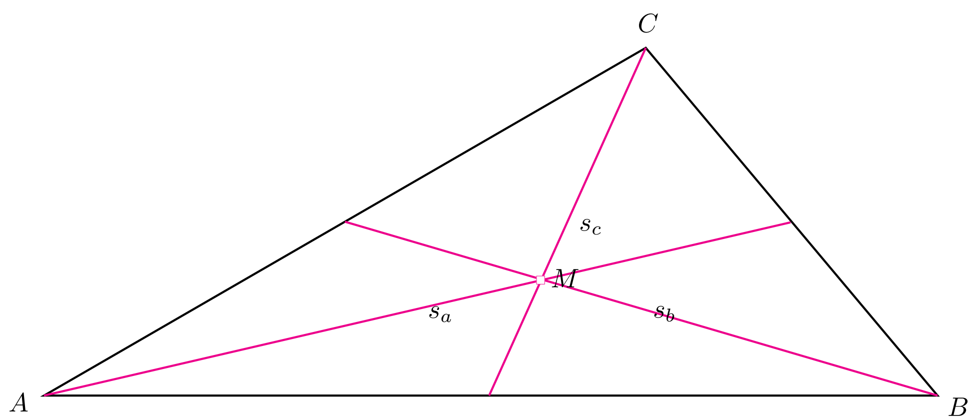
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 50°$
$\gamma = 100°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta$
$b = \frac{a \cdot \sin\beta}{\sin\alpha}$
$b = \frac{6 \cdot \sin 50}{\sin 30}$

$b = 9,19$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 9,19^2 - 2 \cdot 6 \cdot 9,19 \cdot \cos 100^\circ}$
$c = 11,8$
Umfang: $U = a + b + c$
$U = 6 + 9,19 + 11,8$
$U = 27$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 11,8 \cdot \sin 50^\circ$
$h_a = 9,05$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 9,05$
$A = 27,2$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 100^\circ$
$h_b = 5,91$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 9,19 \cdot \sin 30^\circ$
$h_c = 4,6$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{11,8 \cdot \sin 50}{\sin 115}$
$wha = 9,99$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 100}{\sin 55}$

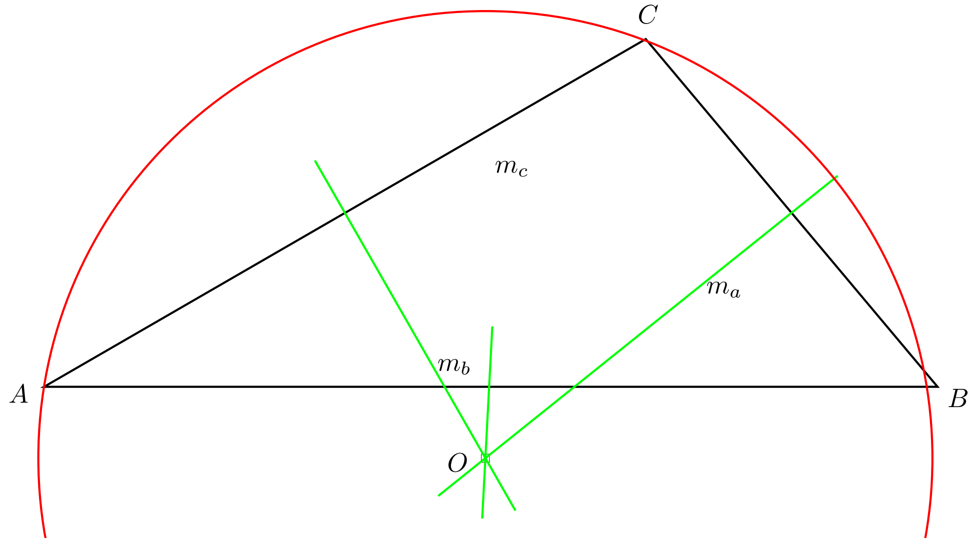
$whb = 7,21$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{9,19 \cdot \sin 30}{\sin 115}$
$whc = 3,31$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(9,19^2 + 11,8^2) - 6^2}$
$s_a = 10,2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 11,8^2) - 9,19^2}$
$s_b = 8,17$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 9,19^2) - 11,8^2}$
$s_c = 6,26$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 30°}$
$r_u = 6$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 27,2}{27}$
$r_i = 2\frac{1}{91}$



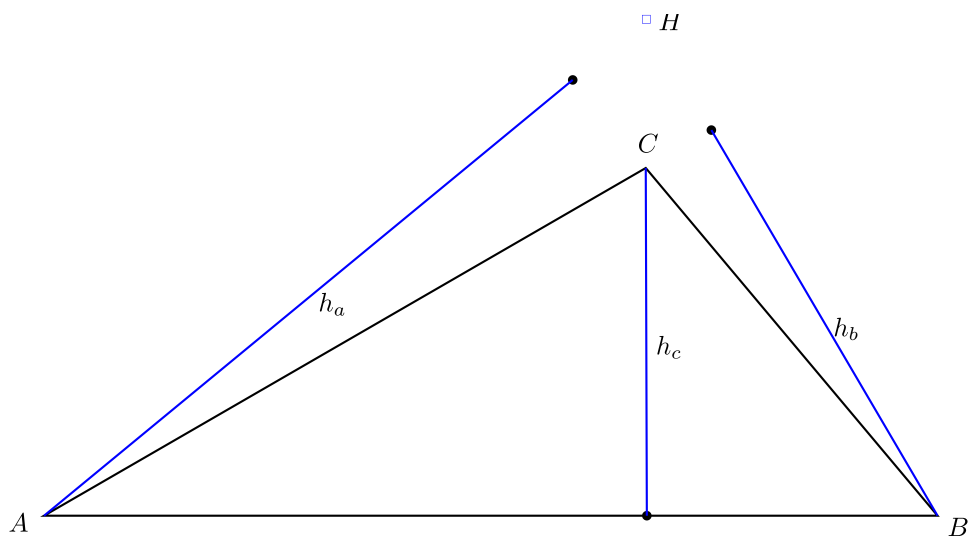
Seitenhalbierende-Schwerpunkt



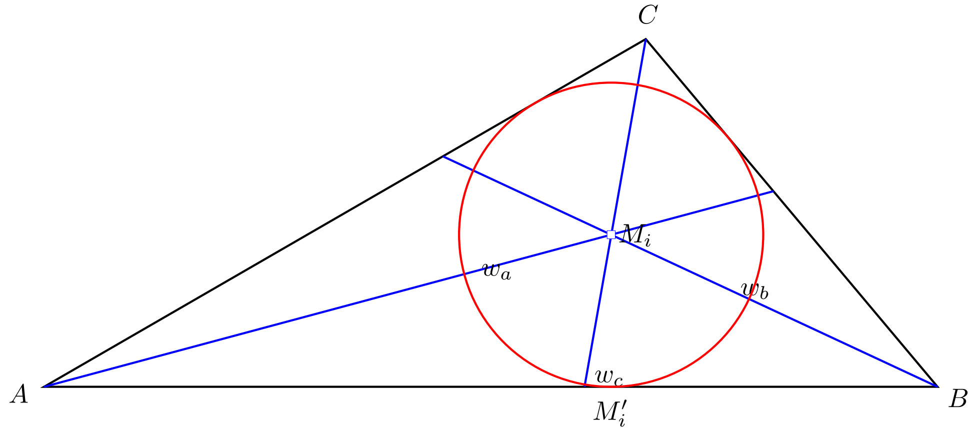
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



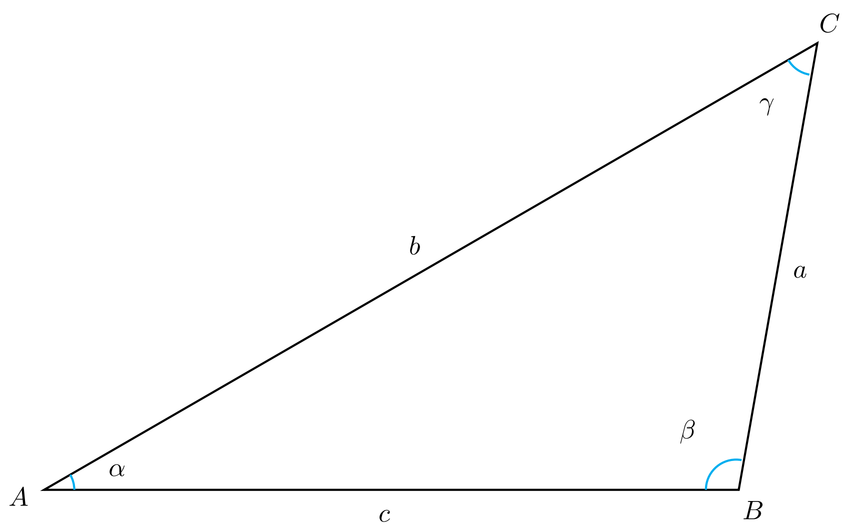
Aufgabe (46)

Winkel-Winkel-Seite
$a = 6 \quad \alpha = 30° \quad \gamma = 50°$

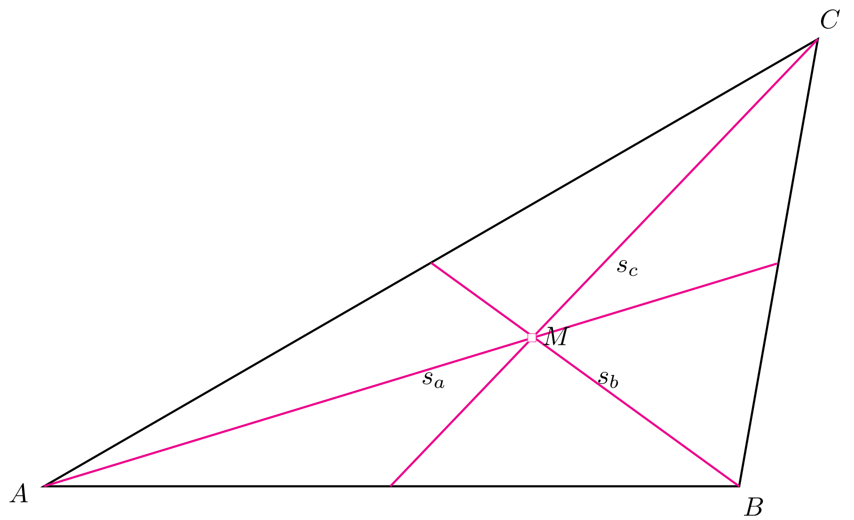
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 30° - 50°$
$\beta = 100°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta$
$b = \frac{a \cdot \sin\beta}{\sin\alpha}$
$b = \frac{6 \cdot \sin 100}{\sin 30}$
$b = 11,8$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 11,8^2 - 2 \cdot 6 \cdot 11,8 \cdot \cos 50°}$
$c = 9,19$
Umfang: $U = a + b + c$
$U = 6 + 11,8 + 9,19$
$U = 27$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$

$h_a = 9,19 \cdot \sin 100°$
$h_a = 9,05$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 9,05$
$A = 27,2$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 50°$
$h_b = 4,6$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 11,8 \cdot \sin 30°$
$h_c = 5,91$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9,19 \cdot \sin 100}{\sin 65}$
$wha = 9,99$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 50}{\sin 80}$
$whb = 4,67$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{11,8 \cdot \sin 30}{\sin 65}$
$whc = 3,31$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

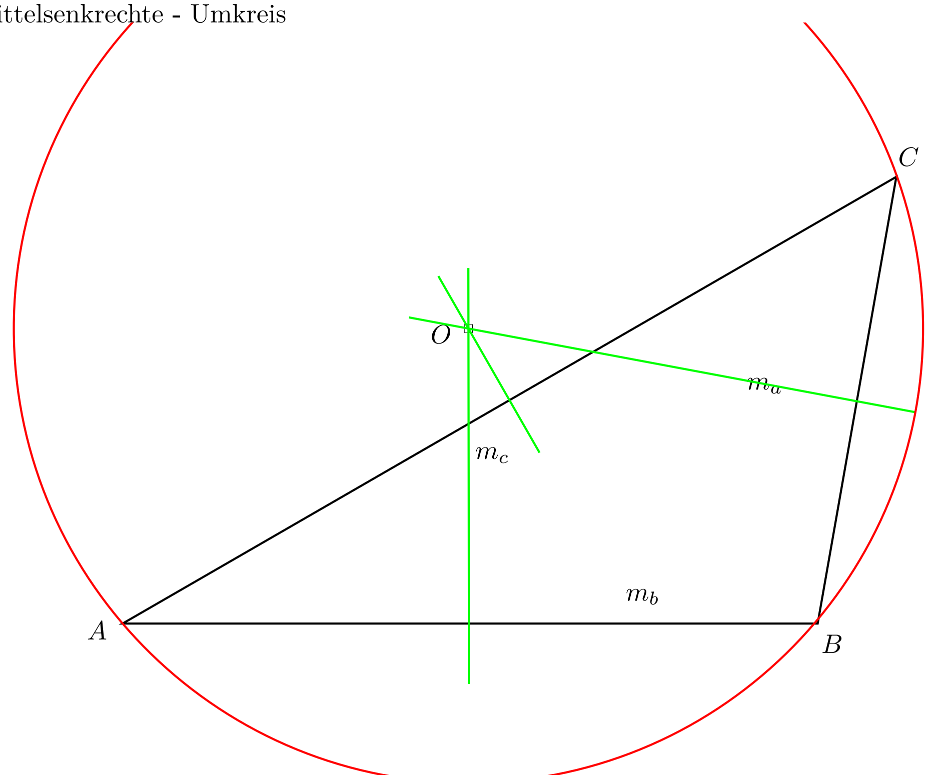
$s_a = \frac{1}{2}\sqrt{2(11,8^2 + 9,19^2) - 6^2}$
$s_a = 10,2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 9,19^2) - 11,8^2}$
$s_b = 5,03$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 11,8^2) - 9,19^2}$
$s_c = 7,27$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 30°}$
$r_u = 6$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 27,2}{27}$
$r_i = 2\frac{1}{91}$



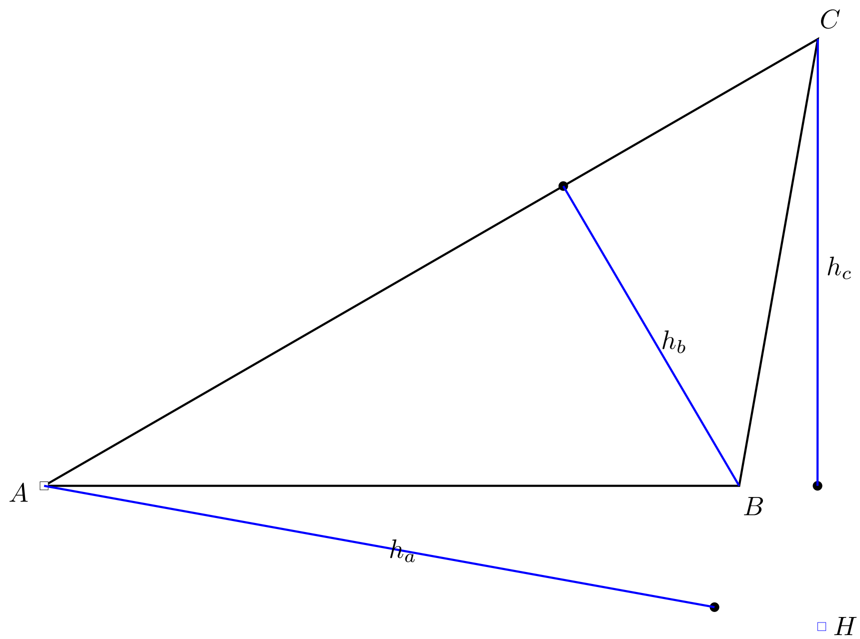
Seitenhalbierende-Schwerpunkt



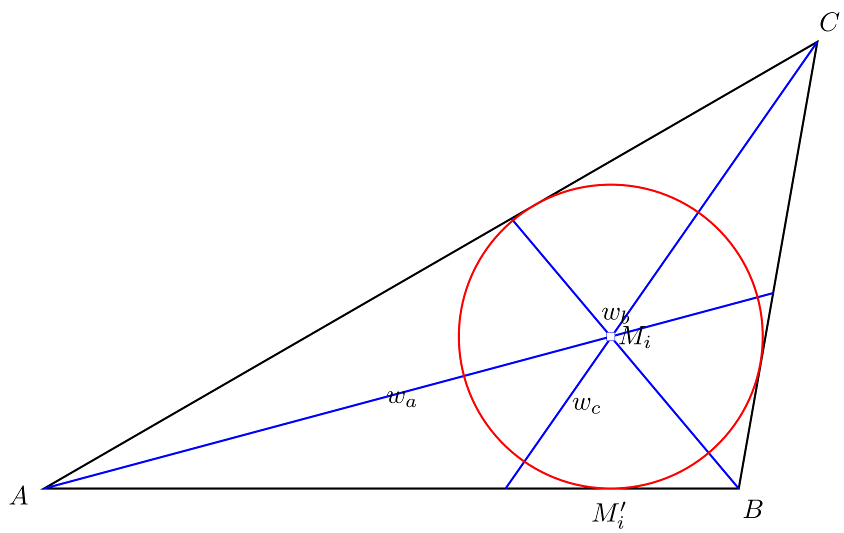
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

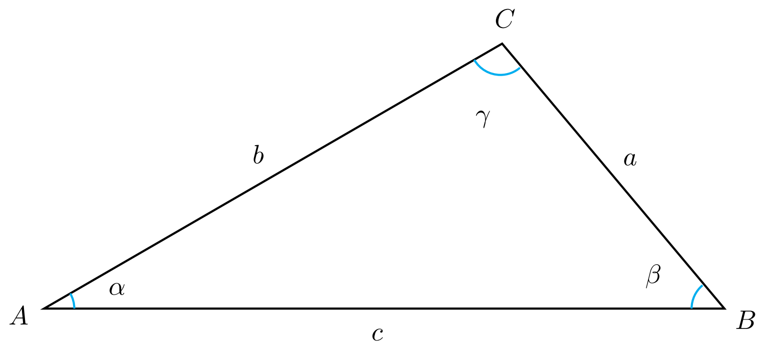


Aufgabe (47)

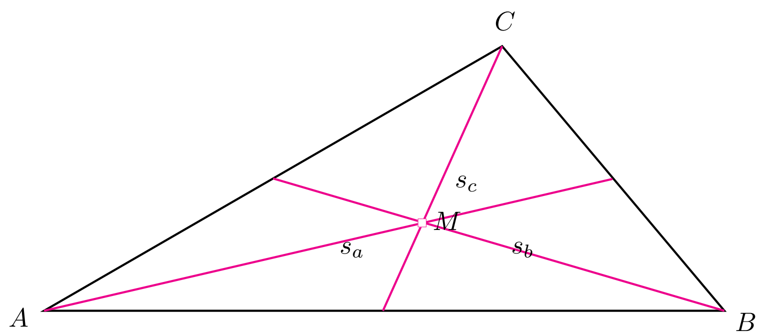
inkel-Winkel-Seite
$b = 7 \quad \alpha = 30° \quad \beta = 50°$

Winkelsumme: $\alpha+\beta+\gamma=180°$
$\alpha+\beta+\gamma=180 \qquad /-\alpha \qquad /-\beta$
$\gamma=180°-\alpha-\beta$
$\gamma=180°-30°-50°$
$\gamma=100°$
Sinus-Satz: $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta} \qquad /\cdot\sin\alpha$
$a=\frac{b\cdot\sin\alpha}{\sin\beta}$
$a=\frac{7\cdot\sin 30}{\sin 50}$
$a=4,57$
Kosinus-Satz: $c^2=a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma$
$c^2=a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma$
$c=\sqrt{a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma}$
$c=\sqrt{4,57^2+7^2-2\cdot 4,57\cdot 7\cdot\cos 100°}$
$c=9$
Umfang: $U=a+b+c$
$U=4,57+7+9$
$U=20,6$
Höhe: h_a
$\sin\beta=\frac{h_a}{c}$
$\sin\beta=\frac{h_a}{c} \quad /\cdot c$
$h_a=c\cdot\sin\beta$
$h_a=9\cdot\sin 50°$
$h_a=6,89$
Flaeche: $\quad A=\frac{1}{2}\cdot a\cdot h_a$
$A=\frac{1}{2}\cdot 4,57\cdot 6,89$
$A=15,7$
Höhe: h_b
$\sin\gamma=\frac{h_b}{a}$
$\sin\gamma=\frac{h_b}{a} \quad /\cdot a$
$h_b=a\cdot\sin\gamma$
$h_b=4,57\cdot\sin 100°$
$h_b=4,5$
Höhe: h_c
$\sin\alpha=\frac{h_c}{b}$
$\sin\alpha=\frac{h_c}{b} \quad /\cdot b$
$h_c=b\cdot\sin\alpha$
$h_c=7\cdot\sin 30°$
$h_c=3\frac{1}{2}$
Winkelhalbierende: α
$\delta=180-\beta-\frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta}=\frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta}=\frac{c}{\sin\delta} \qquad /\cdot\sin\beta$

