

Gleichungen
Aufgaben und Lösungen
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1 Lineare Gleichung

Äquivalenzumformung

Lösen der linearen Gleichung durch Äquivalenzumformung. Auf beiden Seiten denselben Term addieren, subtrahieren, multiplizieren oder dividieren.

$$a \cdot x = b$$

$$a \cdot x = b \quad / : a$$

$$x = \frac{b}{a}$$

$$5 \cdot x = 45 \quad / : 5 \quad -2 \cdot x = -6 \quad / : (-2)$$

$$x = \frac{45}{5} \quad x = \frac{-6}{-2}$$

$$x = 9 \quad x = 3$$

$$x + a = b$$

$$x + a = b \quad / - a$$

$$x = b - a$$

$$x + 2 = 5 \quad / - 2 \quad x + 5 = -7 \quad / - 5$$

$$x = 5 - 2 \quad x = -7 - 5$$

$$x = 3 \quad x = -12$$

$$a \cdot x + b = c$$

$$a \cdot x + b = c \quad / - b$$

$$a \cdot x = c - b \quad / : a$$

$$x = \frac{c - b}{a}$$

$$5 \cdot x - 4 = 6 \quad / + 4 \quad -2 \cdot x + 4 = -6 \quad / - 4$$

$$5 \cdot x = 10 \quad / : 5 \quad -2 \cdot x = -10 \quad / : (-2)$$

$$x = \frac{10}{5} \quad x = \frac{-10}{-2}$$

$$x = 2 \quad x = 5$$

$$\frac{x}{a} = b$$

$$\frac{x}{a} = b \quad / \cdot a$$

$$x = b \cdot a$$

$$\frac{x}{2} = 5 \quad / \cdot 2$$

$$x = 5 \cdot 2$$

$$x = 10$$

$$\frac{x}{5} = -7 \quad / \cdot 5$$

$$x = -7 \cdot 5$$

$$x = -35$$

$$a - x = b$$

$$a - x = b \quad / - a$$

$$-x = b - a \quad / : (-1)$$

$$x = a - b$$

$$2 - x = 5 \quad / - 2 \quad x - 5 = -7 \quad / + 5$$

$$-x = 5 - 2 \quad x = -7 + 5$$

$$-x = 3 / : (-1) \quad x = -2$$

$$x = -3$$

$$x - a = b$$

$$x - a = b \quad / + a$$

$$x = b + a$$

$$x - 2 = 5 \quad / + 2 \quad x - 5 = -7 \quad / + 5$$

$$x = 5 + 2 \quad x = -7 + 5$$

$$x = 7 \quad x = -2$$

$$ax + b = cx + d \quad / - cx$$

$$ax - cx + b = d \quad / - b$$

$$(a - c)x = d - b \quad / : (a - c)$$

$$x = \frac{d - b}{a - c}$$

$$2x + 4 = 6x + 7 \quad / - 6x$$

$$-4x + 4 = 7 \quad / - 4$$

$$-4x = 3 \quad / : (-4)$$

$$x = -\frac{3}{4}$$

1.1 $a \cdot x + b = c$

1.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = c$

Koeffizienten: a, b, c Gesucht: x

- (1) $a = 9$ $b = 7$ $c = 2$
- (2) $a = 5$ $b = 6$ $c = 8$
- (3) $a = 7$ $b = 7$ $c = 5$
- (4) $a = 1\frac{7}{12}$ $b = \frac{12}{19}$ $c = 6$
- (5) $a = \frac{2}{3}$ $b = \frac{5}{7}$ $c = \frac{13}{16}$
- (6) $a = \frac{16}{19}$ $b = 1\frac{6}{7}$ $c = 1\frac{1}{6}$
- (7) $a = -2$ $b = 3$ $c = 4$
- (8) $a = 4$ $b = 5$ $c = 6$

- (9) $a = 4$ $b = \frac{1}{6}$ $c = -3$
- (10) $a = \frac{1}{4}$ $b = 6$ $c = 7$
- (11) $a = -\frac{1}{3}$ $b = 4$ $c = -\frac{1}{5}$
- (12) $a = 1\frac{2}{3}$ $b = -\frac{1}{4}$ $c = 5$
- (13) $a = -\frac{2}{5}$ $b = 3$ $c = \frac{3}{4}$
- (14) $a = \frac{1}{3}$ $b = \frac{1}{3}$ $c = -\frac{4}{7}$

1.1.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 9x + 7 &= 2 && / - 7 \\ 9x &= -5 && / : 9 \\ x &= -\frac{5}{9} \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} -2x + 3 &= 4 && / - 3 \\ -2x &= 1 && / : (-2) \\ x &= -\frac{1}{2} \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} 5x + 6 &= 8 && / - 6 \\ 5x &= 2 && / : 5 \\ x &= \frac{2}{5} \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} 4x + 5 &= 6 && / - 5 \\ 4x &= 1 && / : 4 \\ x &= \frac{1}{4} \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} 7x + 7 &= 5 && / - 7 \\ 7x &= -2 && / : 7 \\ x &= -\frac{2}{7} \end{aligned}$$

Aufgabe (9)

$$\begin{aligned} 4x + \frac{1}{6} &= -3 && / - \frac{1}{6} \\ 4x &= -3\frac{1}{6} && / : 4 \\ x &= -\frac{19}{24} \end{aligned}$$

Aufgabe (4)

$$\begin{aligned} 1\frac{7}{12}x + \frac{12}{19} &= 6 && / - \frac{12}{19} \\ 1\frac{7}{12}x &= 5\frac{7}{19} && / : 1\frac{7}{12} \\ x &= 3,39 \end{aligned}$$

Aufgabe (10)

$$\begin{aligned} \frac{1}{4}x + 6 &= 7 && / - 6 \\ \frac{1}{4}x &= 1 && / : \frac{1}{4} \\ x &= 4 \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} \frac{2}{3}x + \frac{5}{7} &= \frac{13}{16} && / - \frac{5}{7} \\ \frac{2}{3}x &= 0,0982 && / : \frac{2}{3} \\ x &= 0,147 \end{aligned}$$

Aufgabe (11)

$$\begin{aligned} -\frac{1}{3}x + 4 &= -\frac{1}{5} && / - 4 \\ -\frac{1}{3}x &= -4\frac{1}{5} && / : (-\frac{1}{3}) \\ x &= 12\frac{3}{5} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} \frac{16}{19}x + 1\frac{6}{7} &= 1\frac{1}{6} && / - 1\frac{6}{7} \\ \frac{16}{19}x &= -\frac{29}{42} && / : \frac{16}{19} \\ x &= -0,82 \end{aligned}$$

Aufgabe (12)

$$\begin{aligned} 1\frac{2}{3}x - \frac{1}{4} &= 5 && / + \frac{1}{4} \\ 1\frac{2}{3}x &= 5\frac{1}{4} && / : 1\frac{2}{3} \\ x &= 3\frac{3}{20} \end{aligned}$$