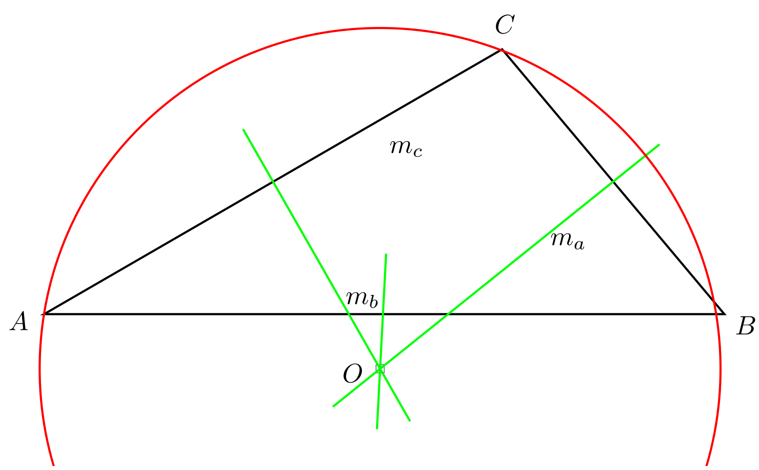
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9 \cdot \sin 50}{\sin 115}$
$wha = 7,61$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4,57 \cdot \sin 100}{\sin 55}$
$whb = 5,49$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 30}{\sin 115}$
$whc = 2,52$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 9^2) - 4,57^2}$
$s_a = 7,73$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4,57^2 + 9^2) - 7^2}$
$s_b = 6,22$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4,57^2 + 7^2) - 9^2}$
$s_c = 4,76$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4,57}{2 \cdot \sin 30°}$
$r_u = 4,57$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15,7}{20,6}$
$r_i = 1,53$



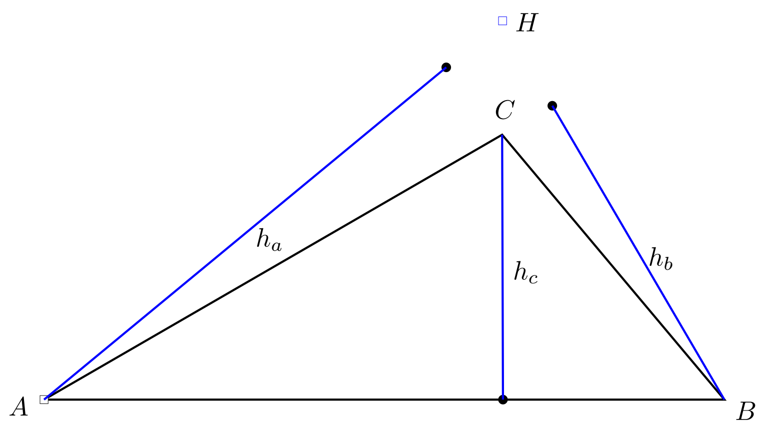
Seitenhalbierende-Schwerpunkt



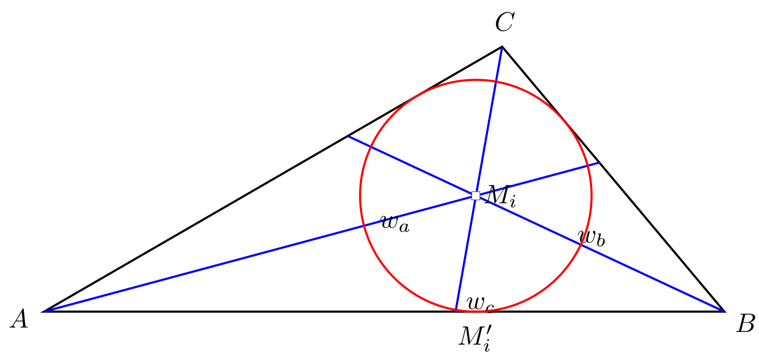
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



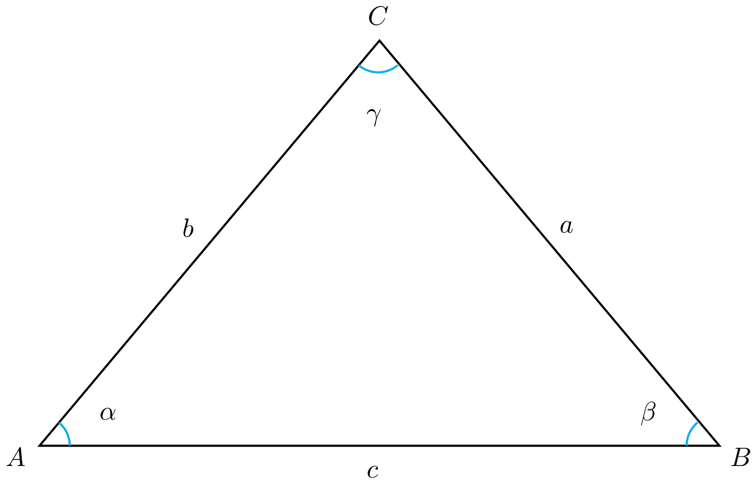
Aufgabe (48)

Winkel-Winkel-Seite
$b = 7 \quad \gamma = 80° \quad \beta = 50°$

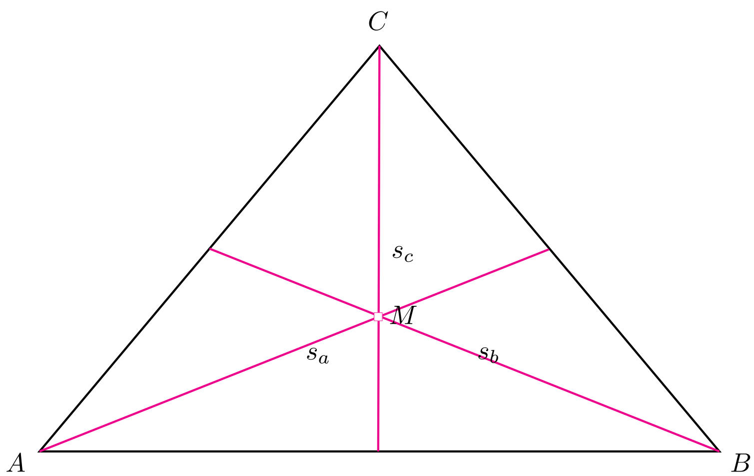
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 50° - 80°$
$\alpha = 50°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$

$a = \frac{7 \cdot \sin 50}{\sin 50}$
$a = 7$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{7^2 + 7^2 - 2 \cdot 7 \cdot 7 \cdot \cos 80°}$
$c = 9$
Umfang: $U = a + b + c$
$U = 7 + 7 + 9$
$U = 23$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 9 \cdot \sin 50°$
$h_a = 6,89$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7 \cdot 6,89$
$A = 24,1$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7 \cdot \sin 80°$
$h_b = 6,89$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 50°$
$h_c = 5,36$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9 \cdot \sin 50}{\sin 105}$
$wha = 7,14$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

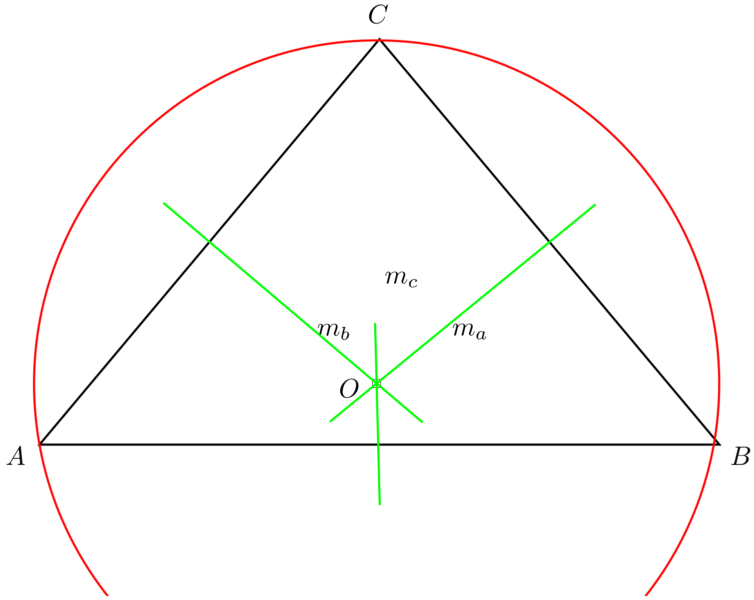
$whb = \frac{7 \cdot \sin 80}{\sin 75}$
$whb = 7,14$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 50}{\sin 105}$
$whc = 5,55$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 9^2) - 7^2}$
$s_a = 7,26$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(7^2 + 9^2) - 7^2}$
$s_b = 7,26$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(7^2 + 7^2) - 9^2}$
$s_c = 6,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7}{2 \cdot \sin 50°}$
$r_u = 4,57$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 24,1}{23}$
$r_i = 2,1$



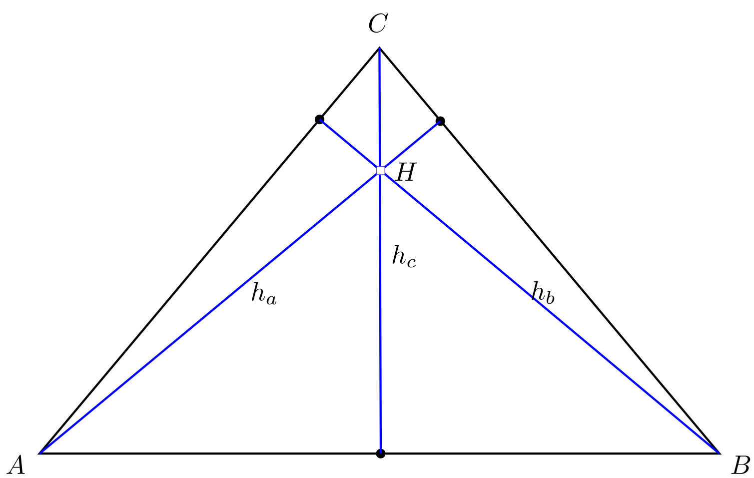
Seitenhalbierende-Schwerpunkt



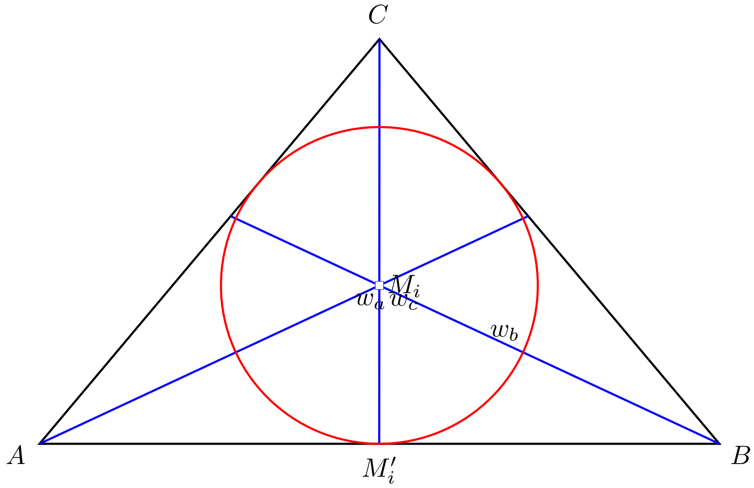
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



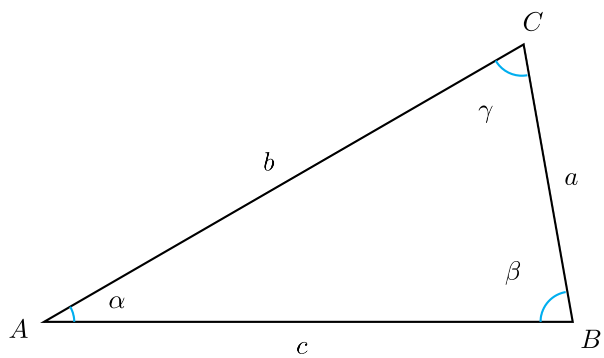
Aufgabe (49)

Winkel-Winkel-Seite
$c = 7 \quad \gamma = 70° \quad \alpha = 30°$

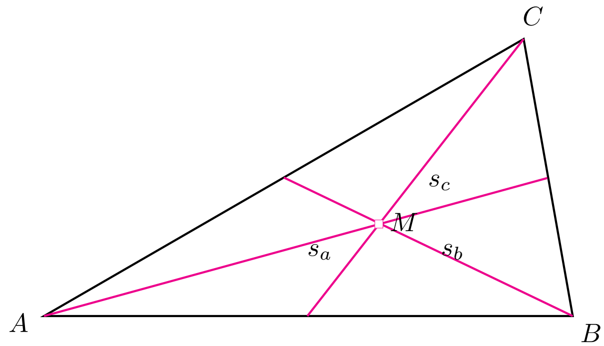
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 30° - 70°$
$\beta = 80°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$
$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\alpha$
$a = \frac{c \cdot \sin\alpha}{\sin\gamma}$
$a = \frac{7 \cdot \sin 30}{\sin 70}$
$a = 3,72$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{3,72^2 + 7^2 - 2 \cdot 3,72 \cdot 7 \cdot \cos 80°}$
$b = 7,34$
Umfang: $U = a + b + c$
$U = 3,72 + 7,34 + 7$
$U = 18,1$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$

$h_a = c \cdot \sin\beta$
$h_a = 7 \cdot \sin 80°$
$h_a = 6,89$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3,72 \cdot 6,89$
$A = 12,8$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3,72 \cdot \sin 70°$
$h_b = 3\frac{1}{2}$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,34 \cdot \sin 30°$
$h_c = 3,67$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{7 \cdot \sin 80}{\sin 85}$
$wha = 6,92$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3,72 \cdot \sin 70}{\sin 70}$
$whb = 3,72$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,34 \cdot \sin 30}{\sin 85}$
$whc = 1,87$
Seitenhalbierende:

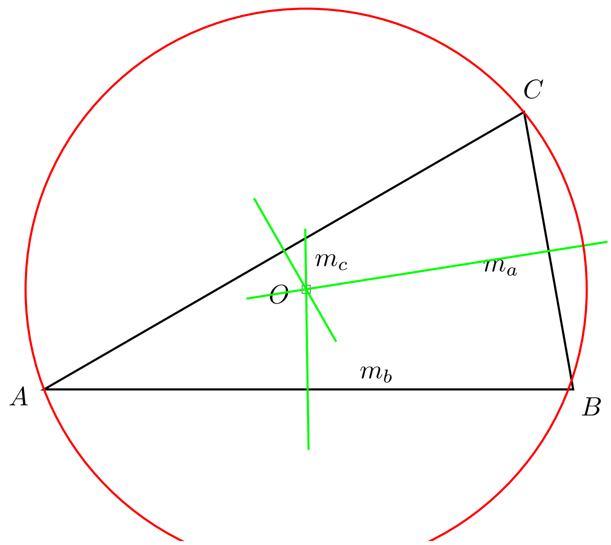
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,34^2 + 7^2) - 3,72^2}$
$s_a = 6,92$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3,72^2 + 7^2) - 7,34^2}$
$s_b = 4,24$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3,72^2 + 7,34^2) - 7^2}$
$s_c = 4,52$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3,72}{2 \cdot \sin 30°}$
$r_u = 3,72$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,8}{18,1}$
$r_i = 1,42$



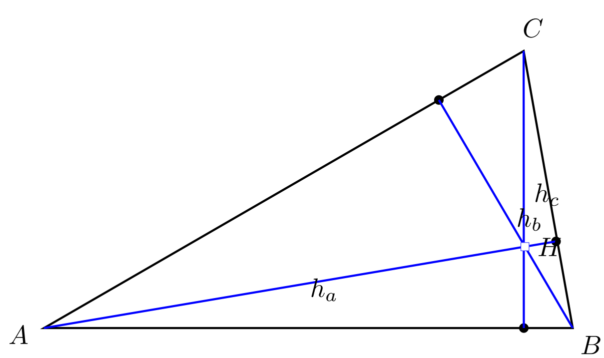
Seitenhalbierende-Schwerpunkt



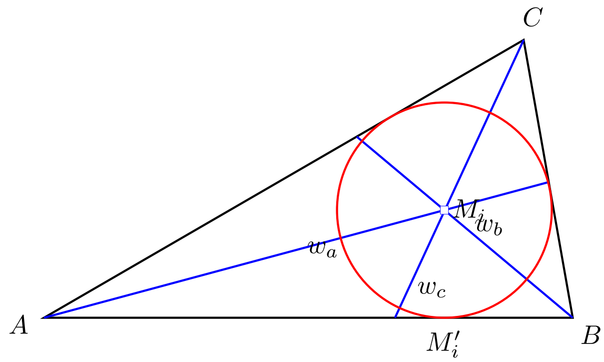
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



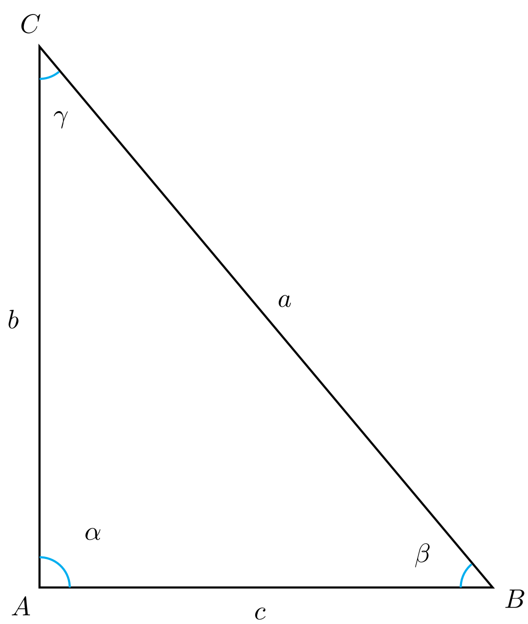
Aufgabe (50)

Winkel-Winkel-Seite
$c = 6 \quad \gamma = 40° \quad \beta = 50°$

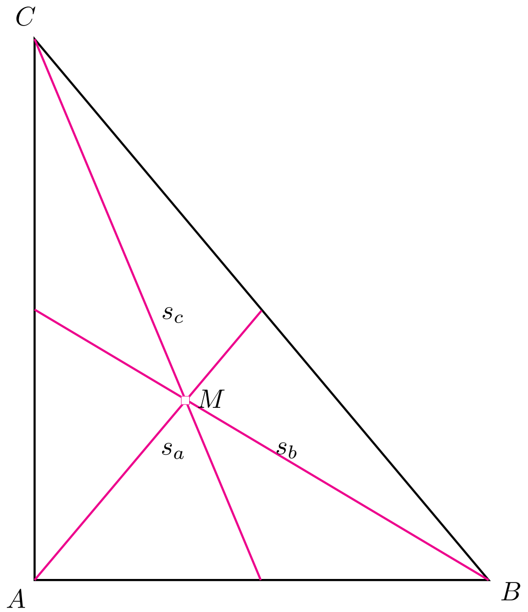
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 50° - 40°$
$\alpha = 90°$
Kosinus: $\cos\beta = \frac{c}{a}$
$\cos\beta = \frac{c}{a} \quad / \cdot a$
$a \cdot \cos\beta = c \quad / : cos\beta$
$a = \frac{c}{\cos\beta}$
$a = \frac{6}{\cos 50}$
$a = 9,33$
Pythagoras: $a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{9,33^2 - 6^2}$
$b = 7,15$
Umfang: $U = a + b + c$
$U = 9,33 + 7,15 + 6$
$U = 22,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 50°$
$h_a = 4,6$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 9,33 \cdot 4,6$
$A = 21,5$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 9,33 \cdot \sin 40°$
$h_b = 6$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,15 \cdot \sin 90°$
$h_c = 7,15$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 50}{\sin 85}$
$wha = 4,61$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{9,33 \cdot \sin 40}{\sin 115}$
$whb = 6,62$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,15 \cdot \sin 90}{\sin 85}$
$whc = 9,37$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,15^2 + 6^2) - 9,33^2}$
$s_a = 4,67$

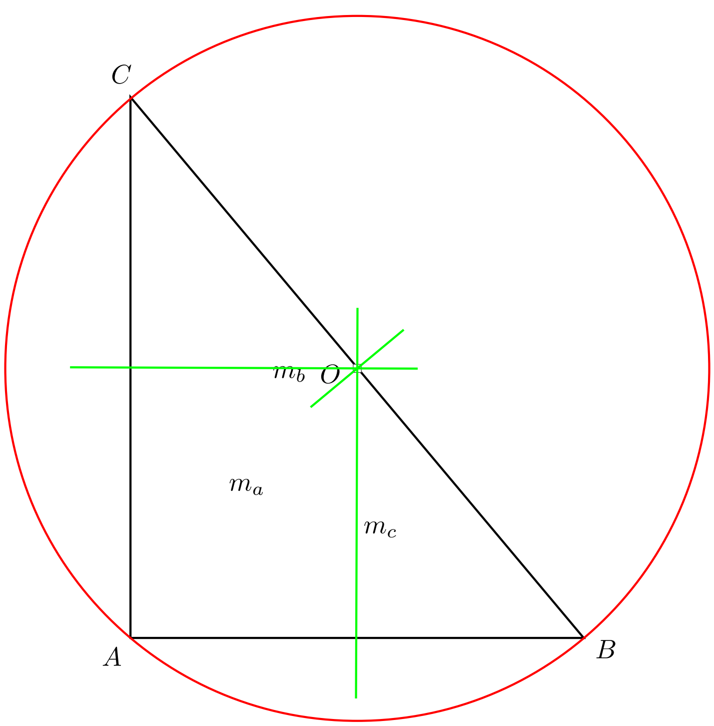
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(9,33^2+6^2)-7,15^2}$
$s_b = 6,98$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(9,33^2+7,15^2)-6^2}$
$s_c = 7,51$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{9,33}{2 \cdot \sin 90°}$
$r_u = 4,67$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 21,5}{22,5}$
$r_i = 1,91$



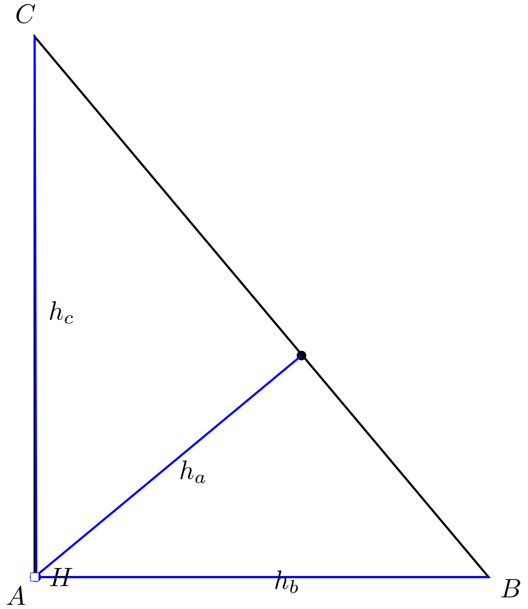
Seitenhalbierende-Schwerpunkt



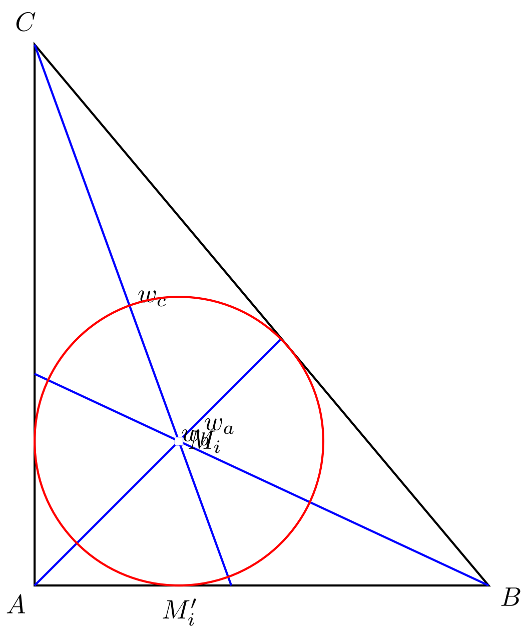
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



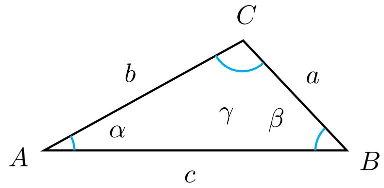
Aufgabe (51)

Seite-Seite-Seite
$a = 2 \quad b = 3 \quad c = 4$

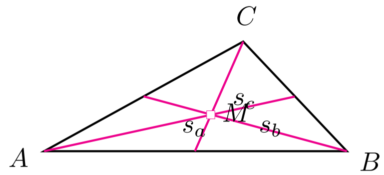
Umfang: $U = a + b + c$
$U = 2 + 3 + 4$
$U = 9$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{3^2 + 4^2 - 2^2}{2 \cdot 3 \cdot 4}$
$\cos\alpha = \dfrac{7}{8}$
$\alpha = \arccos(\dfrac{7}{8})$
$\alpha = 29°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \dfrac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \dfrac{2^2 + 4^2 - 3^2}{2 \cdot 2 \cdot 4}$
$\cos\beta = \dfrac{11}{16}$
$\beta = \arccos(\dfrac{11}{16})$
$\beta = 46,6°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29° - 46,6°$
$\gamma = 104°$
Höhe: h_a
$\sin\beta = \dfrac{h_a}{c}$
$\sin\beta = \dfrac{h_a}{c} \qquad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 46,6°$
$h_a = 2,9$
Flaeche: $\quad A = \dfrac{1}{2} \cdot a \cdot h_a$
$A = \dfrac{1}{2} \cdot 2 \cdot 2,9$
$A = 2,9$
Höhe: h_b
$\sin\gamma = \dfrac{h_b}{a}$
$\sin\gamma = \dfrac{h_b}{a} \qquad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2 \cdot \sin 104°$
$h_b = 1,94$

Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 29°$
$h_c = 1,45$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4 \cdot \sin 46,6}{\sin 119}$
$wha = 3,32$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2 \cdot \sin 104}{\sin 52,2}$
$whb = 2,45$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 29}{\sin 119}$
$whc = 1,11$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2^2}$
$s_a = 3,39$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2^2 + 4^2) - 3^2}$
$s_b = 2,78$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2^2 + 3^2) - 4^2}$
$s_c = 2,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$

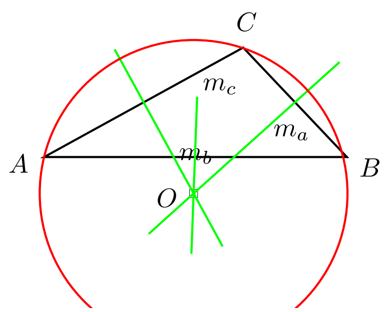
$r_u = \frac{2}{2 \cdot \sin 29°}$
$r_u = 2,07$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,9}{9}$
$r_i = 0,645$



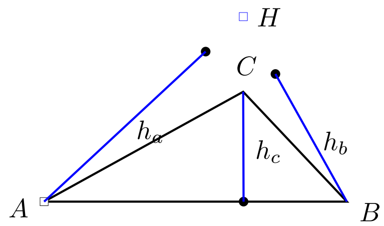
Seitenhalbierende-Schwerpunkt



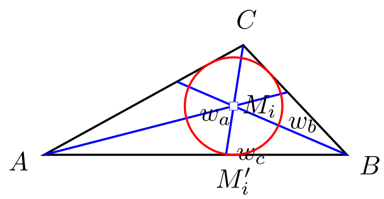
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



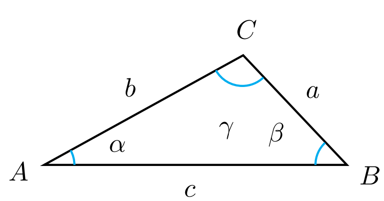
Aufgabe (52)

Seite-Seite-Seite
$a = 2 \quad b = 3 \quad c = 4$

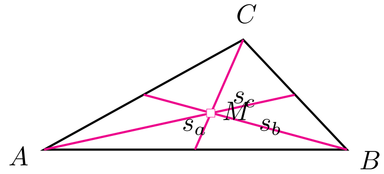
Umfang: $U = a + b + c$
$U = 2 + 3 + 4$
$U = 9$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3^2 + 4^2 - 2^2}{2 \cdot 3 \cdot 4}$
$\cos\alpha = \frac{7}{8}$
$\alpha = \arccos(\frac{7}{8})$
$\alpha = 29°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{2^2 + 4^2 - 3^2}{2 \cdot 2 \cdot 4}$

$\cos\beta = \frac{11}{16}$
$\beta = \arccos(\frac{11}{16})$
$\beta = 46,6°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29° - 46,6°$
$\gamma = 104°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 46,6°$
$h_a = 2,9$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2 \cdot 2,9$
$A = 2,9$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2 \cdot \sin 104°$
$h_b = 1,94$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 29°$
$h_c = 1,45$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4 \cdot \sin 46,6}{\sin 119}$
$wha = 3,32$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2 \cdot \sin 104}{\sin 52,2}$

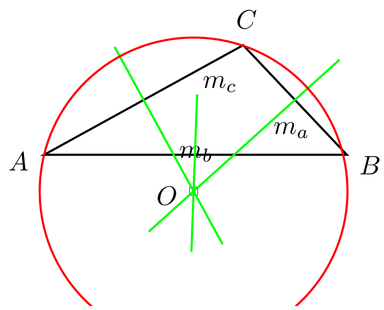
$whb = 2,45$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 29}{\sin 119}$
$whc = 1,11$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2^2}$
$s_a = 3,39$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2^2 + 4^2) - 3^2}$
$s_b = 2,78$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2^2 + 3^2) - 4^2}$
$s_c = 2,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2}{2 \cdot \sin 29°}$
$r_u = 2,07$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,9}{9}$
$r_i = 0,645$



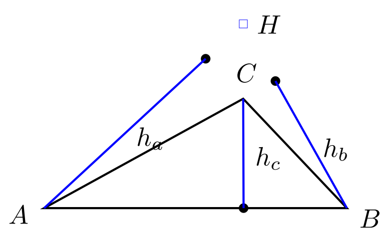
Seitenhalbierende-Schwerpunkt



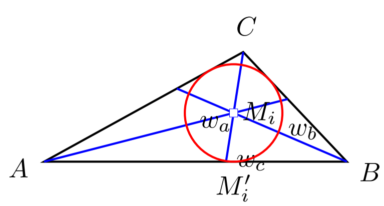
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (53)

Seite-Seite-Seite
$a = 2 \quad b = 3 \quad c = 4$

Umfang: $U = a + b + c$
$U = 2 + 3 + 4$
$U = 9$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3^2 + 4^2 - 2^2}{2 \cdot 3 \cdot 4}$
$\cos\alpha = \frac{7}{8}$
$\alpha = \arccos(\frac{7}{8})$
$\alpha = 29°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{2^2 + 4^2 - 3^2}{2 \cdot 2 \cdot 4}$
$\cos\beta = \frac{11}{16}$
$\beta = \arccos(\frac{11}{16})$
$\beta = 46,6°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29° - 46,6°$
$\gamma = 104°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 46,6°$
$h_a = 2,9$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2 \cdot 2,9$
$A = 2,9$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2 \cdot \sin 104°$

$h_b = 1,94$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 29°$
$h_c = 1,45$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4 \cdot \sin 46,6}{\sin 119}$
$wha = 3,32$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2 \cdot \sin 104}{\sin 52,2}$
$whb = 2,45$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 29}{\sin 119}$
$whc = 1,11$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2^2}$
$s_a = 3,39$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2^2 + 4^2) - 3^2}$
$s_b = 2,78$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2^2 + 3^2) - 4^2}$
$s_c = 2,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

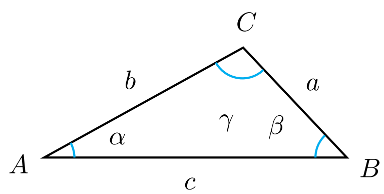
$r_u = \frac{2}{2 \cdot \sin 29^\circ}$

$r_u = 2,07$

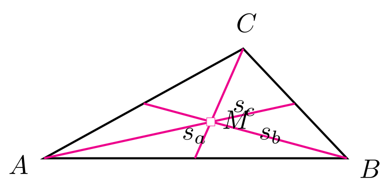
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 2,9}{9}$

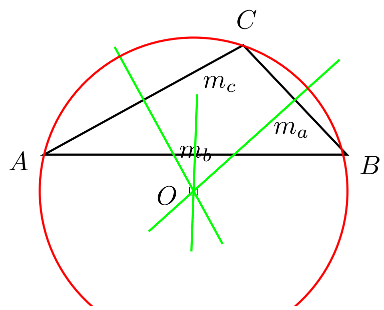
$r_i = 0,645$



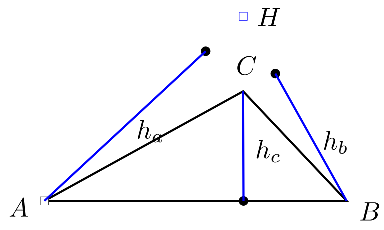
Seitenhalbierende-Schwerpunkt



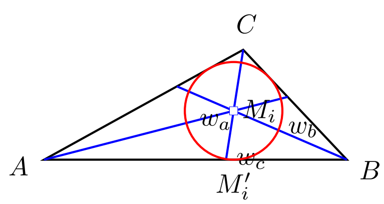
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



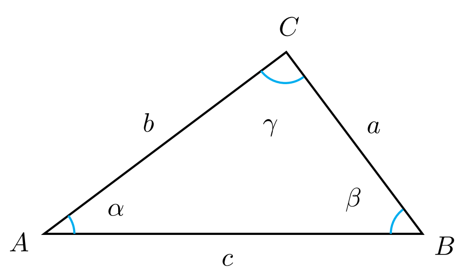
Aufgabe (54)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

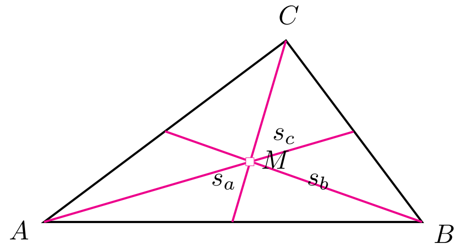
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$

$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$

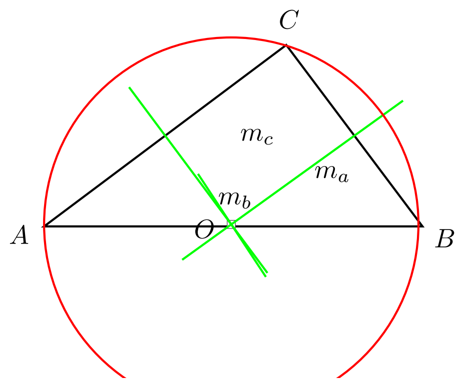
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



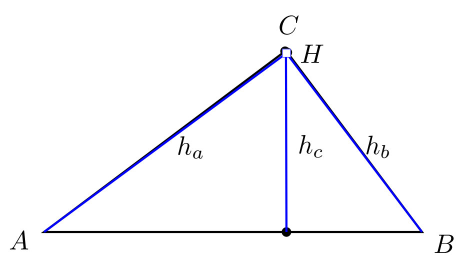
Seitenhalbierende-Schwerpunkt



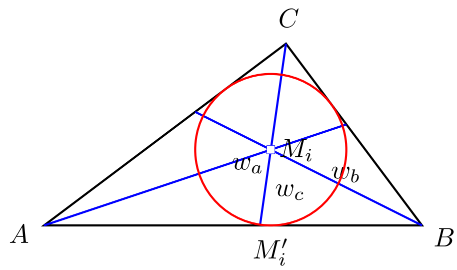
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



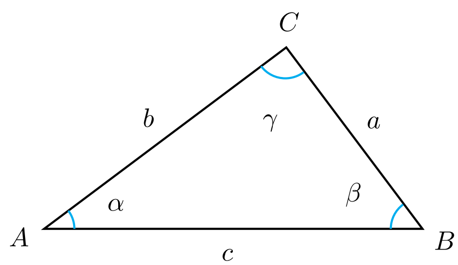
Aufgabe (55)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

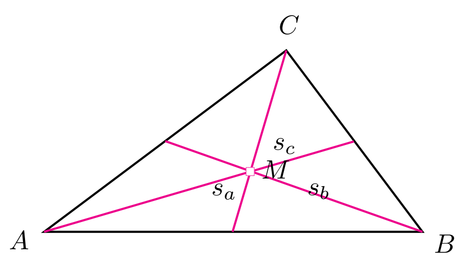
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

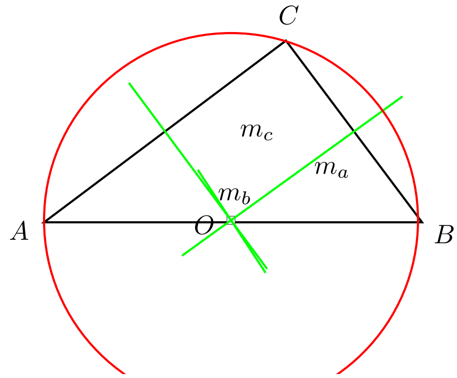
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2\cdot\sin\alpha}$
$r_u = \frac{3}{2\cdot\sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2\cdot A}{U}$
$r_i = \frac{2\cdot 6}{12}$
$r_i = 1$



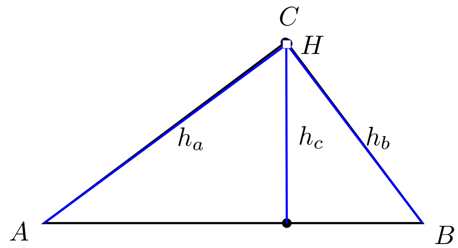
Seitenhalbierende-Schwerpunkt



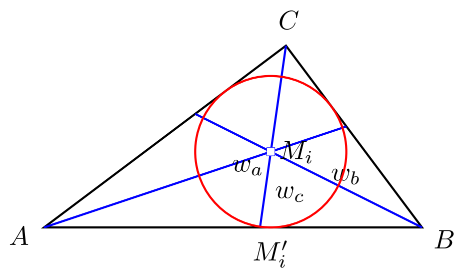
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (56)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$

$whb = 3,35$

Winkelhalbierende: γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$

$whc = 1,9$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$

$s_a = 4,27$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$

$s_b = 3,61$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$

$s_c = 2,92$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

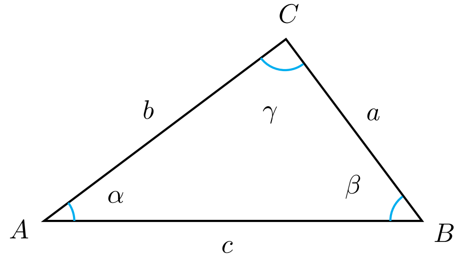
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$

$r_u = 2\frac{1}{2}$

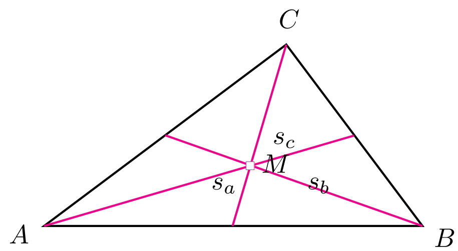
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 6}{12}$

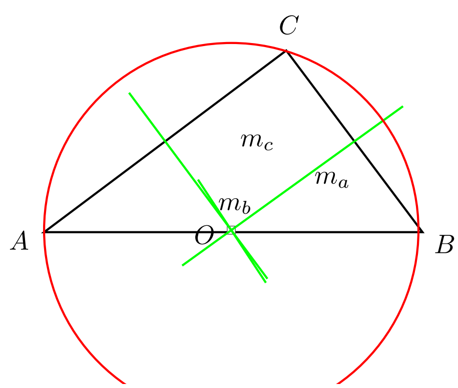
$r_i = 1$



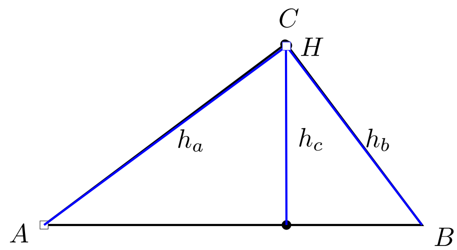
Seitenhalbierende-Schwerpunkt



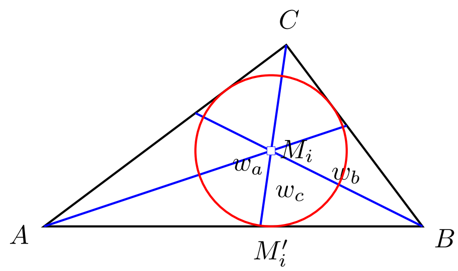
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



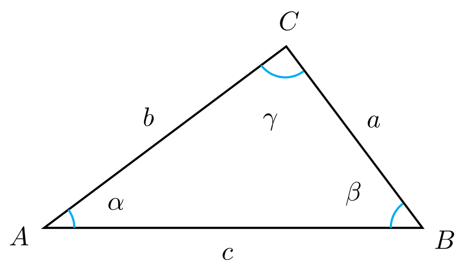
Aufgabe (57)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