Aufgabe (13)

$$\begin{aligned} -\frac{2}{5}x + 3 &= \frac{3}{4} && / -3 \\ -\frac{2}{5}x &= -2\frac{1}{4} && / : (-\frac{2}{5}) \\ x &= 5\frac{5}{8} \end{aligned}$$

$$\begin{aligned} \frac{1}{3}x + \frac{1}{3} &= -\frac{4}{7} && / -\frac{1}{3} \\ \frac{1}{3}x &= -\frac{19}{21} && / : \frac{1}{3} \\ x &= -2\frac{2}{7} \end{aligned}$$

Aufgabe (14)

1.2 $a \cdot x + b = c \cdot x + d$

1.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = c \cdot x + d$

Koeffizienten: a, b, c, d

Gesucht: x

- (1) $a = 9$ $b = 7$ $c = 2$ $d = 4$
- (2) $a = 5$ $b = 6$ $c = 8$ $d = 1$
- (3) $a = 7$ $b = 7$ $c = 5$ $d = 2$
- (4) $a = 1\frac{7}{12}$ $b = \frac{12}{19}$ $c = 6$ $d = -3$
- (5) $a = \frac{2}{3}$ $b = \frac{5}{7}$ $c = \frac{13}{16}$ $d = 1$
- (6) $a = 4$ $b = 5$ $c = 6$ $d = -2$
- (7) $a = 1$ $b = 3$ $c = 2$ $d = 5$

- (8) $a = 1$ $b = 3$ $c = 2$ $d = 3$
- (9) $a = 4$ $b = 5$ $c = 0$ $d = 7$
- (10) $a = \frac{4}{5}$ $b = 5$ $c = \frac{3}{5}$ $d = 7$
- (11) $a = -\frac{4}{9}$ $b = -\frac{5}{6}$ $c = 3$ $d = 7$
- (12) $a = -\frac{4}{5}$ $b = -1\frac{1}{2}$ $c = -3$ $d = 2$
- (13) $a = -\frac{3}{8}$ $b = 1\frac{1}{3}$ $c = 5$ $d = \frac{2}{3}$

1.2.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 9x + 7 &= 2x + 4 && / - 2x \\ 7x + 7 &= 4 && / - 7 \\ 7x &= -3 && / : 7 \\ x &= -\frac{3}{7} \end{aligned}$$

$$\begin{aligned} 4x + 5 &= 6x - 2 && / - 6x \\ -2x + 5 &= -2 && / - 5 \\ -2x &= -7 && / : (-2) \\ x &= 3\frac{1}{2} \end{aligned}$$

Aufgabe (7)

Aufgabe (2)

$$\begin{aligned} 5x + 6 &= 8x + 1 && / - 8x \\ -3x + 6 &= 1 && / - 6 \\ -3x &= -5 && / : (-3) \\ x &= 1\frac{2}{3} \end{aligned}$$

$$\begin{aligned} x + 3 &= 2x + 5 && / - 2x \\ -1x + 3 &= 5 && / - 3 \\ -1x &= 2 && / : (-1) \\ x &= -2 \end{aligned}$$

Aufgabe (8)

Aufgabe (3)

$$\begin{aligned} 7x + 7 &= 5x + 2 && / - 5x \\ 2x + 7 &= 2 && / - 7 \\ 2x &= -5 && / : 2 \\ x &= -2\frac{1}{2} \end{aligned}$$

$$\begin{aligned} x + 3 &= 2x + 3 && / - 2x \\ -1x + 3 &= 3 && / - 3 \\ -1x &= 0 && / : (-1) \\ x &= 0 \end{aligned}$$

Aufgabe (9)

Aufgabe (4)

$$\begin{aligned} 1\frac{7}{12}x + \frac{12}{19} &= 6x - 3 && / - 6x \\ -4\frac{5}{12}x + \frac{12}{19} &= -3 && / - \frac{12}{19} \\ -4\frac{5}{12}x &= -3\frac{12}{19} && / : (-4\frac{5}{12}) \\ x &= 0,822 \end{aligned}$$

$$\begin{aligned} 4x + 5 &= 7 && / - 5 \\ 4x &= 2 && / : 4 \\ x &= \frac{1}{2} \end{aligned}$$

Aufgabe (10)

Aufgabe (5)

$$\begin{aligned} \frac{2}{3}x + \frac{5}{7} &= \frac{13}{16}x + 1 && / - \frac{13}{16}x \\ -\frac{7}{48}x + \frac{5}{7} &= 1 && / - \frac{5}{7} \\ -\frac{7}{48}x &= \frac{2}{7} && / : (-\frac{7}{48}) \\ x &= -1\frac{47}{49} \end{aligned}$$

$$\begin{aligned} \frac{4}{5}x + 5 &= \frac{3}{5}x + 7 && / - \frac{3}{5}x \\ \frac{1}{5}x + 5 &= 7 && / - 5 \\ \frac{1}{5}x &= 2 && / : \frac{1}{5} \\ x &= 10 \end{aligned}$$

Aufgabe (11)

Aufgabe (6)

$$\begin{aligned} -\frac{4}{9}x - \frac{5}{6} &= 3x + 7 && / - 3x \\ -3\frac{4}{9}x - \frac{5}{6} &= 7 && / + \frac{5}{6} \\ -3\frac{4}{9}x &= 7\frac{5}{6} && / : (-3\frac{4}{9}) \end{aligned}$$

$$x = -2\frac{17}{62}$$

Aufgabe (13)

Aufgabe (12)

$$\begin{aligned} -\frac{4}{5}x - 1\frac{1}{2} &= -3x + 2 & / + 3x \\ 2\frac{1}{5}x - 1\frac{1}{2} &= 2 & / + 1\frac{1}{2} \\ 2\frac{1}{5}x &= 3\frac{1}{2} & / : 2\frac{1}{5} \\ x &= 1\frac{13}{22} \end{aligned}$$

$$\begin{aligned} -\frac{3}{8}x + 1\frac{1}{3} &= 5x + \frac{2}{3} & / - 5x \\ -5\frac{3}{8}x + 1\frac{1}{3} &= \frac{2}{3} & / - 1\frac{1}{3} \\ -5\frac{3}{8}x &= -\frac{2}{3} & / : (-5\frac{3}{8}) \\ x &= 0,124 \end{aligned}$$

1.3 $a \cdot x + b = 0$

1.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = 0$

Koeffizienten: a, b

Gesucht: x

- (1) $a = 3$ $b = 9$
- (2) $a = 8$ $b = 1$
- (3) $a = 2$ $b = 3$
- (4) $a = 3$ $b = 5$
- (5) $a = 7$ $b = 7$
- (6) $a = 5$ $b = 5$
- (7) $a = 6$ $b = 6$
- (8) $a = 8$ $b = 6$
- (9) $a = 6$ $b = 4$
- (10) $a = 1$ $b = 2$