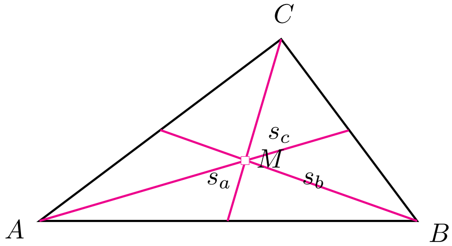
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$

$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

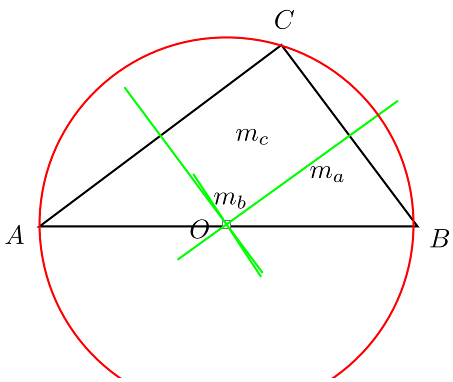
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2+5^2)-3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2+5^2)-4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



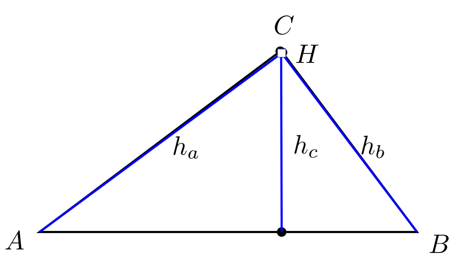
Seitenhalbierende-Schwerpunkt



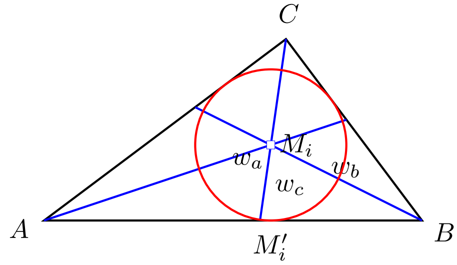
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



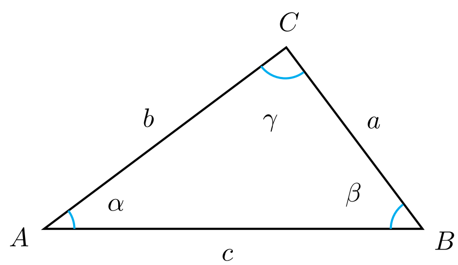
Aufgabe (58)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

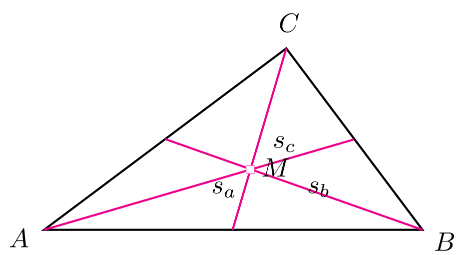
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

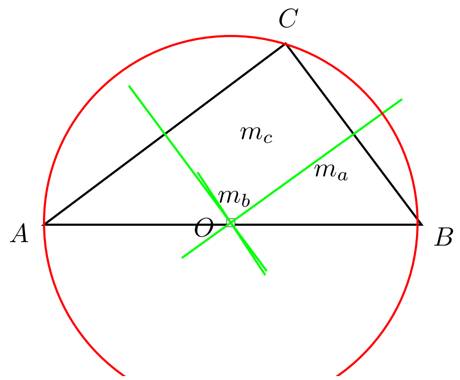
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



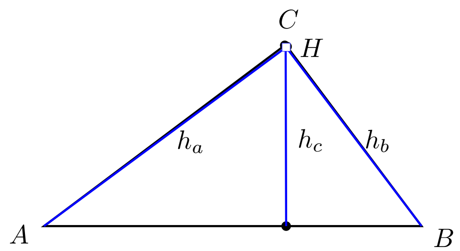
Seitenhalbierende-Schwerpunkt



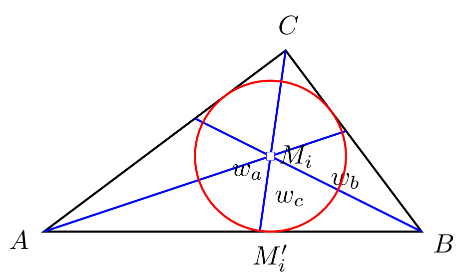
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



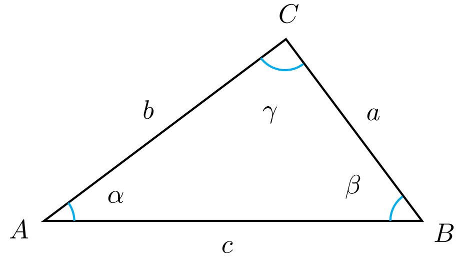
Aufgabe (59)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$

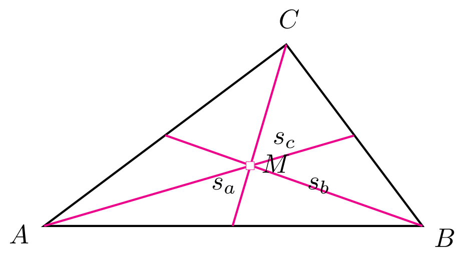
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β

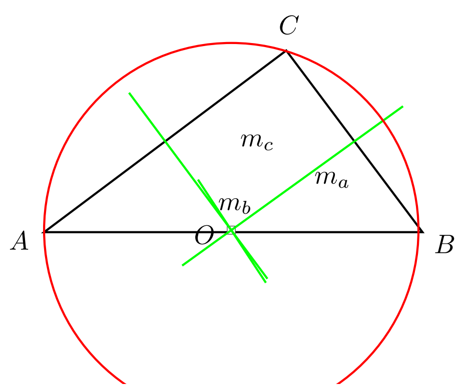
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



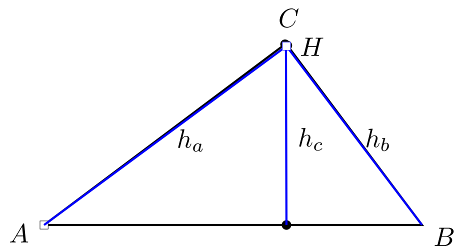
Seitenhalbierende-Schwerpunkt



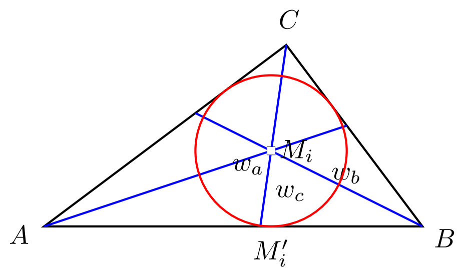
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



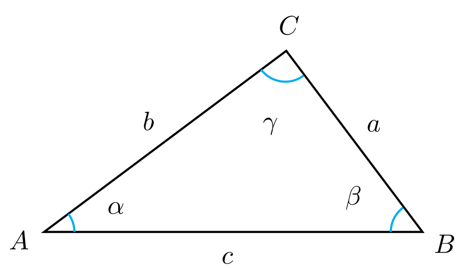
Aufgabe (60)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

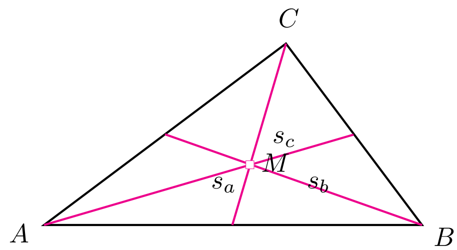
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$

$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

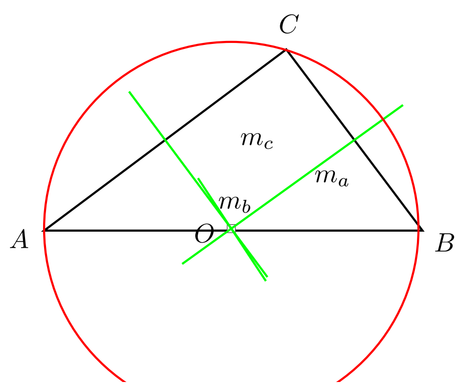
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2+5^2)-3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2+5^2)-4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



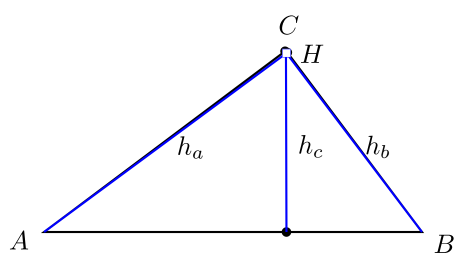
Seitenhalbierende-Schwerpunkt



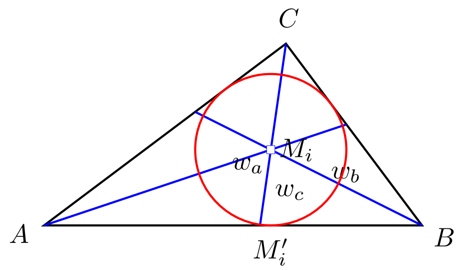
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



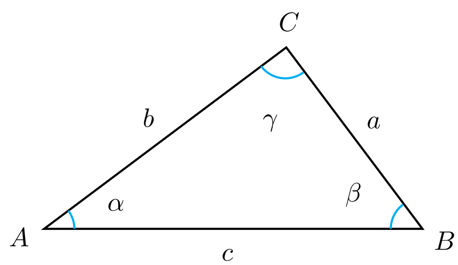
Aufgabe (61)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3 \qquad b = 4$ Hypothenuse: $c = 5 \quad \gamma = 90°$

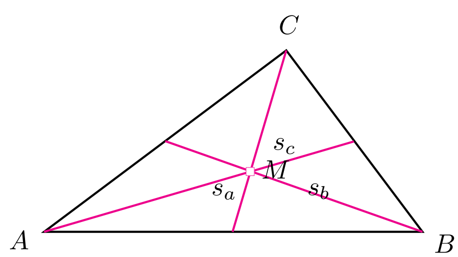
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

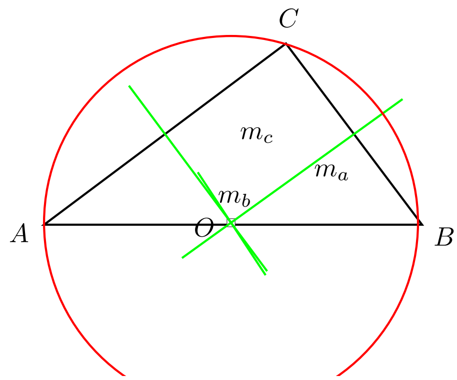
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



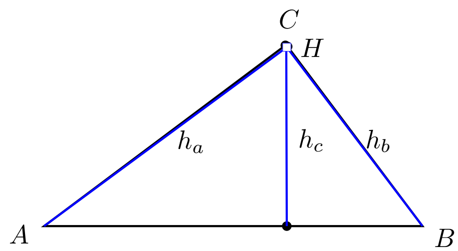
Seitenhalbierende-Schwerpunkt



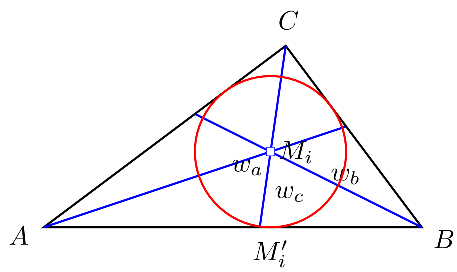
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



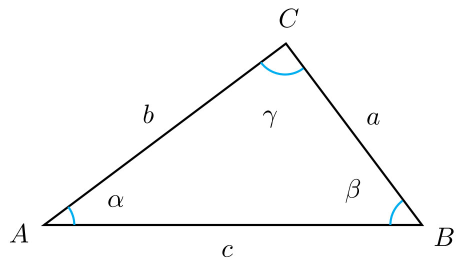
Aufgabe (62)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$

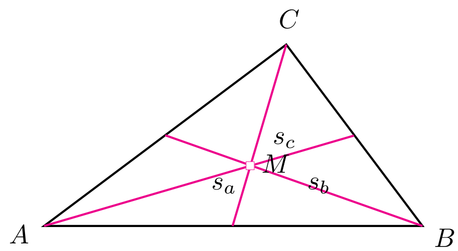
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β

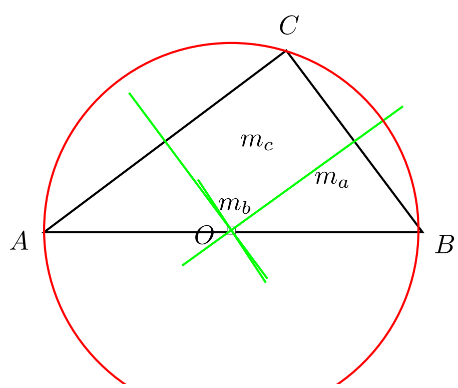
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



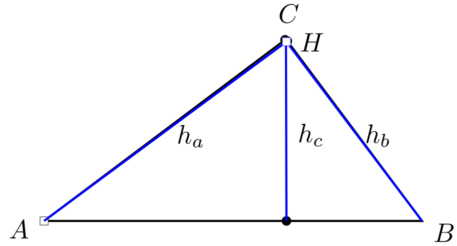
Seitenhalbierende-Schwerpunkt



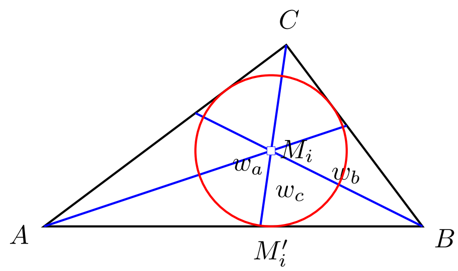
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



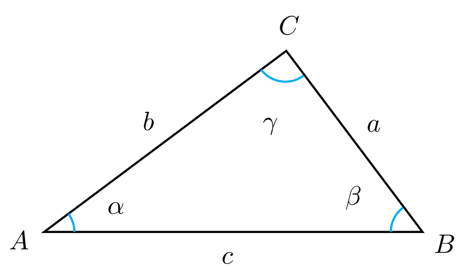
Aufgabe (63)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

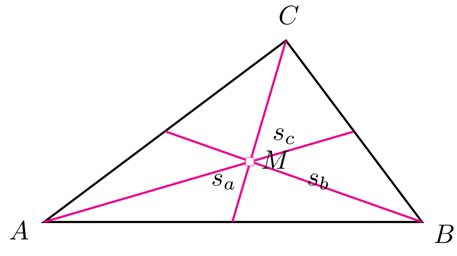
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$

$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

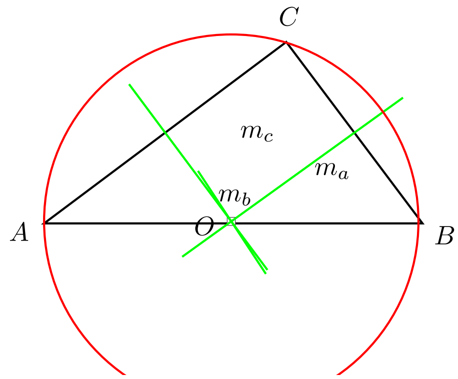
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



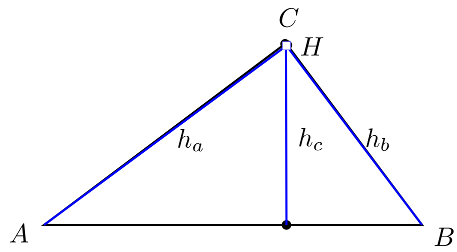
Seitenhalbierende-Schwerpunkt



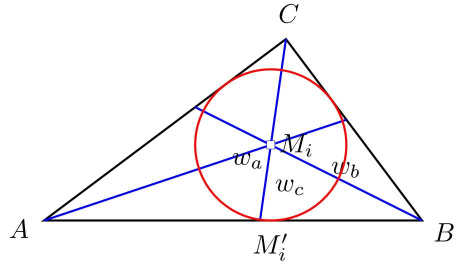
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



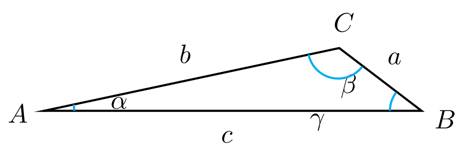
Aufgabe (64)

Seite-Winkel-Seite
$b = 4 \quad c = 5 \quad \alpha = 12°$

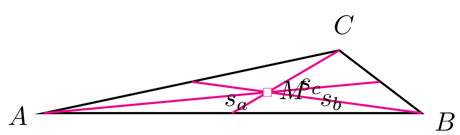
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 12°}$
$a = 1,37$
Umfang: $U = a + b + c$
$U = 1,37 + 4 + 5$
$U = 10,4$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{1,37^2 + 5^2 - 4^2}{2 \cdot 1,37 \cdot 5}$
$\cos\beta = 0,794$
$\beta = \arccos(0,794)$
$\beta = 37,4°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 12° - 37,4°$
$\gamma = 131°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 37,4°$
$h_a = 3,04$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$

$A = \frac{1}{2} \cdot 1,37 \cdot 3,04$
$A = 2,08$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 1,37 \cdot \sin 131°$
$h_b = 1,04$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 12°$
$h_c = 0,832$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 37,4}{\sin 137}$
$wha = 4,42$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{1,37 \cdot \sin 131}{\sin 30,7}$
$whb = 2,04$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 12}{\sin 137}$
$whc = 0,414$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 1,37^2}$
$s_a = 4,48$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

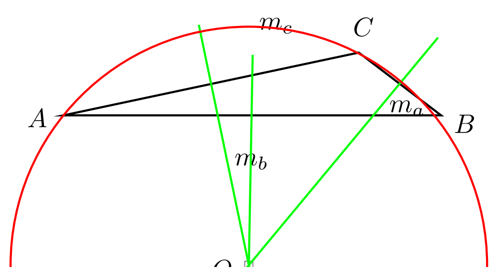
$s_b = \frac{1}{2}\sqrt{2(1,37^2 + 5^2) - 4^2}$
$s_b = 3,07$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(1,37^2 + 4^2) - 5^2}$
$s_c = 2,22$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{1,37}{2 \cdot \sin 12°}$
$r_u = 3,29$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,08}{10,4}$
$r_i = 0,401$



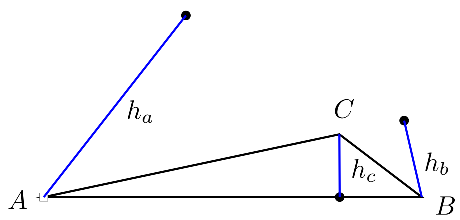
Seitenhalbierende-Schwerpunkt



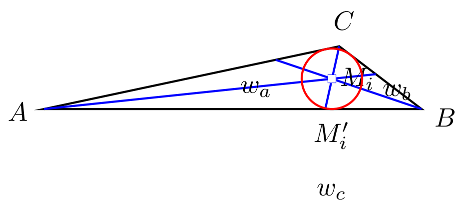
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



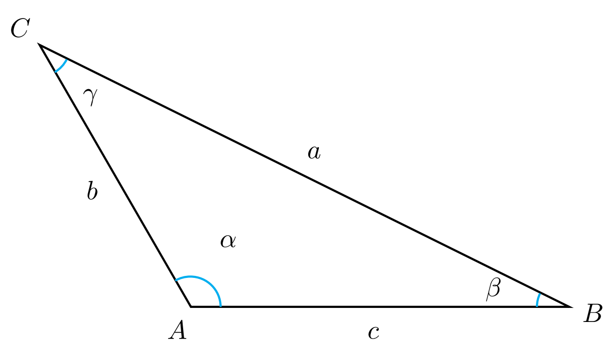
Aufgabe (65)

Seite-Winkel-Seite
$b = 4 \quad c = 5 \quad \alpha = 120°$

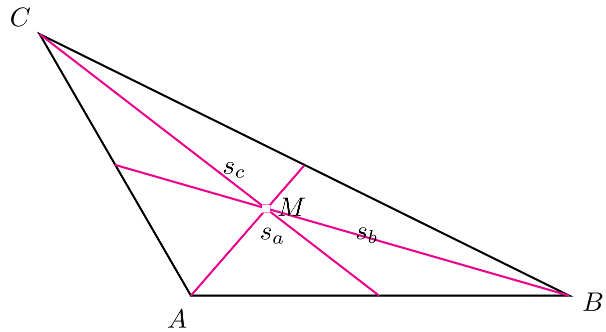
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 120°}$
$a = 7,81$
Umfang: $U = a + b + c$
$U = 7,81 + 4 + 5$
$U = 16,8$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{7,81^2 + 5^2 - 4^2}{2 \cdot 7,81 \cdot 5}$
$\cos\beta = 0,896$
$\beta = \arccos(0,896)$
$\beta = 26,3°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 120° - 26,3°$
$\gamma = 33,7°$
Höhe: h_a

$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 26,3°$
$h_a = 2,22$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7,81 \cdot 2,22$
$A = 8,66$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7,81 \cdot \sin 33,7°$
$h_b = 4,33$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 120°$
$h_c = 3,46$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 26,3}{\sin 93,7}$
$wha = 2\frac{2}{9}$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7,81 \cdot \sin 33,7}{\sin 133}$
$whb = 5,94$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

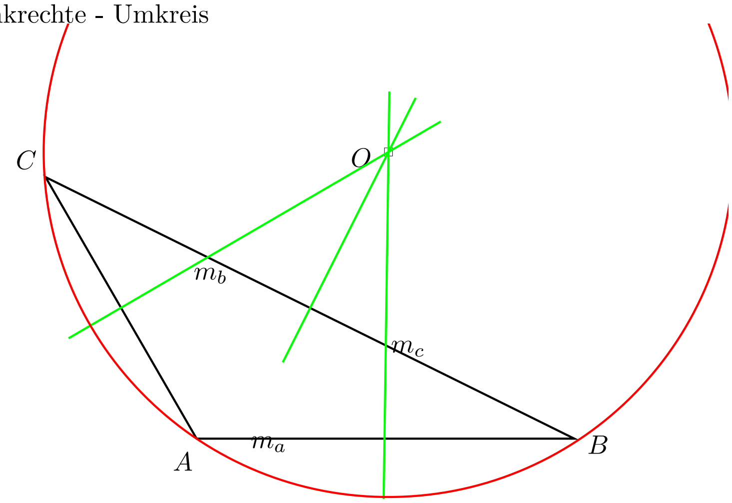
$whc = \frac{4 \cdot \sin 120}{\sin 93,7}$
$whc = 6\frac{7}{9}$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2+5^2)-7,81^2}$
$s_a = 2,29$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(7,81^2+5^2)-4^2}$
$s_b = 6,24$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(7,81^2+4^2)-5^2}$
$s_c = 5,87$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7,81}{2 \cdot \sin 120°}$
$r_u = 4,51$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8,66}{16,8}$
$r_i = 1,03$



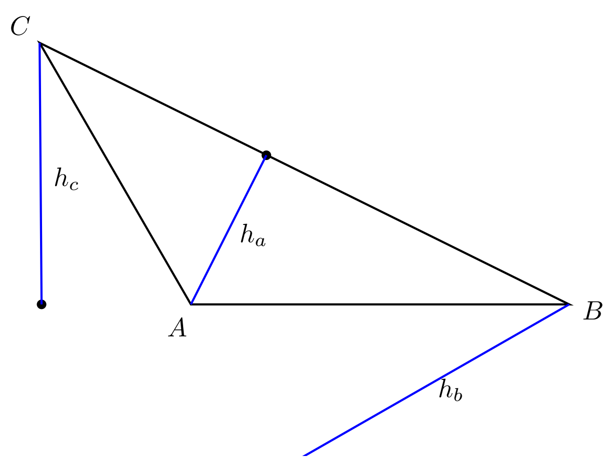
Seitenhalbierende-Schwerpunkt



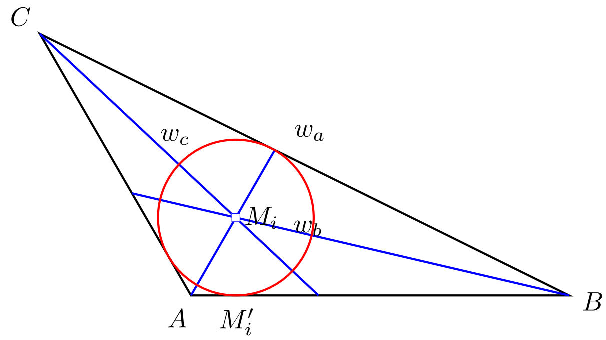
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



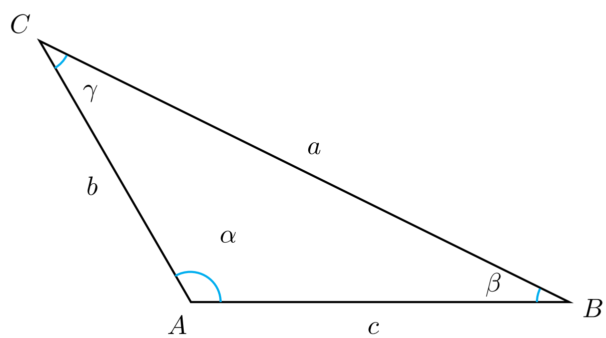
Aufgabe (66)

Seite-Winkel-Seite
$b = 4 \quad c = 5 \quad \alpha = 120°$

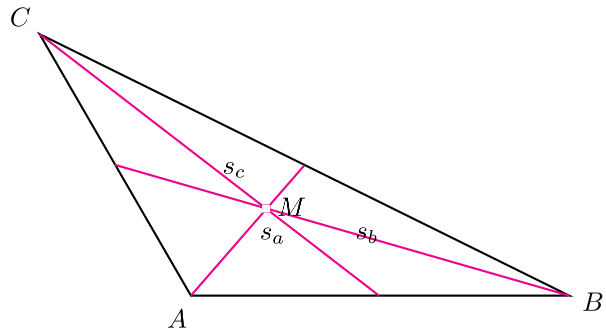
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 120°}$
$a = 7,81$
Umfang: $U = a + b + c$
$U = 7,81 + 4 + 5$
$U = 16,8$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{7,81^2 + 5^2 - 4^2}{2 \cdot 7,81 \cdot 5}$
$\cos\beta = 0,896$
$\beta = \arccos(0,896)$
$\beta = 26,3°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 120° - 26,3°$
$\gamma = 33,7°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 26,3°$
$h_a = 2,22$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7,81 \cdot 2,22$
$A = 8,66$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7,81 \cdot \sin 33,7°$
$h_b = 4,33$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 120°$
$h_c = 3,46$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 26,3}{\sin 93,7}$
$wha = 2\frac{2}{9}$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7,81 \cdot \sin 33,7}{\sin 133}$
$whb = 5,94$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 120}{\sin 93,7}$
$whc = 6\frac{7}{9}$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

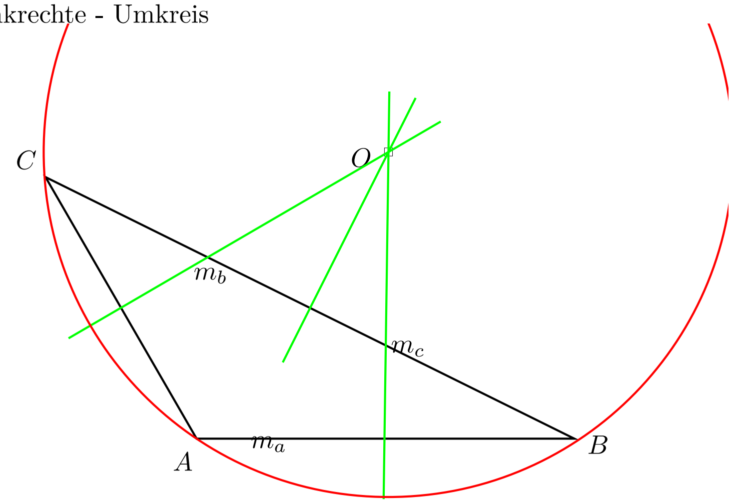
$s_a = \frac{1}{2}\sqrt{2(4^2+5^2)-7,81^2}$
$s_a = 2,29$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(7,81^2+5^2)-4^2}$
$s_b = 6,24$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(7,81^2+4^2)-5^2}$
$s_c = 5,87$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7,81}{2 \cdot \sin 120°}$
$r_u = 4,51$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8,66}{16,8}$
$r_i = 1,03$



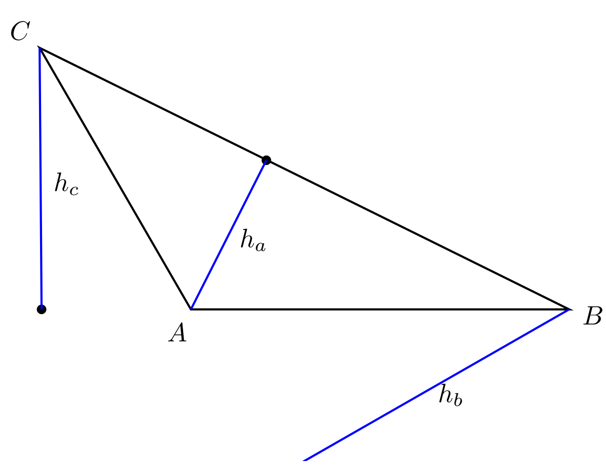
Seitenhalbierende-Schwerpunkt



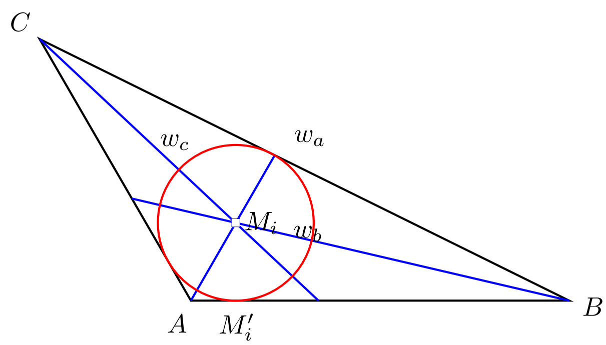
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (67)

Winkel-Seite-Winkel
$b = 0 \quad \alpha = 120° \quad \gamma = 3°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 120° - 3°$
$\beta = 57°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 120}{\sin 57}$
$a = 4,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4,13^2 + 4^2 - 2 \cdot 4,13 \cdot 4 \cdot \cos 3°}$
$c = 0,25$
Umfang: $U = a + b + c$
$U = 4,13 + 4 + 0,25$
$U = 8,38$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 0,25 \cdot \sin 57°$
$h_a = 0,209$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,13 \cdot 0,209$
$A = 0,432$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,13 \cdot \sin 3°$
$h_b = 0,216$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 120°$
$h_c = 3,46$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{0,25 \cdot \sin 57}{\sin 63}$
$wha = 0,235$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4,13 \cdot \sin 3}{\sin 148\frac{1}{2}}$
$whb = 0,414$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 120}{\sin 63}$
$whc = 4,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 0,25^2) - 4,13^2}$
$s_a = 1,94$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$

$s_b = \frac{1}{2}\sqrt{2(4,13^2+0,25^2)-4^2}$

$s_b = 2,14$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$

$s_c = \frac{1}{2}\sqrt{2(4,13^2+4^2)-0,25^2}$

$s_c = 3,54$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2\cdot\sin\alpha}$

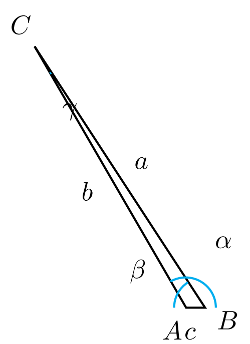
$r_u = \frac{4,13}{2\cdot\sin 120^\circ}$

$r_u = 2,38$

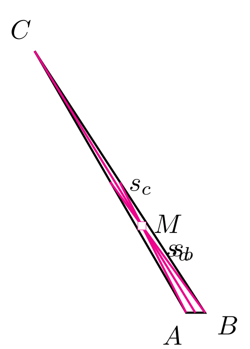
Inkreisradius: $r_i = \frac{2\cdot A}{U}$

$r_i = \frac{2\cdot 0,432}{8,38}$

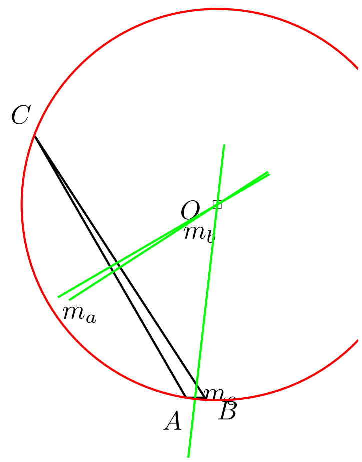
$r_i = 0,103$



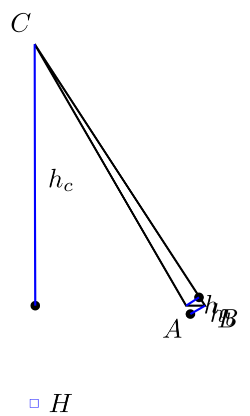
Seitenhalbierende-Schwerpunkt



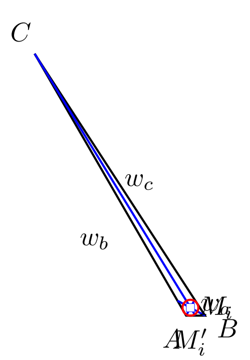
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

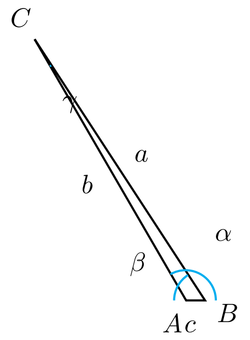


Aufgabe (68)

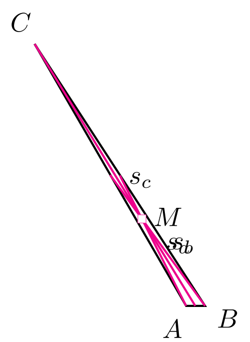
Winkel-Seite-Winkel
$b = 0 \quad \alpha = 120° \quad \gamma = 3°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 120° - 3°$
$\beta = 57°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 120}{\sin 57}$
$a = 4,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4,13^2 + 4^2 - 2 \cdot 4,13 \cdot 4 \cdot \cos 3°}$
$c = 0,25$
Umfang: $U = a + b + c$
$U = 4,13 + 4 + 0,25$
$U = 8,38$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 0,25 \cdot \sin 57°$
$h_a = 0,209$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,13 \cdot 0,209$
$A = 0,432$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,13 \cdot \sin 3°$
$h_b = 0,216$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 120°$
$h_c = 3,46$

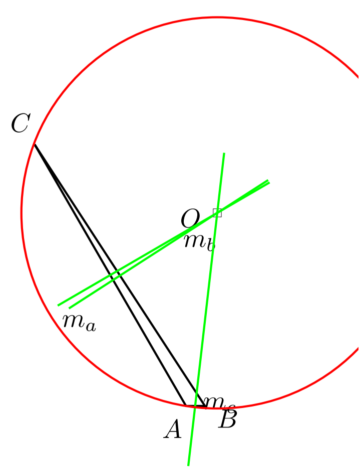
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{0,25 \cdot \sin 57}{\sin 63}$
$wha = 0,235$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4,13 \cdot \sin 3}{\sin 148\frac{1}{2}}$
$whb = 0,414$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 120}{\sin 63}$
$whc = 4,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 0,25^2) - 4,13^2}$
$s_a = 1,94$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4,13^2 + 0,25^2) - 4^2}$
$s_b = 2,14$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4,13^2 + 4^2) - 0,25^2}$
$s_c = 3,54$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4,13}{2 \cdot \sin 120°}$
$r_u = 2,38$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 0,432}{8,38}$
$r_i = 0,103$



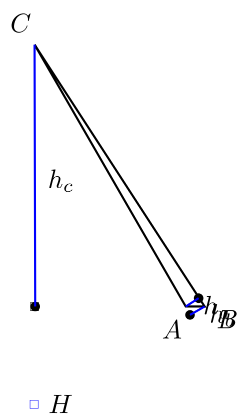
Seitenhalbierende-Schwerpunkt



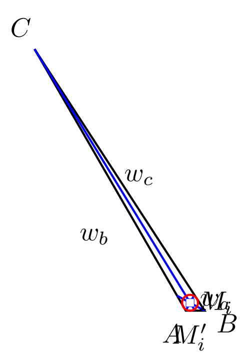
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



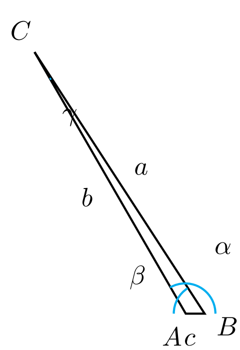
Aufgabe (69)

Winkel-Seite-Winkel
$b = 0 \quad \alpha = 120° \quad \gamma = 3°$

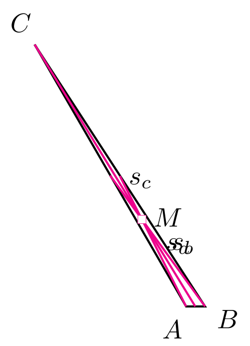
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 120° - 3°$
$\beta = 57°$
Sinus-Satz: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$
$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \dfrac{b \cdot \sin\alpha}{\sin\beta}$

$a = \frac{4 \cdot \sin 120}{\sin 57}$
$a = 4,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4,13^2 + 4^2 - 2 \cdot 4,13 \cdot 4 \cdot \cos 3^\circ}$
$c = 0,25$
Umfang: $U = a + b + c$
$U = 4,13 + 4 + 0,25$
$U = 8,38$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 0,25 \cdot \sin 57^\circ$
$h_a = 0,209$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,13 \cdot 0,209$
$A = 0,432$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,13 \cdot \sin 3^\circ$
$h_b = 0,216$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 120^\circ$
$h_c = 3,46$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{0,25 \cdot \sin 57}{\sin 63}$
$wha = 0,235$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

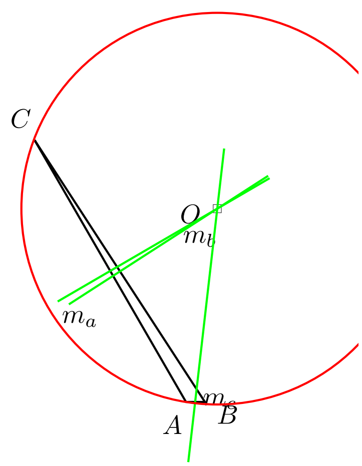
$whb = \frac{4,13 \cdot \sin 3}{\sin 148\frac{1}{2}}$
$whb = 0,414$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 120}{\sin 63}$
$whc = 4,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 0,25^2) - 4,13^2}$
$s_a = 1,94$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4,13^2 + 0,25^2) - 4^2}$
$s_b = 2,14$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4,13^2 + 4^2) - 0,25^2}$
$s_c = 3,54$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4,13}{2 \cdot \sin 120°}$
$r_u = 2,38$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 0,432}{8,38}$
$r_i = 0,103$



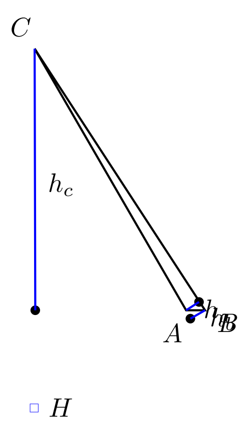
Seitenhalbierende-Schwerpunkt



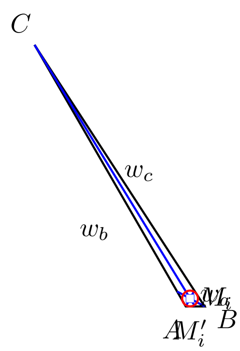
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



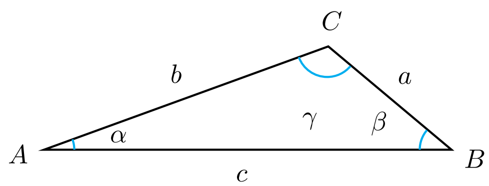
Aufgabe (70)

inkel-Winkel-Seite
$b = 4 \quad \alpha = 20° \quad \beta = 40°$

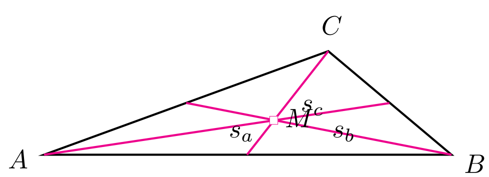
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 20° - 40°$
$\gamma = 120°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 20}{\sin 40}$
$a = 2,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{2,13^2 + 4^2 - 2 \cdot 2,13 \cdot 4 \cdot \cos 120°}$
$c = 5,39$
Umfang: $U = a + b + c$
$U = 2,13 + 4 + 5,39$
$U = 11,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,39 \cdot \sin 40°$
$h_a = 3,46$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,13 \cdot 3,46$
$A = 3,69$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,13 \cdot \sin 120°$
$h_b = 1,84$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 20°$
$h_c = 1,37$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,39 \cdot \sin 40}{\sin 130}$
$wha = 4,52$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,13 \cdot \sin 120}{\sin 40}$
$whb = 2,87$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 20}{\sin 130}$
$whc = 0,95$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5,39^2) - 2,13^2}$
$s_a = 4,62$