- (11) $a = 4$ $b = 7$
- (12) $a = 2$ $b = 0$
- (13) $a = -\frac{1}{2}$ $b = 0$
- (14) $a = 6$ $b = -36$
- (15) $a = 3$ $b = 3$
- (16) $a = -\frac{1}{2}$ $b = 4\frac{1}{2}$
- (17) $a = -\frac{2}{3}$ $b = \frac{1}{6}$
- (18) $a = \frac{1}{4}$ $b = -2$
- (19) $a = \frac{1}{4}$ $b = -3$

1.3.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 3x + 9 &= 0 && / -9 \\ 3x &= -9 && / : 3 \\ x &= -3 \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} 6x + 6 &= 0 && / -6 \\ 6x &= -6 && / : 6 \\ x &= -1 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} 8x + 1 &= 0 && / -1 \\ 8x &= -1 && / : 8 \\ x &= -\frac{1}{8} \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} 8x + 6 &= 0 && / -6 \\ 8x &= -6 && / : 8 \\ x &= -\frac{3}{4} \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} 2x + 3 &= 0 && / -3 \\ 2x &= -3 && / : 2 \\ x &= -1\frac{1}{2} \end{aligned}$$

Aufgabe (9)

$$\begin{aligned} 6x + 4 &= 0 && / -4 \\ 6x &= -4 && / : 6 \\ x &= -\frac{2}{3} \end{aligned}$$

Aufgabe (4)

$$\begin{aligned} 3x + 5 &= 0 && / -5 \\ 3x &= -5 && / : 3 \\ x &= -1\frac{2}{3} \end{aligned}$$

Aufgabe (10)

$$\begin{aligned} x + 2 &= 0 && / -2 \\ x &= -2 \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} 7x + 7 &= 0 && / -7 \\ 7x &= -7 && / : 7 \\ x &= -1 \end{aligned}$$

Aufgabe (11)

$$\begin{aligned} 4x + 7 &= 0 && / -7 \\ 4x &= -7 && / : 4 \\ x &= -1\frac{3}{4} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} 5x + 5 &= 0 && / -5 \\ 5x &= -5 && / : 5 \\ x &= -1 \end{aligned}$$

Aufgabe (12)

$$\begin{aligned} 2x &= 0 && / : 2 \\ x &= 0 \end{aligned}$$

Aufgabe (13)

$$-\frac{1}{2}x = 0 \quad / : (-\frac{1}{2})$$

$$x = 0$$

Aufgabe (17)

Aufgabe (14)

$$6x - 36 = 0 \quad / + 36$$

$$6x = 36 \quad / : 6$$

$$x = 6$$

$$-\frac{2}{3}x + \frac{1}{6} = 0 \quad / -\frac{1}{6}$$

$$-\frac{2}{3}x = -\frac{1}{6} \quad / : (-\frac{2}{3})$$

$$x = \frac{1}{4}$$

Aufgabe (18)

Aufgabe (15)

$$3x + 3 = 0 \quad / - 3$$

$$3x = -3 \quad / : 3$$

$$x = -1$$

$$\frac{1}{4}x - 2 = 0 \quad / + 2$$

$$\frac{1}{4}x = 2 \quad / : \frac{1}{4}$$

$$x = 8$$

Aufgabe (19)

Aufgabe (16)

$$-\frac{1}{2}x + 4\frac{1}{2} = 0 \quad / - 4\frac{1}{2}$$

$$-\frac{1}{2}x = -4\frac{1}{2} \quad / : (-\frac{1}{2})$$

$$x = 9$$

$$\frac{1}{4}x - 3 = 0 \quad / + 3$$

$$\frac{1}{4}x = 3 \quad / : \frac{1}{4}$$

$$x = 12$$

1.4 $a \cdot x = d$

1.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x = d$

Koeffizienten: a, d

Gesucht: x

(1) $a = 3$ $d = 9$

(2) $a = 8$ $d = 1$

(3) $a = 2$ $d = 3$

(4) $a = 3$ $d = 5$

(5) $a = 7$ $d = 7$

(6) $a = 5$ $d = 5$

(7) $a = 6$ $d = 6$

(8) $a = 8$ $d = 6$

(9) $a = 6$ $d = 4$

(10) $a = 1$ $d = 2$

(11) $a = 4$ $d = 7$

(12) $a = 2$ $d = 0$

(13) $a = -\frac{1}{2}$ $d = 0$

(14) $a = 6$ $d = -36$

(15) $a = 3$ $d = 3$

(16) $a = -\frac{1}{2}$ $d = 4\frac{1}{2}$

(17) $a = -\frac{2}{3}$ $d = \frac{1}{6}$

(18) $a = \frac{1}{4}$ $d = -2$

(19) $a = \frac{1}{4}$ $d = -3$

(20) $a = -2$ $d = 4$

(21) $a = 1$ $d = -2$

(22) $a = -1\frac{1}{4}$ $d = -10$

(23) $a = 4$ $d = -8$

(24) $a = -\frac{24}{49}$ $d = 2\frac{22}{49}$

(25) $a = \frac{8}{27}$ $d = 2\frac{2}{3}$

(26) $a = \frac{20}{81}$ $d = 2\frac{2}{9}$

1.4.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 3x &= 9 & / : 3 \\ x &= 3 \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} 8x &= 6 & / : 8 \\ x &= \frac{3}{4} \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} 8x &= 1 & / : 8 \\ x &= \frac{1}{8} \end{aligned}$$

Aufgabe (9)

$$\begin{aligned} 6x &= 4 & / : 6 \\ x &= \frac{2}{3} \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} 2x &= 3 & / : 2 \\ x &= 1\frac{1}{2} \end{aligned}$$

Aufgabe (10)

$$x = 2$$

Aufgabe (4)

$$\begin{aligned} 3x &= 5 & / : 3 \\ x &= 1\frac{2}{3} \end{aligned}$$

Aufgabe (11)

$$\begin{aligned} 4x &= 7 & / : 4 \\ x &= 1\frac{3}{4} \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} 7x &= 7 & / : 7 \\ x &= 1 \end{aligned}$$

Aufgabe (12)

$$\begin{aligned} 2x &= 0 & / : 2 \\ x &= 0 \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} 5x &= 5 & / : 5 \\ x &= 1 \end{aligned}$$

Aufgabe (13)

$$\begin{aligned} -\frac{1}{2}x &= 0 & / : (-\frac{1}{2}) \\ x &= 0 \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} 6x &= 6 & / : 6 \\ x &= 1 \end{aligned}$$

Aufgabe (14)

$$\begin{aligned} 6x &= -36 & / : 6 \\ x &= -6 \end{aligned}$$

Aufgabe (15)

$$\begin{aligned} 3x &= 3 & / : 3 \\ x &= 1 \end{aligned}$$

Aufgabe (21)

$$x = -2$$

Aufgabe (16)

$$\begin{aligned} -\frac{1}{2}x &= 4\frac{1}{2} & / : (-\frac{1}{2}) \\ x &= -9 \end{aligned}$$

Aufgabe (22)

$$\begin{aligned} -1\frac{1}{4}x &= -10 & / : (-1\frac{1}{4}) \\ x &= 8 \end{aligned}$$