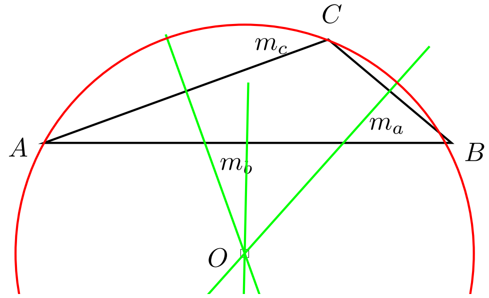
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,13^2+5,39^2)-4^2}$
$s_b = 3,58$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,13^2+4^2)-5,39^2}$
$s_c = 2,5$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,13}{2 \cdot \sin 20°}$
$r_u = 3,11$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,69}{11,5}$
$r_i = 0,64$



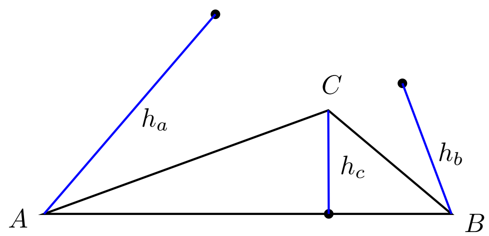
Seitenhalbierende-Schwerpunkt



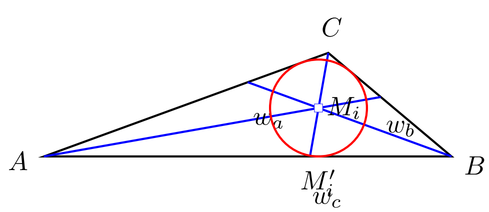
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (71)

inkel-Winkel-Seite
$b = 4 \quad \alpha = 20° \quad \beta = 40°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 20° - 40°$
$\gamma = 120°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 20}{\sin 40}$
$a = 2,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{2,13^2 + 4^2 - 2 \cdot 2,13 \cdot 4 \cdot \cos 120^\circ}$
$c = 5,39$
Umfang: $U = a + b + c$
$U = 2,13 + 4 + 5,39$
$U = 11,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,39 \cdot \sin 40^\circ$
$h_a = 3,46$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,13 \cdot 3,46$
$A = 3,69$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,13 \cdot \sin 120^\circ$
$h_b = 1,84$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 20^\circ$
$h_c = 1,37$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,39 \cdot \sin 40}{\sin 130}$
$wha = 4,52$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{2,13 \cdot \sin 120}{\sin 40}$

$whb = 2,87$

Winkelhalbierende: γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{4 \cdot \sin 20}{\sin 130}$

$whc = 0,95$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(4^2 + 5,39^2) - 2,13^2}$

$s_a = 4,62$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(2,13^2 + 5,39^2) - 4^2}$

$s_b = 3,58$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(2,13^2 + 4^2) - 5,39^2}$

$s_c = 2,5$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

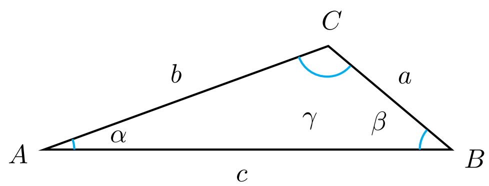
$r_u = \frac{2,13}{2 \cdot \sin 20°}$

$r_u = 3,11$

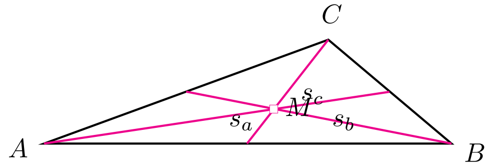
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 3,69}{11,5}$

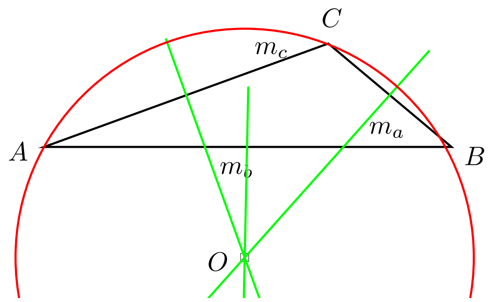
$r_i = 0,64$



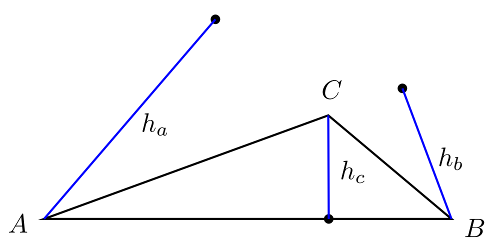
Seitenhalbierende-Schwerpunkt



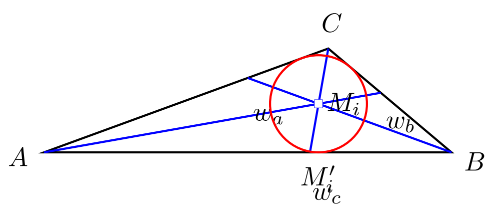
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

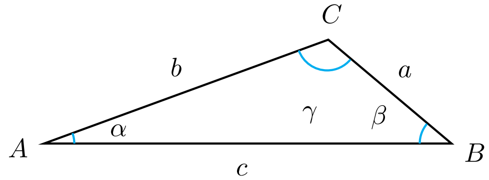


Aufgabe (72)

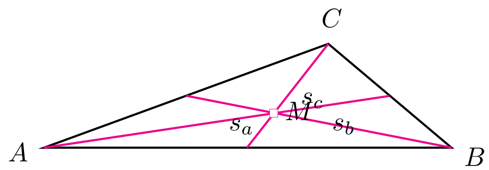
inkel-Winkel-Seite
$b = 4 \quad \alpha = 20° \quad \beta = 40°$

Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 20° - 40°$
$\gamma = 120°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 20}{\sin 40}$
$a = 2,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{2,13^2 + 4^2 - 2 \cdot 2,13 \cdot 4 \cdot \cos 120°}$
$c = 5,39$
Umfang: $U = a + b + c$
$U = 2,13 + 4 + 5,39$
$U = 11,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,39 \cdot \sin 40°$
$h_a = 3,46$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,13 \cdot 3,46$
$A = 3,69$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,13 \cdot \sin 120°$
$h_b = 1,84$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 20°$
$h_c = 1,37$
Winkelhalbierende: α

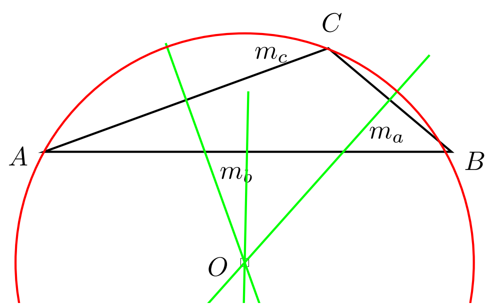
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,39 \cdot \sin 40}{\sin 130}$
$wha = 4,52$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,13 \cdot \sin 120}{\sin 40}$
$whb = 2,87$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 20}{\sin 130}$
$whc = 0,95$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5,39^2) - 2,13^2}$
$s_a = 4,62$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,13^2 + 5,39^2) - 4^2}$
$s_b = 3,58$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,13^2 + 4^2) - 5,39^2}$
$s_c = 2,5$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,13}{2 \cdot \sin 20°}$
$r_u = 3,11$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,69}{11,5}$
$r_i = 0,64$



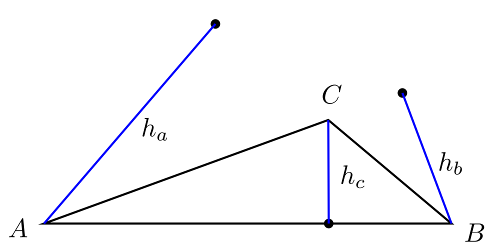
Seitenhalbierende-Schwerpunkt



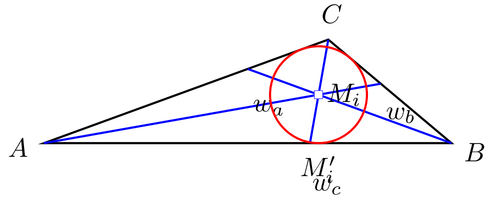
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



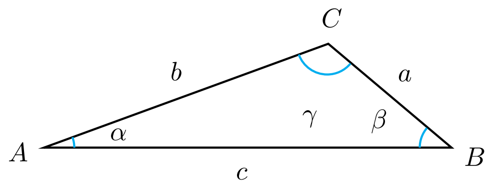
Aufgabe (73)

inkel-Winkel-Seite
$b = 4 \quad \alpha = 20° \quad \beta = 40°$

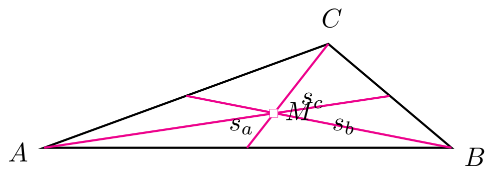
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 20° - 40°$
$\gamma = 120°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{4 \cdot \sin 20}{\sin 40}$
$a = 2,13$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{2,13^2 + 4^2 - 2 \cdot 2,13 \cdot 4 \cdot \cos 120°}$
$c = 5,39$
Umfang: $U = a + b + c$
$U = 2,13 + 4 + 5,39$
$U = 11,5$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,39 \cdot \sin 40°$
$h_a = 3,46$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,13 \cdot 3,46$
$A = 3,69$
Höhe: h_b

$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,13 \cdot \sin 120°$
$h_b = 1,84$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 20°$
$h_c = 1,37$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,39 \cdot \sin 40}{\sin 130}$
$wha = 4,52$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,13 \cdot \sin 120}{\sin 40}$
$whb = 2,87$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 20}{\sin 130}$
$whc = 0,95$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5,39^2) - 2,13^2}$
$s_a = 4,62$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,13^2 + 5,39^2) - 4^2}$
$s_b = 3,58$

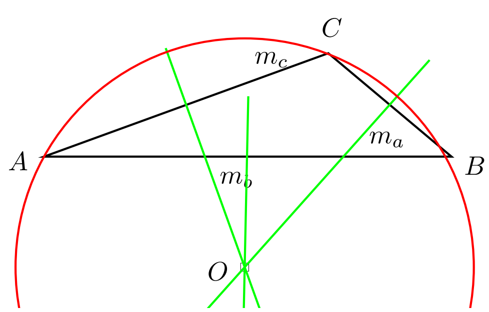
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,13^2+4^2)-5,39^2}$
$s_c = 2,5$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2\cdot\sin\alpha}$
$r_u = \frac{2,13}{2\cdot\sin 20^\circ}$
$r_u = 3,11$
Inkreisradius: $r_i = \frac{2\cdot A}{U}$
$r_i = \frac{2\cdot 3,69}{11,5}$
$r_i = 0,64$



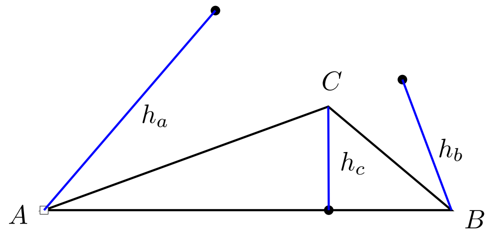
Seitenhalbierende-Schwerpunkt



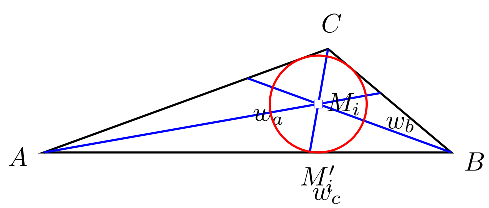
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



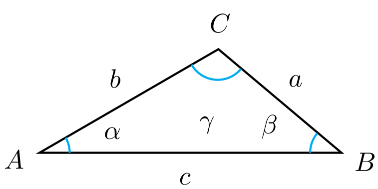
Aufgabe (74)

Winkel-Seite-Winkel
$c = 4 \quad \alpha = 30° \quad \beta = 40°$

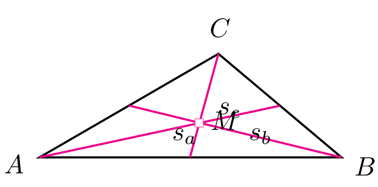
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 40°$
$\gamma = 110°$
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$
$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\alpha$
$a = \frac{c \cdot \sin\alpha}{\sin\gamma}$
$a = \frac{4 \cdot \sin 30}{\sin 110}$
$a = 2,13$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{2,13^2 + 4^2 - 2 \cdot 2,13 \cdot 4 \cdot \cos 40°}$
$b = 2,74$
Umfang: $U = a + b + c$
$U = 2,13 + 2,74 + 4$

$U = 8,86$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 40^\circ$
$h_a = 2,57$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,13 \cdot 2,57$
$A = 2,74$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,13 \cdot \sin 110^\circ$
$h_b = 2$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 2,74 \cdot \sin 30^\circ$
$h_c = 1,37$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4 \cdot \sin 40}{\sin 125}$
$wha = 3,14$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,13 \cdot \sin 110}{\sin 50}$
$whb = 2,61$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

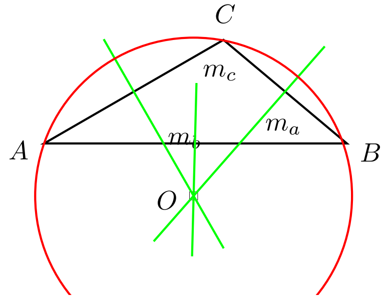
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{2,74 \cdot \sin 30}{\sin 125}$
$whc = 1,3$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(2,74^2 + 4^2) - 2,13^2}$
$s_a = 3,26$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,13^2 + 4^2) - 2,74^2}$
$s_b = 2,9$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,13^2 + 2,74^2) - 4^2}$
$s_c = 2,03$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,13}{2 \cdot \sin 30°}$
$r_u = 2,13$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,74}{8,86}$
$r_i = 0,617$



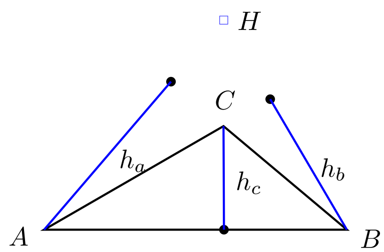
Seitenhalbierende-Schwerpunkt



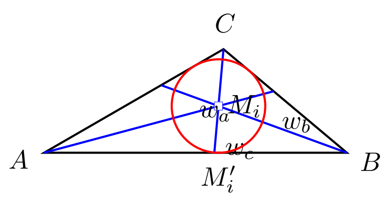
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



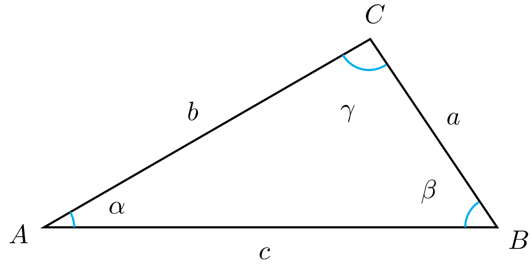
Aufgabe (75)

Seite-Winkel-Seite
$a = 3 \quad c = 6 \quad \beta = 56°$

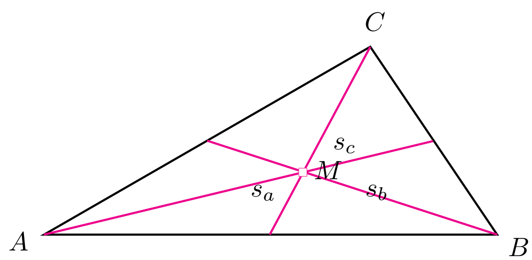
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta}$
$b = \sqrt{3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 56°}$
$b = 4,99$
Umfang: $U = a + b + c$

$U = 3 + 4,99 + 6$
$U = 14$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{4,99^2 + 6^2 - 3^2}{2 \cdot 4,99 \cdot 6}$
$\cos\alpha = 0,867$
$\alpha = \arccos(0,867)$
$\alpha = 29,9°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29,9° - 56°$
$\gamma = 94,1°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 56°$
$h_a = 4,97$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4,97$
$A = 7,46$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 94,1°$
$h_b = 2,99$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,99 \cdot \sin 29,9°$
$h_c = 2,49$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 56}{\sin 109}$
$wha = 5,26$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$

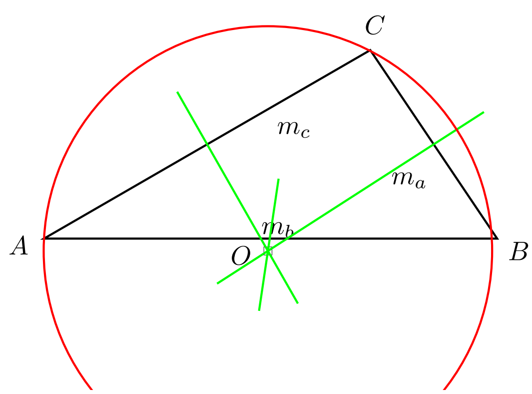
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 94,1}{\sin 57,9}$
$whb = 3,53$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,99 \cdot \sin 29,9}{\sin 109}$
$whc = 1,58$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,99^2 + 6^2) - 3^2}$
$s_a = 5,31$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 6^2) - 4,99^2}$
$s_b = 4,04$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4,99^2) - 6^2}$
$s_c = 3,27$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 29,9^\circ}$
$r_u = 3,01$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,46}{14}$
$r_i = 1,07$



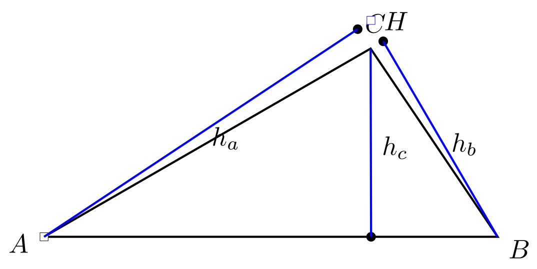
Seitenhalbierende-Schwerpunkt



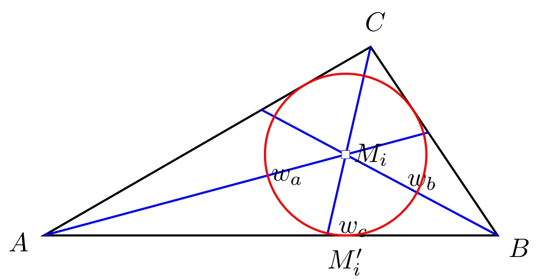
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (76)

Seite-Winkel-Seite
$a = 3 \quad c = 6 \quad \beta = 56°$

Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 56°}$
$b = 4,99$
Umfang: $U = a + b + c$
$U = 3 + 4,99 + 6$
$U = 14$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{4,99^2 + 6^2 - 3^2}{2 \cdot 4,99 \cdot 6}$
$\cos\alpha = 0,867$

$\alpha = \arccos(0,867)$
$\alpha = 29,9°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29,9° - 56°$
$\gamma = 94,1°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 56°$
$h_a = 4,97$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4,97$
$A = 7,46$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 94,1°$
$h_b = 2,99$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,99 \cdot \sin 29,9°$
$h_c = 2,49$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 56}{\sin 109}$
$wha = 5,26$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 94,1}{\sin 57,9}$
$whb = 3,53$
Winkelhalbierende: γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{4,99 \cdot \sin 29,9}{\sin 109}$

$whc = 1,58$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(4,99^2 + 6^2) - 3^2}$

$s_a = 5,31$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(3^2 + 6^2) - 4,99^2}$

$s_b = 4,04$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(3^2 + 4,99^2) - 6^2}$

$s_c = 3,27$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

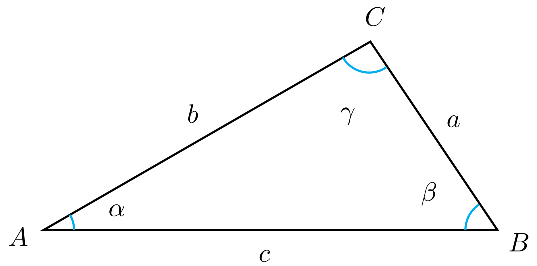
$r_u = \frac{3}{2 \cdot \sin 29,9^\circ}$

$r_u = 3,01$

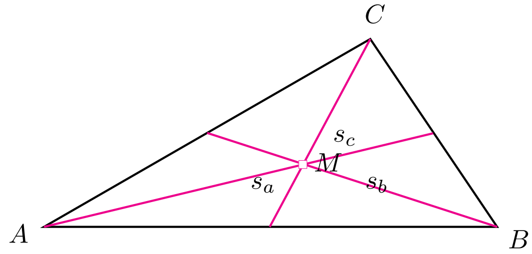
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 7,46}{14}$