Aufgabe (17)

$$\begin{aligned} -\frac{2}{3}x &= \frac{1}{6} & / : (-\frac{2}{3}) \\ x &= -\frac{1}{4} \end{aligned}$$

Aufgabe (23)

$$\begin{aligned} 4x &= -8 & / : 4 \\ x &= -2 \end{aligned}$$

Aufgabe (18)

$$\begin{aligned} \frac{1}{4}x &= -2 & / : \frac{1}{4} \\ x &= -8 \end{aligned}$$

Aufgabe (24)

$$\begin{aligned} -\frac{24}{49}x &= 2\frac{22}{49} & / : (-\frac{24}{49}) \\ x &= -5 \end{aligned}$$

Aufgabe (19)

$$\begin{aligned} \frac{1}{4}x &= -3 & / : \frac{1}{4} \\ x &= -12 \end{aligned}$$

Aufgabe (25)

$$\begin{aligned} \frac{8}{27}x &= 2\frac{2}{3} & / : \frac{8}{27} \\ x &= 9 \end{aligned}$$

Aufgabe (20)

$$\begin{aligned} -2x &= 4 & / : (-2) \\ x &= -2 \end{aligned}$$

Aufgabe (26)

$$\begin{aligned} \frac{20}{81}x &= 2\frac{2}{9} & / : \frac{20}{81} \\ x &= 9 \end{aligned}$$

2 Quadratische Gleichung

Umformen: $ax^2 + c = 0$

$$ax^2 + c = 0 \quad / -c$$

$$ax^2 = -c \quad / : a$$

$$x_{1/2} = \pm \sqrt{\frac{-c}{a}}$$

Diskriminante:

$$D = \frac{-c}{a}$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$-\frac{2}{3}x^2 + \frac{1}{6} = 0 \quad / -\frac{1}{6}$$

$$-\frac{2}{3}x^2 = -\frac{1}{6} \quad / : \left(-\frac{2}{3}\right)$$

$$x^2 = \frac{-\frac{1}{6}}{-\frac{2}{3}}$$

$$x = \pm \sqrt{\frac{1}{4}}$$

$$x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$$

Faktorisieren: $ax^2 + bx = 0$

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

$$x_1 = 0 \quad \vee \quad x_2 = \frac{-b}{a}$$

$$-2x^2 - 8x = 0$$

$$x(-2x - 8) = 0$$

$$x_1 = 0$$

$$-2x - 8 = 0 \quad / +8$$

$$-2x = 8 \quad / : (-2)$$

$$x = \frac{8}{-2}$$

$$x_2 = -4$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x_1 = 0$$

$$x - 1 = 0 \quad / +1$$

$$x = 1$$

$$x_2 = 1$$

Lösungsformel (Mitternachtsformel): $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Diskriminante:

$$D = b^2 - 4 \cdot a \cdot c$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{49}}{2}$$

$$x_{1/2} = \frac{-3 \pm 7}{2}$$

$$x_1 = \frac{-3 + 7}{2} \quad x_2 = \frac{-3 - 7}{2}$$

$$x_1 = 2 \quad x_2 = -5$$

p-q Formel: $x^2 + px + q = 0$

$$x^2 + px + q = 0$$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Diskriminante:

$$D = \left(\frac{p}{2}\right)^2 - q$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$$

$$x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = 2 \quad x_2 = -5$$

Satz von Vieta: $x^2 + px + q = 0$

$$x^2 + px + q = 0$$

x_1, x_2 sind die Lösungen der Gleichung

$$(x - x_1) \cdot (x - x_2) = 0$$

$$x^2 - x_2 \cdot x - x_1 \cdot x + x_1 \cdot x_2 = 0$$

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$$

$$x_1 + x_2 = -p$$

$$x_1 \cdot x_2 = q$$

$$x^2 + 3x - 10 = 0$$

$$p = 3 \quad q = -10$$

$$x_1 + x_2 = -3$$

$$x_1 \cdot x_2 = 10$$

$$2 - 5 = -3$$

$$2 \cdot (-5) = -10$$

$$x_1 = 2 \quad x_2 = -5$$

$$(x - 2) \cdot (x + 5) = 0$$

2.1 $ax^2 + bx + c = 0$

2.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ax^2 + bx + c = 0$

Gesucht:

Lösung der Gleichung

(1) $3x^2 + 3 = 0$

(2) $-\frac{1}{2}x^2 + 4\frac{1}{2} = 0$

(3) $-\frac{2}{3}x^2 + \frac{1}{6} = 0$

(4) $\frac{1}{4}x^2 - 2 = 0$

(5) $\frac{1}{4}x^2 - 3 = 0$

(6) $-2x^2 + 4 = 0$

(7) $x^2 - 2 = 0$

(8) $-\frac{1}{3}x^2 + 2x = 0$

(9) $-2x^2 - 8x = 0$

(10) $x^2 - 1x = 0$

(11) $\frac{1}{2}x^2 - \frac{2}{3}x = 0$

(12) $2x^2 - 5x = 0$

(13) $x^2 + 2x - 24 = 0$

(14) $x^2 + 3x - 10 = 0$

(15) $x^2 - 1x = 0$

(16) $x^2 - 8x - 20 = 0$

(17) $x^2 - 8x + 15 = 0$

(18) $-\frac{1}{3}x^2 - 2x + 3 = 0$

(19) $x^2 - 4x + 7 = 0$

(20) $-1x^2 + 4x - 7 = 0$

(21) $2x^2 + 4x = 0$

(22) $-\frac{1}{2}x^2 + 2x + 5 = 0$

(23) $-2x^2 + 3x + 4 = 0$

(24) $x^2 + 6x - 2 = 0$

(25) $-\frac{1}{3}x^2 + 2x + 5 = 0$

(26) $\frac{1}{2}x^2 - 1x + 4 = 0$

(27) $-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$

(28) $-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$

(29) $-1\frac{1}{4}x^2 + 5x = 0$

(30) $-\frac{3}{4}x^2 - 3x = 0$

(31) $\frac{5}{9}x^2 - 5 = 0$

(32) $12x^2 + 12x = 0$

(33) $-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$

(34) $-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$

(35) $-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$

(36) $\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$

(37) $-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$

(38) $-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0$

(39) $-\frac{7}{9}x^2 + 4\frac{2}{3}x = 0$

(40) $\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0$

(41) $\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$

(42) $-1\frac{1}{4}x^2 - 10x - 15 = 0$

(43) $4x^2 - 8x = 0$

(44) $-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$

(45) $\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$

(46) $\frac{20}{81}x^2 + 2\frac{2}{9}x = 0$

(47) $1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$

2.1.2 Lösungen

Aufgabe (1)

<p>Umformen</p> $\begin{aligned} 3x^2 + 3 &= 0 & / -3 \\ 3x^2 &= -3 & / : 3 \\ x^2 &= \frac{-3}{3} \\ \text{keine Lösung} \end{aligned}$	<p>a-b-c Formel</p> $\begin{aligned} 3x^2 + 0x + 3 &= 0 \\ x_{1/2} &= \frac{-0 \pm \sqrt{0^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} \\ x_{1/2} &= \frac{-0 \pm \sqrt{-36}}{6} \\ \text{Diskriminante negativ keine Lösung} \end{aligned}$	<p>p-q Formel</p> $\begin{aligned} 3x^2 + 0x + 3 &= 0 & / : 3 \\ x^2 + 0x + 1 &= 0 \\ x_{1/2} &= -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - 1} \\ x_{1/2} &= 0 \pm \sqrt{-1} \\ \text{Diskriminante negativ keine Lösung} \end{aligned}$
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Aufgabe (2)

<p>Umformen</p> $\begin{aligned} -\frac{1}{2}x^2 + 4\frac{1}{2} &= 0 & / -4\frac{1}{2} \\ -\frac{1}{2}x^2 &= -4\frac{1}{2} & / : \left(-\frac{1}{2}\right) \\ x^2 &= \frac{-4\frac{1}{2}}{-\frac{1}{2}} \\ x &= \pm\sqrt{9} \\ x_1 &= 3 & x_2 = -3 \end{aligned}$	<p>a-b-c Formel</p> $\begin{aligned} -\frac{1}{2}x^2 + 0x + 4\frac{1}{2} &= 0 \\ x_{1/2} &= \frac{-0 \pm \sqrt{0^2 - 4 \cdot \left(-\frac{1}{2}\right) \cdot 4\frac{1}{2}}}{2 \cdot \left(-\frac{1}{2}\right)} \\ x_{1/2} &= \frac{-0 \pm \sqrt{9}}{-1} \\ x_{1/2} &= \frac{0 \pm 3}{-1} \\ x_1 &= \frac{0+3}{-1} & x_2 = \frac{0-3}{-1} \\ x_1 &= -3 & x_2 = 3 \end{aligned}$	<p>p-q Formel</p> $\begin{aligned} -\frac{1}{2}x^2 + 0x + 4\frac{1}{2} &= 0 & / : -\frac{1}{2} \\ x^2 + 0x - 9 &= 0 \\ x_{1/2} &= -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)} \\ x_{1/2} &= 0 \pm \sqrt{9} \\ x_{1/2} &= 0 \pm 3 \\ x_1 &= 3 & x_2 = -3 \end{aligned}$
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Aufgabe (3)

<p>Umformen</p> $\begin{aligned} -\frac{2}{3}x^2 + \frac{1}{6} &= 0 & / -\frac{1}{6} \\ -\frac{2}{3}x^2 &= -\frac{1}{6} & / : \left(-\frac{2}{3}\right) \\ x^2 &= \frac{-\frac{1}{6}}{-\frac{2}{3}} \\ x &= \pm\sqrt{\frac{1}{4}} \\ x_1 &= \frac{1}{2} & x_2 = -\frac{1}{2} \end{aligned}$	<p>a-b-c Formel</p> $\begin{aligned} -\frac{2}{3}x^2 + 0x + \frac{1}{6} &= 0 \\ x_{1/2} &= \frac{-0 \pm \sqrt{0^2 - 4 \cdot \left(-\frac{2}{3}\right) \cdot \frac{1}{6}}}{2 \cdot \left(-\frac{2}{3}\right)} \\ x_{1/2} &= \frac{-0 \pm \sqrt{\frac{4}{9}}}{-1\frac{1}{3}} \\ x_{1/2} &= \frac{0 \pm \frac{2}{3}}{-1\frac{1}{3}} \\ x_1 &= \frac{0 + \frac{2}{3}}{-1\frac{1}{3}} & x_2 = \frac{0 - \frac{2}{3}}{-1\frac{1}{3}} \\ x_1 &= -\frac{1}{2} & x_2 = \frac{1}{2} \end{aligned}$	<p>p-q Formel</p> $\begin{aligned} -\frac{2}{3}x^2 + 0x + \frac{1}{6} &= 0 & / : -\frac{2}{3} \\ x^2 + 0x - \frac{1}{4} &= 0 \\ x_{1/2} &= -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - \left(-\frac{1}{4}\right)} \\ x_{1/2} &= 0 \pm \sqrt{\frac{1}{4}} \\ x_{1/2} &= 0 \pm \frac{1}{2} \\ x_1 &= \frac{1}{2} & x_2 = -\frac{1}{2} \end{aligned}$
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Aufgabe (4)

<p>Umformen</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 - 2 = 0 \quad / + 2$ $\frac{1}{4}x^2 = 2 \quad / : \frac{1}{4}$ $x^2 = \frac{2}{\frac{1}{4}}$ $x = \pm\sqrt{8}$ $x_1 = 2,83 \quad x_2 = -2,83$	<p>a-b-c Formel</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 + 0x - 2 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-2)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{2}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm 1,41}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,41}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,41}{\frac{1}{2}}$ $x_1 = 2,83 \quad x_2 = -2,83$	<p>p-q Formel</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 + 0x - 2 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 8 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-8)}$ $x_{1/2} = 0 \pm \sqrt{8}$ $x_{1/2} = 0 \pm 2,83$ $x_1 = 2,83 \quad x_2 = -2,83$
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Aufgabe (5)

<p>Umformen</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 - 3 = 0 \quad / + 3$ $\frac{1}{4}x^2 = 3 \quad / : \frac{1}{4}$ $x^2 = \frac{3}{\frac{1}{4}}$ $x = \pm\sqrt{12}$ $x_1 = 3,46 \quad x_2 = -3,46$	<p>a-b-c Formel</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 + 0x - 3 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-3)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{3}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm 1,73}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,73}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,73}{\frac{1}{2}}$ $x_1 = 3,46 \quad x_2 = -3,46$	<p>p-q Formel</p> <hr style="border: 0.5px solid black;"/> $\frac{1}{4}x^2 + 0x - 3 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 12 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-12)}$ $x_{1/2} = 0 \pm \sqrt{12}$ $x_{1/2} = 0 \pm 3,46$ $x_1 = 3,46 \quad x_2 = -3,46$
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Aufgabe (6)

<p>Umformen</p> <hr style="border: 0.5px solid black;"/> $-2x^2 + 4 = 0 \quad / - 4$ $-2x^2 = -4 \quad / : (-2)$ $x^2 = \frac{-4}{-2}$ $x = \pm\sqrt{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>a-b-c Formel</p> <hr style="border: 0.5px solid black;"/> $-2x^2 + 0x + 4 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)}$ $x_{1/2} = \frac{-0 \pm \sqrt{32}}{-4}$ $x_{1/2} = \frac{0 \pm 5,66}{-4}$ $x_1 = \frac{0 + 5,66}{-4} \quad x_2 = \frac{0 - 5,66}{-4}$ $x_1 = -1,41 \quad x_2 = 1,41$	<p>p-q Formel</p> <hr style="border: 0.5px solid black;"/> $-2x^2 + 0x + 4 = 0 \quad / : -2$ $x^2 + 0x - 2 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)}$ $x_{1/2} = 0 \pm \sqrt{2}$ $x_{1/2} = 0 \pm 1,41$ $x_1 = 1,41 \quad x_2 = -1,41$
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Aufgabe (7)