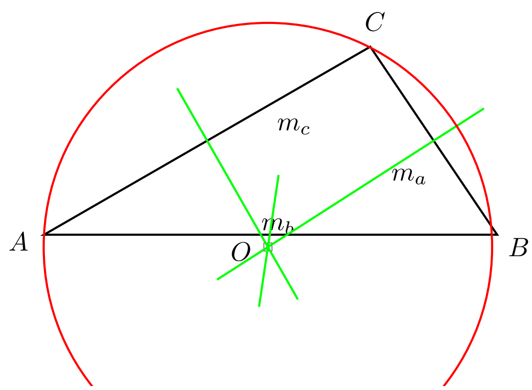
$r_i = 1,07$



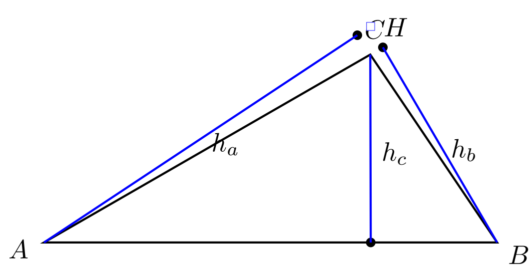
Seitenhalbierende-Schwerpunkt



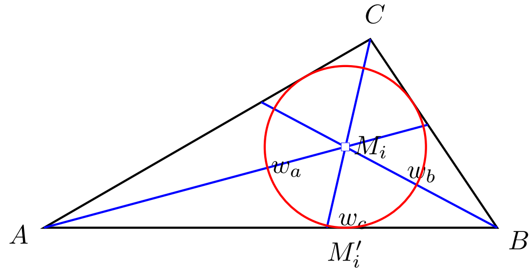
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



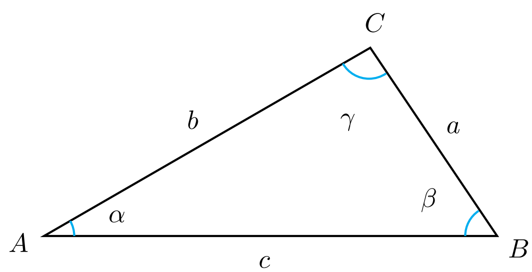
Aufgabe (77)

Seite-Winkel-Seite
$a = 3 \quad c = 6 \quad \beta = 56°$

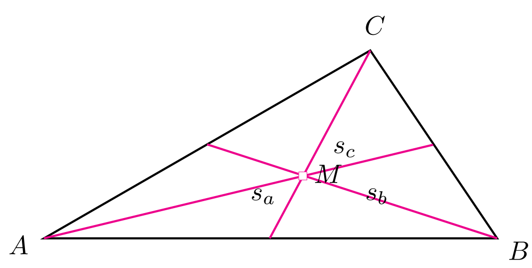
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 56°}$
$b = 4,99$
Umfang: $U = a + b + c$
$U = 3 + 4,99 + 6$
$U = 14$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{4,99^2 + 6^2 - 3^2}{2 \cdot 4,99 \cdot 6}$
$\cos\alpha = 0,867$
$\alpha = \arccos(0,867)$
$\alpha = 29,9°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 29,9° - 56°$
$\gamma = 94,1°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 56°$
$h_a = 4,97$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$

$A = \frac{1}{2} \cdot 3 \cdot 4,97$
$A = 7,46$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 94,1°$
$h_b = 2,99$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,99 \cdot \sin 29,9°$
$h_c = 2,49$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 56}{\sin 109}$
$wha = 5,26$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 94,1}{\sin 57,9}$
$whb = 3,53$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,99 \cdot \sin 29,9}{\sin 109}$
$whc = 1,58$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,99^2 + 6^2) - 3^2}$
$s_a = 5,31$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

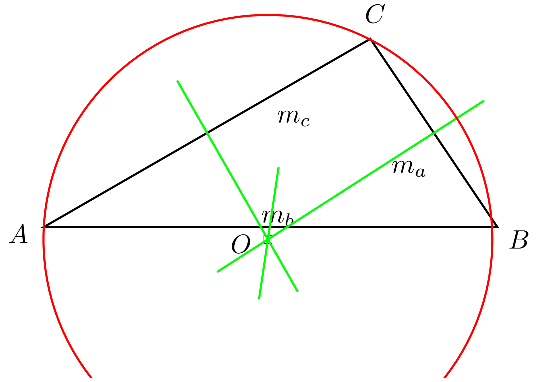
$s_b = \frac{1}{2}\sqrt{2(3^2 + 6^2) - 4,99^2}$
$s_b = 4,04$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4,99^2) - 6^2}$
$s_c = 3,27$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 29,9°}$
$r_u = 3,01$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,46}{14}$
$r_i = 1,07$



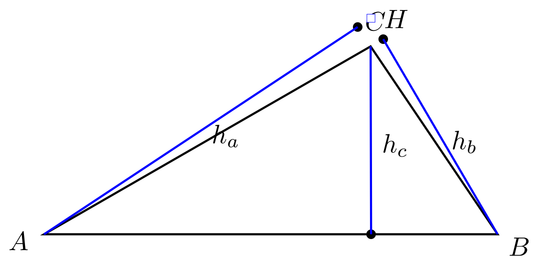
Seitenhalbierende-Schwerpunkt



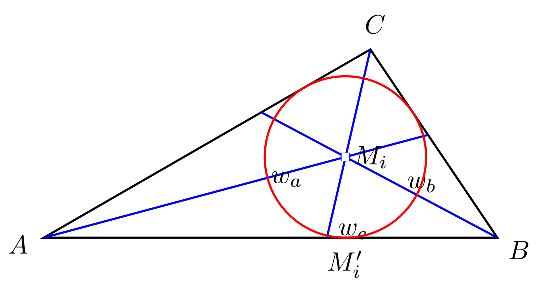
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



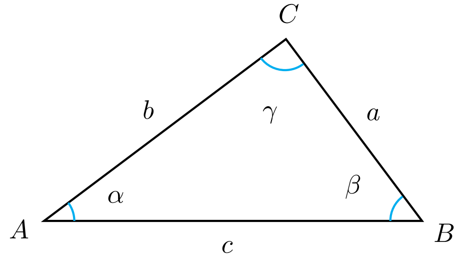
Aufgabe (78)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$

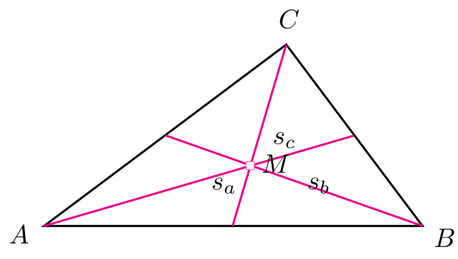
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180$ $/ - \alpha$ $/ - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c}$ $/ \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a}$ $/ \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b}$ $/ \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$ $/ \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β

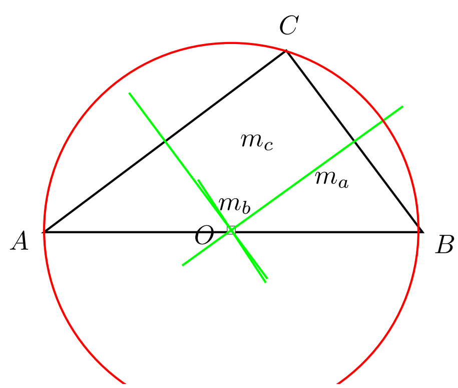
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



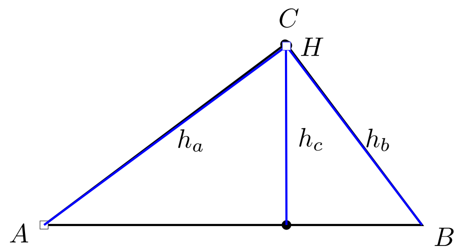
Seitenhalbierende-Schwerpunkt



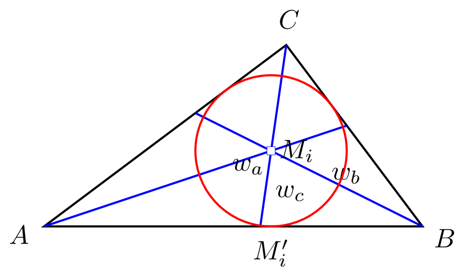
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



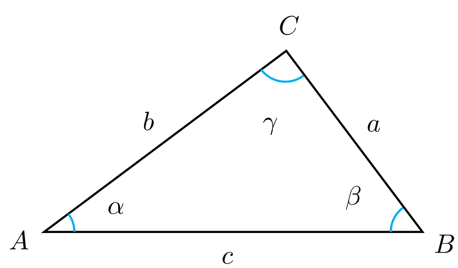
Aufgabe (79)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

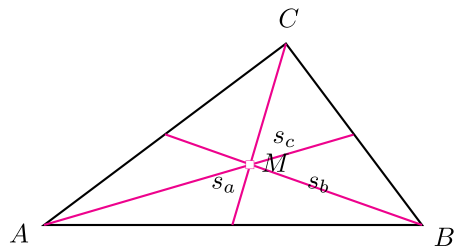
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$

$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

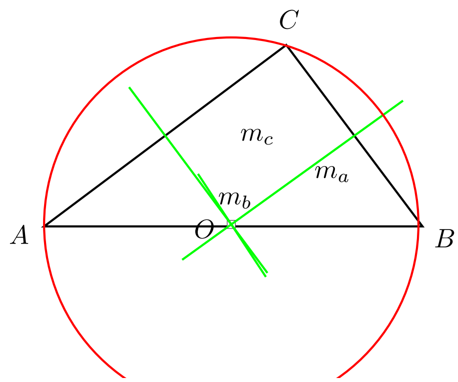
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



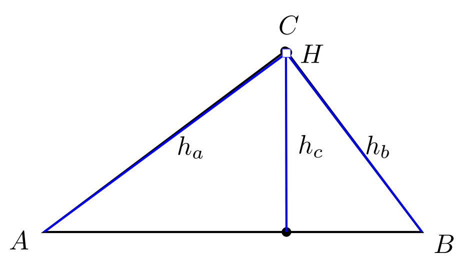
Seitenhalbierende-Schwerpunkt



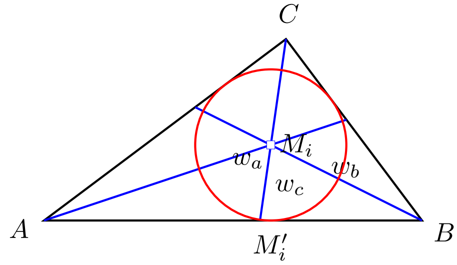
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



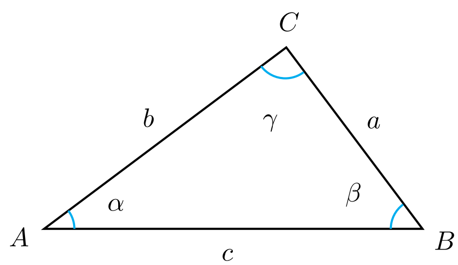
Aufgabe (80)

Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5$ $\gamma = 90°$

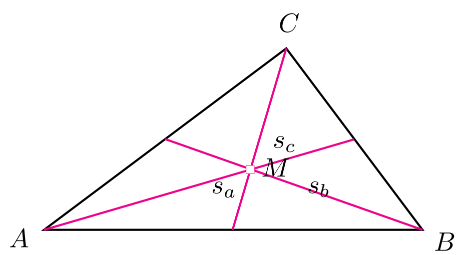
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$

$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90^\circ$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9^\circ$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

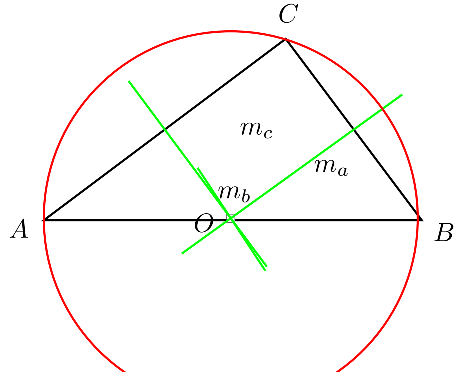
$s_c = \frac{1}{2}\sqrt{2(3^2+4^2)-5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9°}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



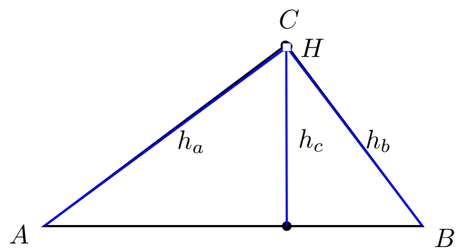
Seitenhalbierende-Schwerpunkt



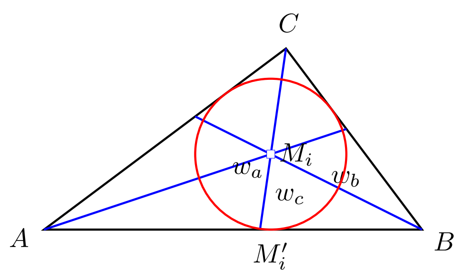
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

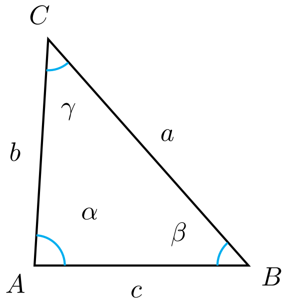


Aufgabe (81)

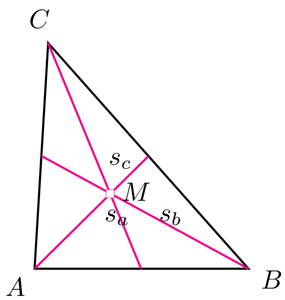
Seite-Winkel-Seite
$a = 4 \quad b = 3 \quad \gamma = 45°$

Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos 45^\circ}$
$c = 2,83$
Umfang: $U = a + b + c$
$U = 4 + 3 + 2,83$
$U = 9,83$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3^2 + 2,83^2 - 4^2}{2 \cdot 3 \cdot 2,83}$
$\cos\alpha = 0,0605$
$\alpha = \arccos(0,0605)$
$\alpha = 86,5^\circ$
Winkelsumme: $\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180^\circ - \alpha - \gamma$
$\beta = 180^\circ - 86,5^\circ - 45^\circ$
$\beta = 48,5^\circ$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,83 \cdot \sin 48,5^\circ$
$h_a = 2,12$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 2,12$
$A = 4,24$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 45^\circ$
$h_b = 2,83$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 86,5^\circ$
$h_c = 2,99$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

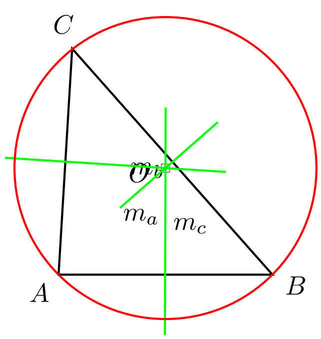
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,83 \cdot \sin 48,5}{\sin 88,3}$
$wha = 2,12$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 45}{\sin 111}$
$whb = 3,02$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 86,5}{\sin 88,3}$
$whc = 3,99$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 2,83^2) - 4^2}$
$s_a = 2,12$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 2,83^2) - 3^2}$
$s_b = 3,12$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 2,83^2}$
$s_c = 3,2$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 86,5^\circ}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 4,24}{9,83}$
$r_i = 0,863$



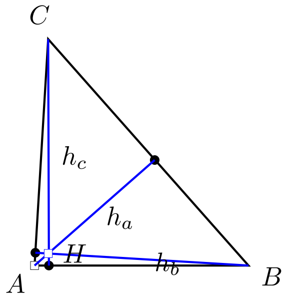
Seitenhalbierende-Schwerpunkt



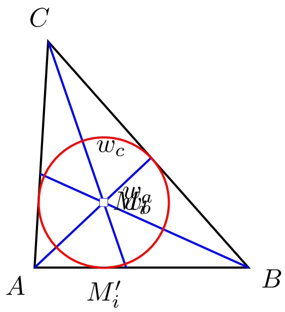
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



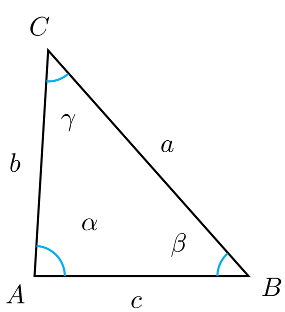
Aufgabe (82)

Seite-Winkel-Seite
$a = 4 \quad b = 3 \quad \gamma = 45°$

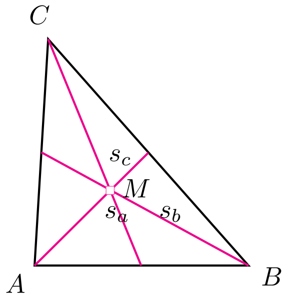
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos 45°}$
$c = 2,83$
Umfang: $U = a + b + c$
$U = 4 + 3 + 2,83$
$U = 9,83$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$

$\cos\alpha = \dfrac{3^2 + 2,83^2 - 4^2}{2 \cdot 3 \cdot 2,83}$
$\cos\alpha = 0,0605$
$\alpha = \arccos(0,0605)$
$\alpha = 86,5°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 86,5° - 45°$
$\beta = 48,5°$
Höhe: h_a
$\sin\beta = \dfrac{h_a}{c}$
$\sin\beta = \dfrac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,83 \cdot \sin 48,5°$
$h_a = 2,12$
Flaeche: $\quad A = \dfrac{1}{2} \cdot a \cdot h_a$
$A = \dfrac{1}{2} \cdot 4 \cdot 2,12$
$A = 4,24$
Höhe: h_b
$\sin\gamma = \dfrac{h_b}{a}$
$\sin\gamma = \dfrac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 45°$
$h_b = 2,83$
Höhe: h_c
$\sin\alpha = \dfrac{h_c}{b}$
$\sin\alpha = \dfrac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 86,5°$
$h_c = 2,99$
Winkelhalbierende: α
$\delta = 180 - \beta - \dfrac{\alpha}{2}$
Sinus-Satz: $\dfrac{wha}{\sin\beta} = \dfrac{c}{\sin\delta}$
$\dfrac{wha}{\sin\beta} = \dfrac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \dfrac{c \cdot \sin\beta}{\sin\delta}$
$wha = \dfrac{2,83 \cdot \sin 48,5}{\sin 88,3}$
$wha = 2,12$
Winkelhalbierende: β
$\delta = 180 - \dfrac{\beta}{2} - \gamma$
Sinus-Satz: $\dfrac{whb}{\sin\gamma} = \dfrac{a}{\sin\delta}$
$\dfrac{whb}{\sin\gamma} = \dfrac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \dfrac{a \cdot \sin\gamma}{\sin\delta}$

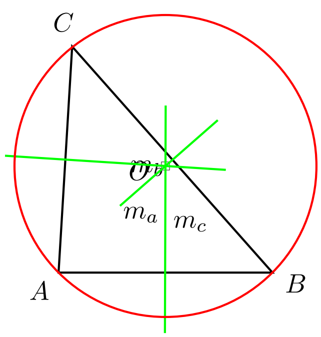
$whb = \frac{4 \cdot \sin 45}{\sin 111}$
$whb = 3,02$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 86,5}{\sin 88,3}$
$whc = 3,99$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 2,83^2) - 4^2}$
$s_a = 2,12$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 2,83^2) - 3^2}$
$s_b = 3,12$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 2,83^2}$
$s_c = 3,2$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 86,5°}$
$r_u = 2$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 4,24}{9,83}$
$r_i = 0,863$



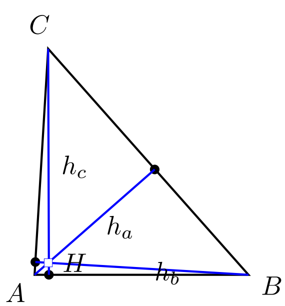
Seitenhalbierende-Schwerpunkt



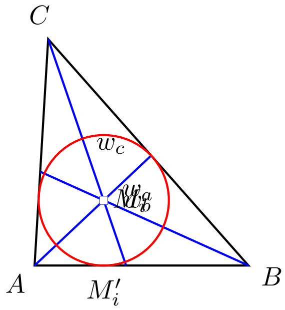
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (83)

Seite-Seite-Winkel
Gegebener Winkel muß der groesseren Seite gegenüber liegen.
$b > a + c$
Zeichnung nicht möglich

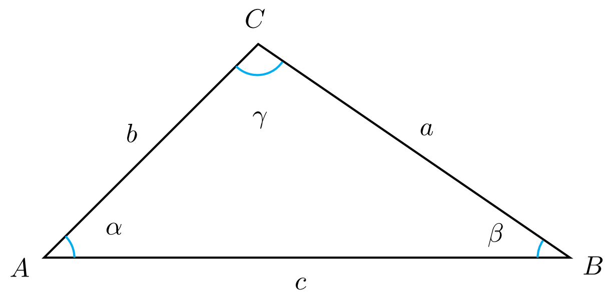
Aufgabe (84)