<p>Umformen</p> <hr/> $1x^2 - 2 = 0 \quad / + 2$ $1x^2 = 2 \quad / : 1$ $x^2 = \frac{2}{1}$ $x = \pm\sqrt{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>a-b-c Formel</p> <hr/> $1x^2 + 0x - 2 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$ $x_{1/2} = \frac{-0 \pm \sqrt{8}}{2}$ $x_{1/2} = \frac{0 \pm 2,83}{2}$ $x_1 = \frac{0 + 2,83}{2} \quad x_2 = \frac{0 - 2,83}{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>p-q Formel</p> <hr/> $x^2 + 0x - 2 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)}$ $x_{1/2} = 0 \pm \sqrt{2}$ $x_{1/2} = 0 \pm 1,41$ $x_1 = 1,41 \quad x_2 = -1,41$
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Aufgabe (8)

<p>x-Ausklammern</p> <hr/> $-\frac{1}{3}x^2 + 2x = 0$ $x(-\frac{1}{3}x + 2) = 0$ $-\frac{1}{3}x + 2 = 0 \quad / - 2$ $-\frac{1}{3}x = -2 \quad / : (-\frac{1}{3})$ $x = \frac{-2}{-\frac{1}{3}}$ $x_1 = 0$ $x_2 = 6$	<p>a-b-c Formel</p> <hr/> $-\frac{1}{3}x^2 + 2x + 0 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{3}) \cdot 0}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{-2 \pm \sqrt{4}}{-\frac{2}{3}}$ $x_{1/2} = \frac{-2 \pm 2}{-\frac{2}{3}}$ $x_1 = \frac{-2 + 2}{-\frac{2}{3}} \quad x_2 = \frac{-2 - 2}{-\frac{2}{3}}$ $x_1 = 0 \quad x_2 = 6$	<p>p-q Formel</p> <hr/> $-\frac{1}{3}x^2 + 2x + 0 = 0 \quad / : -\frac{1}{3}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$
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Aufgabe (9)

<p>x-Ausklammern</p> <hr/> $-2x^2 - 8x = 0$ $x(-2x - 8) = 0$ $-2x - 8 = 0 \quad / + 8$ $-2x = 8 \quad / : (-2)$ $x = \frac{8}{-2}$ $x_1 = 0$ $x_2 = -4$	<p>a-b-c Formel</p> <hr/> $-2x^2 - 8x + 0 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot (-2) \cdot 0}}{2 \cdot (-2)}$ $x_{1/2} = \frac{+8 \pm \sqrt{64}}{-4}$ $x_{1/2} = \frac{8 \pm 8}{-4}$ $x_1 = \frac{8 + 8}{-4} \quad x_2 = \frac{8 - 8}{-4}$ $x_1 = -4 \quad x_2 = 0$	<p>p-q Formel</p> <hr/> $-2x^2 - 8x + 0 = 0 \quad / : -2$ $x^2 + 4x + 0 = 0$ $x_{1/2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0}$ $x_{1/2} = -2 \pm \sqrt{4}$ $x_{1/2} = -2 \pm 2$ $x_1 = 0 \quad x_2 = -4$
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Aufgabe (10)

x-Ausklammern	a-b-c Formel	p-q Formel
$1x^2 - 1x = 0$ $x(1x - 1) = 0$ $1x - 1 = 0 \quad / + 1$ $1x = 1 \quad / : 1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	$1x^2 - 1x + 0 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1+1}{2} \quad x_2 = \frac{1-1}{2}$ $x_1 = 1 \quad x_2 = 0$	$x^2 - 1x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{(-1)}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$

Aufgabe (11)

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{1}{2}x^2 - \frac{2}{3}x = 0$ $x(\frac{1}{2}x - \frac{2}{3}) = 0$ $\frac{1}{2}x - \frac{2}{3} = 0 \quad / + \frac{2}{3}$ $\frac{1}{2}x = \frac{2}{3} \quad / : \frac{1}{2}$ $x = \frac{\frac{2}{3}}{\frac{1}{2}}$ $x_1 = 0$ $x_2 = 1\frac{1}{3}$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{(-\frac{2}{3})^2 - 4 \cdot \frac{1}{2} \cdot 0}}{2 \cdot \frac{1}{2}}$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9}}}{1}$ $x_{1/2} = \frac{\frac{2}{3} \pm \frac{2}{3}}{1}$ $x_1 = \frac{\frac{2}{3} + \frac{2}{3}}{1} \quad x_2 = \frac{\frac{2}{3} - \frac{2}{3}}{1}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0 \quad / : \frac{1}{2}$ $x^2 - 1\frac{1}{3}x + 0 = 0$ $x_{1/2} = -\frac{-1\frac{1}{3}}{2} \pm \sqrt{\left(\frac{(-1\frac{1}{3})}{2}\right)^2 - 0}$ $x_{1/2} = \frac{2}{3} \pm \sqrt{\frac{4}{9}}$ $x_{1/2} = \frac{2}{3} \pm \frac{2}{3}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$

Aufgabe (12)

x-Ausklammern	a-b-c Formel	p-q Formel
$2x^2 - 5x = 0$ $x(2x - 5) = 0$ $2x - 5 = 0 \quad / + 5$ $2x = 5 \quad / : 2$ $x = \frac{5}{2}$ $x_1 = 0$ $x_2 = 2\frac{1}{2}$	$2x^2 - 5x + 0 = 0$ $x_{1/2} = \frac{+5 \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 0}}{2 \cdot 2}$ $x_{1/2} = \frac{+5 \pm \sqrt{25}}{4}$ $x_{1/2} = \frac{5 \pm 5}{4}$ $x_1 = \frac{5+5}{4} \quad x_2 = \frac{5-5}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$	$2x^2 - 5x + 0 = 0 \quad / : 2$ $x^2 - 2\frac{1}{2}x + 0 = 0$ $x_{1/2} = -\frac{-2\frac{1}{2}}{2} \pm \sqrt{\left(\frac{(-2\frac{1}{2})}{2}\right)^2 - 0}$ $x_{1/2} = 1\frac{1}{4} \pm \sqrt{1\frac{9}{16}}$ $x_{1/2} = 1\frac{1}{4} \pm 1\frac{1}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$

Aufgabe (13)

a-b-c Formel	p-q Formel
$1x^2 + 2x - 24 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-24)}}{2 \cdot 1}$ $x_{1/2} = \frac{-2 \pm \sqrt{100}}{2}$ $x_{1/2} = \frac{-2 \pm 10}{2}$ $x_1 = \frac{-2 + 10}{2} \quad x_2 = \frac{-2 - 10}{2}$ $x_1 = 4 \quad x_2 = -6$	$x^2 + 2x - 24 = 0$ $x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-24)}$ $x_{1/2} = -1 \pm \sqrt{25}$ $x_{1/2} = -1 \pm 5$ $x_1 = 4 \quad x_2 = -6$