Seite-Seite-Winkel
$a = 5 \quad b = 4 \quad \alpha = 45°$

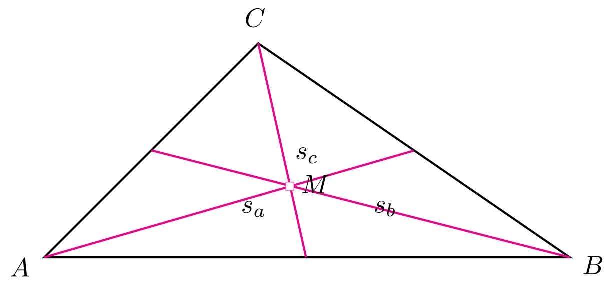
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{4 \cdot \sin 45°}{5}$
$\sin\beta = 0,566$
$\beta = \arcsin(0,566)$
$\beta = 34,4°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 45° - 34,4°$
$\gamma = 101°$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos 101°}$
$c = 6,95$

Umfang: $U = a + b + c$
$U = 5 + 4 + 6,95$
$U = 16$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6,95 \cdot \sin 34,4°$
$h_a = 3,93$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 3,93$
$A = 9,83$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 101°$
$h_b = 4,92$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 45°$
$h_c = 2,83$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6,95 \cdot \sin 34,4}{\sin 123}$
$wha = 4,69$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 101}{\sin 62,2}$
$whb = 5,56$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

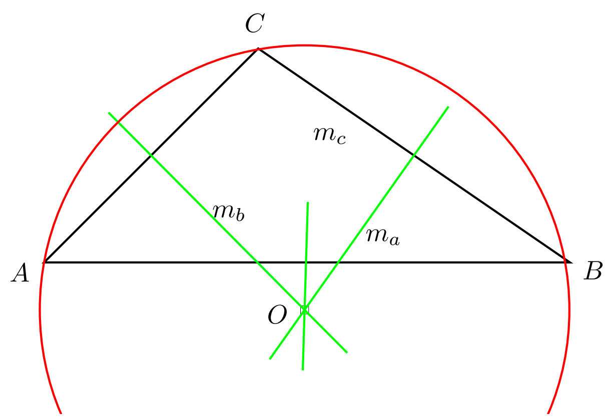
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 45}{\sin 123}$
$whc = 4,22$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2+6,95^2)-5^2}$
$s_a = 5,09$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2+6,95^2)-4^2}$
$s_b = 5,72$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2+4^2)-6,95^2}$
$s_c = 4,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 45°}$
$r_u = 3,54$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,83}{16}$
$r_i = 1,23$



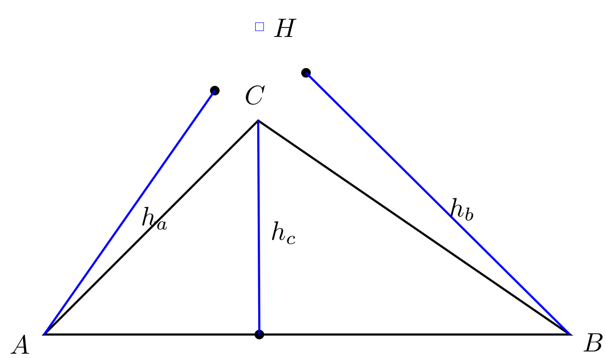
Seitenhalbierende-Schwerpunkt



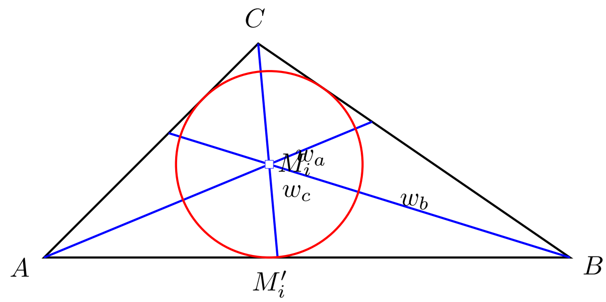
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



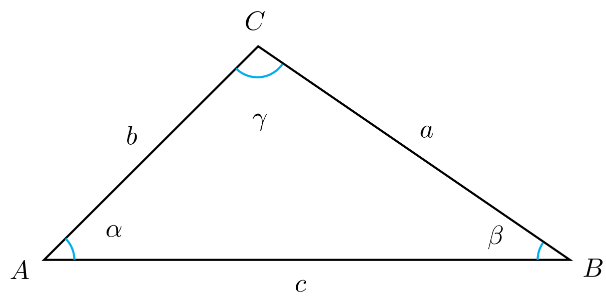
Aufgabe (85)

Seite-Seite-Winkel
$a = 5 \quad b = 4 \quad \alpha = 45°$

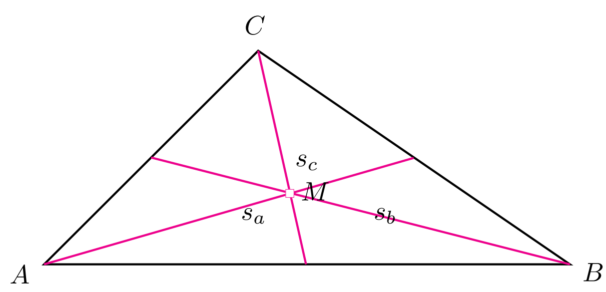
Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{4 \cdot \sin 45°}{5}$
$\sin\beta = 0,566$
$\beta = \arcsin(0,566)$
$\beta = 34,4°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / -\alpha \qquad / -\beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 45° - 34,4°$
$\gamma = 101°$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cdot \cos 101°}$
$c = 6,95$
Umfang: $U = a + b + c$
$U = 5 + 4 + 6,95$
$U = 16$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \qquad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6,95 \cdot \sin 34,4°$
$h_a = 3,93$

Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 3,93$
$A = 9,83$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 101°$
$h_b = 4,92$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 45°$
$h_c = 2,83$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6,95 \cdot \sin 34,4}{\sin 123}$
$wha = 4,69$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 101}{\sin 62,2}$
$whb = 5,56$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 45}{\sin 123}$
$whc = 4,22$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 6,95^2) - 5^2}$
$s_a = 5,09$

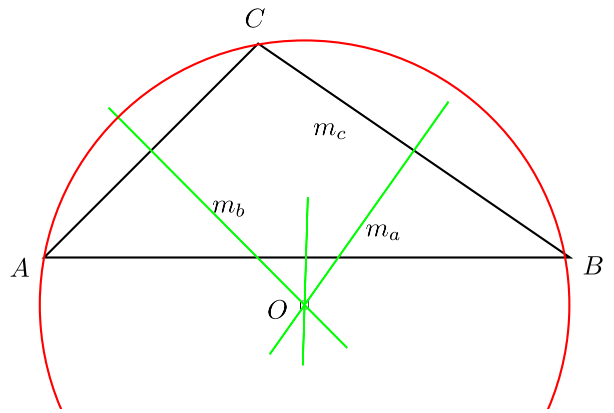
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2+c^2)-b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2+6,95^2)-4^2}$
$s_b = 5,72$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2+4^2)-6,95^2}$
$s_c = 4,06$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 45°}$
$r_u = 3,54$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,83}{16}$
$r_i = 1,23$



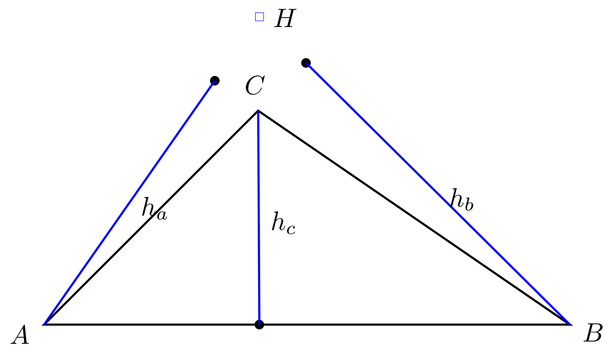
Seitenhalbierende-Schwerpunkt



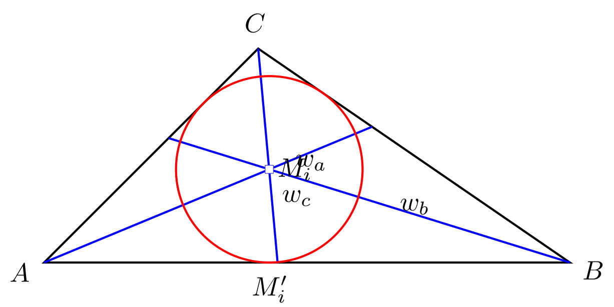
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



Aufgabe (86)

Seite-Seite-Seite
$a = 4 \quad b = 5 \quad c = 6$

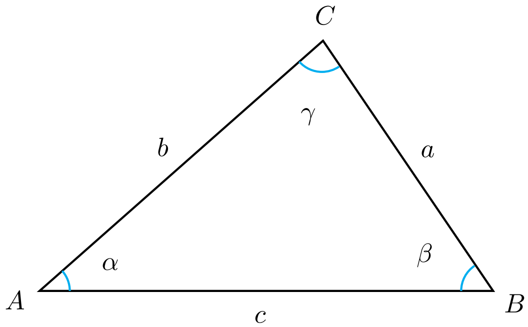
Umfang: $U = a + b + c$
$U = 4 + 5 + 6$
$U = 15$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{5^2 + 6^2 - 4^2}{2 \cdot 5 \cdot 6}$
$\cos\alpha = \frac{3}{4}$
$\alpha = \arccos(\frac{3}{4})$
$\alpha = 41,4°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{4^2 + 6^2 - 5^2}{2 \cdot 4 \cdot 6}$
$\cos\beta = \frac{9}{16}$
$\beta = \arccos(\frac{9}{16})$
$\beta = 55,8°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 41,4° - 55,8°$
$\gamma = 82,8°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 55,8°$
$h_a = 4\frac{49}{51}$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 4\frac{49}{51}$
$A = 9,92$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 82,8°$
$h_b = 3,97$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$

$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 41,4°$
$h_c = 3,31$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 55,8}{\sin 104}$
$wha = 5,1$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 82,8}{\sin 69,3}$
$whb = 4,24$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 41,4}{\sin 104}$
$whc = 2,72$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 6^2) - 4^2}$
$s_a = 5,15$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 6^2) - 5^2}$
$s_b = 4,44$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 6^2}$
$s_c = 3,77$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 41,4°}$
$r_u = 3,02$

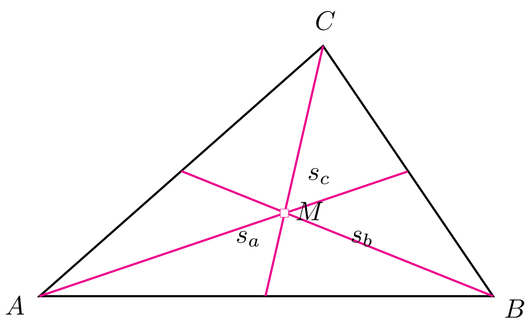
Inkreisradius: $r_i = \dfrac{2 \cdot A}{U}$

$r_i = \dfrac{2 \cdot 9,92}{15}$

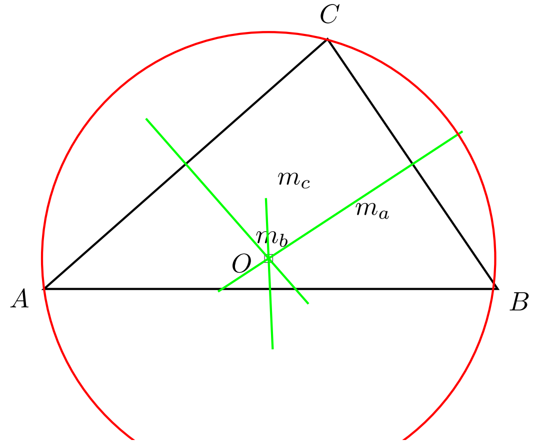
$r_i = 1,32$



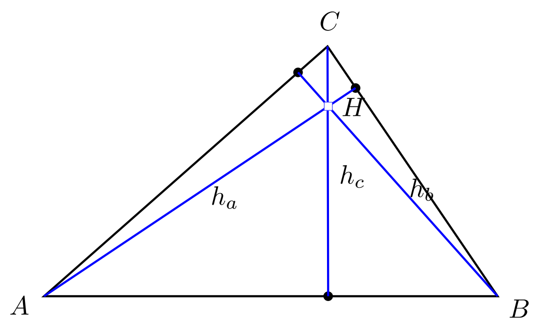
Seitenhalbierende-Schwerpunkt



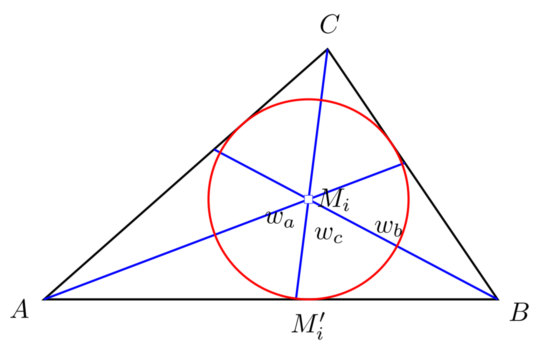
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis



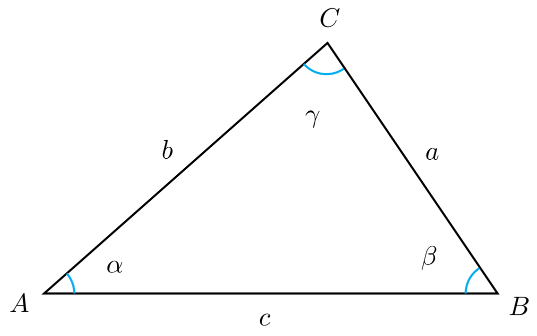
Aufgabe (87)

Seite-Seite-Seite
$a = 4 \quad b = 5 \quad c = 6$

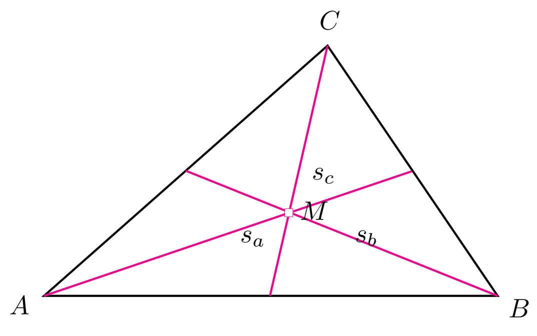
Umfang: $U = a + b + c$
$U = 4 + 5 + 6$
$U = 15$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{5^2 + 6^2 - 4^2}{2 \cdot 5 \cdot 6}$
$\cos\alpha = \frac{3}{4}$
$\alpha = \arccos(\frac{3}{4})$
$\alpha = 41,4°$
Kosinus-Satz: $b^2 = a^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta \qquad / - b^2 \qquad / + 2 \cdot a \cdot c \cdot \cos\beta$
$2 \cdot a \cdot c \cdot \cos\beta = a^2 + c^2 - b^2 \qquad / : (2 \cdot a \cdot c)$
$\cos\beta = \frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$
$\cos\beta = \frac{4^2 + 6^2 - 5^2}{2 \cdot 4 \cdot 6}$
$\cos\beta = \frac{9}{16}$
$\beta = \arccos(\frac{9}{16})$
$\beta = 55,8°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 41,4° - 55,8°$
$\gamma = 82,8°$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 55,8°$
$h_a = 4\frac{49}{51}$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 4\frac{49}{51}$
$A = 9,92$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 82,8°$

$h_b = 3,97$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 41,4°$
$h_c = 3,31$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 55,8}{\sin 104}$
$wha = 5,1$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 82,8}{\sin 69,3}$
$whb = 4,24$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 41,4}{\sin 104}$
$whc = 2,72$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 6^2) - 4^2}$
$s_a = 5,15$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 6^2) - 5^2}$
$s_b = 4,44$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 6^2}$
$s_c = 3,77$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

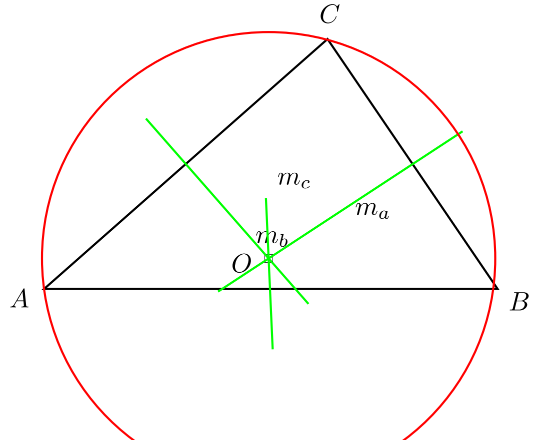
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 41,4°}$
$r_u = 3,02$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,92}{15}$
$r_i = 1,32$



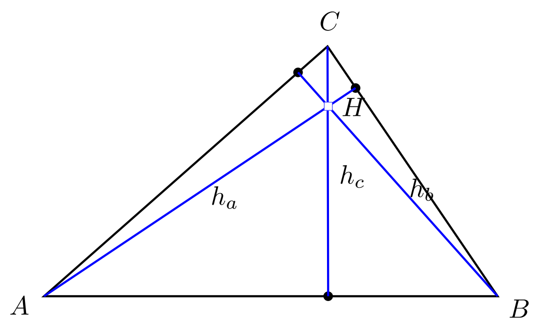
Seitenhalbierende-Schwerpunkt



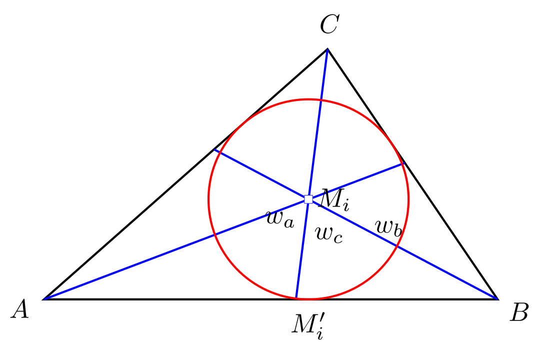
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis

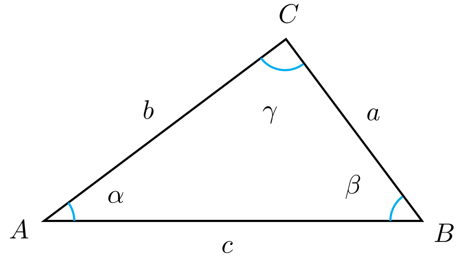


Aufgabe (88)

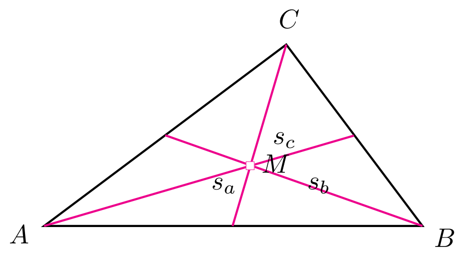
Seite-Seite-Seite
$a = 3 \quad b = 4 \quad c = 5$
Pythagoras: $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkliges Dreieck
Kathete: $a = 3$ $b = 4$ Hypothenuse: $c = 5 \quad \gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,9$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,9° - 90°$
$\beta = 53,1°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe: h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,1°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe: h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe: h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,9°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende: α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

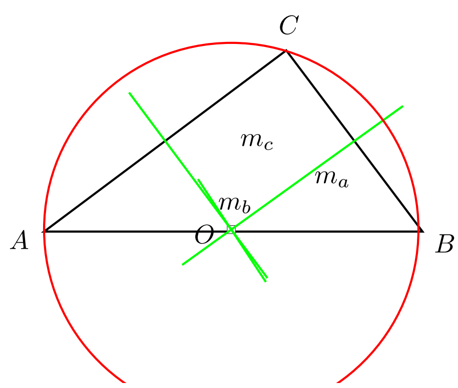
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,1}{\sin 108}$
$wha = 4,22$
Winkelhalbierende: β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,4}$
$whb = 3,35$
Winkelhalbierende: γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,9}{\sin 108}$
$whc = 1,9$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,27$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,61$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,92$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,9^\circ}$
$r_u = 2\frac{1}{2}$
Inkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



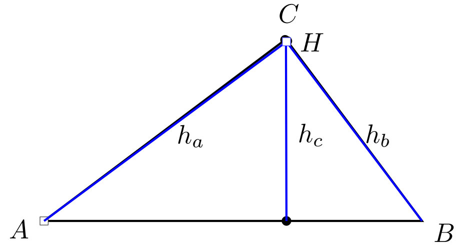
Seitenhalbierende-Schwerpunkt



Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Inkreis