Aufgabe (14)

a-b-c Formel	p-q Formel
$1x^2 + 3x - 10 = 0$ $x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$ $x_{1/2} = \frac{-3 \pm \sqrt{49}}{2}$ $x_{1/2} = \frac{-3 \pm 7}{2}$ $x_1 = \frac{-3 + 7}{2} \quad x_2 = \frac{-3 - 7}{2}$ $x_1 = 2 \quad x_2 = -5$	$x^2 + 3x - 10 = 0$ $x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$ $x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = 2 \quad x_2 = -5$

Aufgabe (15)

x-Ausklammern	a-b-c Formel	p-q Formel
$1x^2 - 1x + 0 = 0$ $x(1x - 1) = 0$ $1x - 1 = 0 \quad / + 1$ $1x = 1 \quad / : 1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	$1x^2 - 1x + 0 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1 + 1}{2} \quad x_2 = \frac{1 - 1}{2}$ $x_1 = 1 \quad x_2 = 0$	$x^2 - 1x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$

Aufgabe (16)

a-b-c Formel

$$1x^2 - 8x - 20 = 0$$

$$x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (-20)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+8 \pm \sqrt{144}}{2}$$

$$x_{1/2} = \frac{8 \pm 12}{2}$$

$$x_1 = \frac{8 + 12}{2} \quad x_2 = \frac{8 - 12}{2}$$

$$x_1 = 10 \quad x_2 = -2$$

p-q Formel

$$x^2 - 8x - 20 = 0$$

$$x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{(-8)}{2}\right)^2 - (-20)}$$

$$x_{1/2} = 4 \pm \sqrt{36}$$

$$x_{1/2} = 4 \pm 6$$

$$x_1 = 10 \quad x_2 = -2$$

Aufgabe (17)

a-b-c Formel

$$1x^2 - 8x + 15 = 0$$

$$x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+8 \pm \sqrt{4}}{2}$$

$$x_{1/2} = \frac{8 \pm 2}{2}$$

$$x_1 = \frac{8 + 2}{2} \quad x_2 = \frac{8 - 2}{2}$$

$$x_1 = 5 \quad x_2 = 3$$

p-q Formel

$$x^2 - 8x + 15 = 0$$

$$x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{(-8)}{2}\right)^2 - 15}$$

$$x_{1/2} = 4 \pm \sqrt{1}$$

$$x_{1/2} = 4 \pm 1$$

$$x_1 = 5 \quad x_2 = 3$$

Aufgabe (18)

a-b-c Formel

$$-\frac{1}{3}x^2 - 2x + 3 = 0$$

$$x_{1/2} = \frac{+2 \pm \sqrt{(-2)^2 - 4 \cdot (-\frac{1}{3}) \cdot 3}}{2 \cdot (-\frac{1}{3})}$$

$$x_{1/2} = \frac{+2 \pm \sqrt{8}}{-\frac{2}{3}}$$

$$x_{1/2} = \frac{2 \pm 2,83}{-\frac{2}{3}}$$

$$x_1 = \frac{2 + 2,83}{-\frac{2}{3}} \quad x_2 = \frac{2 - 2,83}{-\frac{2}{3}}$$

$$x_1 = -7,24 \quad x_2 = 1,24$$

p-q Formel

$$-\frac{1}{3}x^2 - 2x + 3 = 0 \quad / : -\frac{1}{3}$$

$$x^2 + 6x - 9 = 0$$

$$x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-9)}$$

$$x_{1/2} = -3 \pm \sqrt{18}$$

$$x_{1/2} = -3 \pm 4,24$$

$$x_1 = 1,24 \quad x_2 = -7,24$$

Aufgabe (19)

a-b-c Formel

$$1x^2 - 4x + 7 = 0$$

$$x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+4 \pm \sqrt{-12}}{2}$$

Diskriminante negativ keine Lösung

p-q Formel

$$x^2 - 4x + 7 = 0$$

$$x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$$

$$x_{1/2} = 2 \pm \sqrt{-3}$$

Diskriminante negativ keine Lösung

Aufgabe (20)

a-b-c Formel

$$-1x^2 + 4x - 7 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-7)}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{-12}}{-2}$$

Diskriminante negativ keine Lösung

p-q Formel

$$-1x^2 + 4x - 7 = 0 \quad / : -1$$

$$x^2 - 4x + 7 = 0$$

$$x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$$

$$x_{1/2} = 2 \pm \sqrt{-3}$$

Diskriminante negativ keine Lösung

Aufgabe (21)

x-Ausklammern

$$2x^2 + 4x = 0$$

$$x(2x + 4) = 0$$

$$2x + 4 = 0 \quad / -4$$

$$2x = -4 \quad / : 2$$

$$x = \frac{-4}{2}$$

$$x_1 = 0$$

$$x_2 = -2$$

a-b-c Formel

$$2x^2 + 4x + 0 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 0}}{2 \cdot 2}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{16}}{4}$$

$$x_{1/2} = \frac{-4 \pm 4}{4}$$

$$x_1 = \frac{-4 + 4}{4} \quad x_2 = \frac{-4 - 4}{4}$$

$$x_1 = 0 \quad x_2 = -2$$

p-q Formel

$$2x^2 + 4x + 0 = 0 \quad / : 2$$

$$x^2 + 2x + 0 = 0$$

$$x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 0}$$

$$x_{1/2} = -1 \pm \sqrt{1}$$

$$x_{1/2} = -1 \pm 1$$

$$x_1 = 0 \quad x_2 = -2$$

Aufgabe (22)

a-b-c Formel

$$-\frac{1}{2}x^2 + 2x + 5 = 0$$

$$x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{2}) \cdot 5}}{2 \cdot (-\frac{1}{2})}$$

$$x_{1/2} = \frac{-2 \pm \sqrt{14}}{-1}$$

$$x_{1/2} = \frac{-2 \pm 3,74}{-1}$$

$$x_1 = \frac{-2 + 3,74}{-1} \quad x_2 = \frac{-2 - 3,74}{-1}$$

$$x_1 = -1,74 \quad x_2 = 5,74$$

p-q Formel

$$-\frac{1}{2}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{2}$$

$$x^2 - 4x - 10 = 0$$

$$x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - (-10)}$$

$$x_{1/2} = 2 \pm \sqrt{14}$$

$$x_{1/2} = 2 \pm 3,74$$

$$x_1 = 5,74 \quad x_2 = -1,74$$

Aufgabe (23)

a-b-c Formel	p-q Formel
$-2x^2 + 3x + 4 = 0$ $x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)}$ $x_{1/2} = \frac{-3 \pm \sqrt{41}}{-4}$ $x_{1/2} = \frac{-3 \pm 6,4}{-4}$ $x_1 = \frac{-3 + 6,4}{-4} \quad x_2 = \frac{-3 - 6,4}{-4}$ $x_1 = -0,851 \quad x_2 = 2,35$	$-2x^2 + 3x + 4 = 0 \quad / : -2$ $x^2 - 1\frac{1}{2}x - 2 = 0$ $x_{1/2} = -\frac{-1\frac{1}{2}}{2} \pm \sqrt{\left(\frac{(-1\frac{1}{2})}{2}\right)^2 - (-2)}$ $x_{1/2} = \frac{3}{4} \pm \sqrt{2\frac{9}{16}}$ $x_{1/2} = \frac{3}{4} \pm 1,6$ $x_1 = 2,35 \quad x_2 = -0,851$

Aufgabe (24)

a-b-c Formel	p-q Formel
$1x^2 + 6x - 2 = 0$ $x_{1/2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$ $x_{1/2} = \frac{-6 \pm \sqrt{44}}{2}$ $x_{1/2} = \frac{-6 \pm 6,63}{2}$ $x_1 = \frac{-6 + 6,63}{2} \quad x_2 = \frac{-6 - 6,63}{2}$ $x_1 = 0,317 \quad x_2 = -6,32$	$x^2 + 6x - 2 = 0$ $x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-2)}$ $x_{1/2} = -3 \pm \sqrt{11}$ $x_{1/2} = -3 \pm 3,32$ $x_1 = 0,317 \quad x_2 = -6,32$

Aufgabe (25)

a-b-c Formel	p-q Formel
$-\frac{1}{3}x^2 + 2x + 5 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{3}) \cdot 5}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{-2 \pm \sqrt{10\frac{2}{3}}}{-\frac{2}{3}}$ $x_{1/2} = \frac{-2 \pm 3,27}{-\frac{2}{3}}$ $x_1 = \frac{-2 + 3,27}{-\frac{2}{3}} \quad x_2 = \frac{-2 - 3,27}{-\frac{2}{3}}$ $x_1 = -1,9 \quad x_2 = 7,9$	$-\frac{1}{3}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{3}$ $x^2 - 6x - 15 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{(-6)}{2}\right)^2 - (-15)}$ $x_{1/2} = 3 \pm \sqrt{24}$ $x_{1/2} = 3 \pm 4,9$ $x_1 = 7,9 \quad x_2 = -1,9$

Aufgabe (26)

a-b-c Formel

$$\begin{aligned} \frac{1}{2}x^2 - 1x + 4 &= 0 \\ x_{1/2} &= \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{2} \cdot 4}}{2 \cdot \frac{1}{2}} \\ x_{1/2} &= \frac{+1 \pm \sqrt{-7}}{1} \end{aligned}$$

Diskriminante negativ keine Lösung

p-q Formel

$$\begin{aligned} \frac{1}{2}x^2 - 1x + 4 &= 0 \quad / : \frac{1}{2} \\ x^2 - 2x + 8 &= 0 \\ x_{1/2} &= -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - 8} \\ x_{1/2} &= 1 \pm \sqrt{-7} \end{aligned}$$

Diskriminante negativ keine Lösung

Aufgabe (27)

a-b-c Formel

$$\begin{aligned} -\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} &= 0 \\ x_{1/2} &= \frac{+\frac{24}{49} \pm \sqrt{\left(-\frac{24}{49}\right)^2 - 4 \cdot \left(-\frac{8}{49}\right) \cdot 1\frac{31}{49}}}{2 \cdot \left(-\frac{8}{49}\right)} \\ x_{1/2} &= \frac{+\frac{24}{49} \pm \sqrt{1\frac{15}{49}}}{-\frac{16}{49}} \\ x_{1/2} &= \frac{\frac{24}{49} \pm 1\frac{1}{7}}{-\frac{16}{49}} \\ x_1 &= \frac{\frac{24}{49} + 1\frac{1}{7}}{-\frac{16}{49}} \quad x_2 = \frac{\frac{24}{49} - 1\frac{1}{7}}{-\frac{16}{49}} \\ x_1 &= -5 \quad x_2 = 2 \end{aligned}$$

p-q Formel

$$\begin{aligned} -\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} &= 0 \quad / : -\frac{8}{49} \\ x^2 + 3x - 10 &= 0 \\ x_{1/2} &= -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)} \\ x_{1/2} &= -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}} \\ x_{1/2} &= -1\frac{1}{2} \pm 3\frac{1}{2} \\ x_1 &= 2 \quad x_2 = -5 \end{aligned}$$

Aufgabe (28)

a-b-c Formel

$$\begin{aligned} -\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} &= 0 \\ x_{1/2} &= \frac{+\frac{32}{81} \pm \sqrt{\left(-\frac{32}{81}\right)^2 - 4 \cdot \left(-\frac{32}{81}\right) \cdot 7\frac{73}{81}}}{2 \cdot \left(-\frac{32}{81}\right)} \\ x_{1/2} &= \frac{+\frac{32}{81} \pm \sqrt{12\frac{52}{81}}}{-\frac{64}{81}} \\ x_{1/2} &= \frac{\frac{32}{81} \pm 3\frac{5}{9}}{-\frac{64}{81}} \\ x_1 &= \frac{\frac{32}{81} + 3\frac{5}{9}}{-\frac{64}{81}} \quad x_2 = \frac{\frac{32}{81} - 3\frac{5}{9}}{-\frac{64}{81}} \\ x_1 &= -5 \quad x_2 = 4 \end{aligned}$$

p-q Formel

$$\begin{aligned} -\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} &= 0 \quad / : -\frac{32}{81} \\ x^2 + 1x - 20 &= 0 \\ x_{1/2} &= -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-20)} \\ x_{1/2} &= -\frac{1}{2} \pm \sqrt{20\frac{1}{4}} \\ x_{1/2} &= -\frac{1}{2} \pm 4\frac{1}{2} \\ x_1 &= 4 \quad x_2 = -5 \end{aligned}$$

Aufgabe (29)

<p><u>x-Ausklammern</u></p> $-1\frac{1}{4}x^2 + 5x = 0$ $x(-1\frac{1}{4}x + 5) = 0$ $-1\frac{1}{4}x + 5 = 0 \quad / -5$ $-1\frac{1}{4}x = -5 \quad / : (-1\frac{1}{4})$ $x = \frac{-5}{-1\frac{1}{4}}$ $x_1 = 0$ $x_2 = 4$	<p><u>a-b-c Formel</u></p> $-1\frac{1}{4}x^2 + 5x + 0 = 0$ $x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-1\frac{1}{4}) \cdot 0}}{2 \cdot (-1\frac{1}{4})}$ $x_{1/2} = \frac{-5 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{-5 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{-5 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{-5 - 5}{-2\frac{1}{2}}$ $x_1 = 0 \quad x_2 = 4$	<p><u>p-q Formel</u></p> $-1\frac{1}{4}x^2 + 5x + 0 = 0 \quad / : -1\frac{1}{4}$ $x^2 - 4x + 0 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - 0}$ $x_{1/2} = 2 \pm \sqrt{4}$ $x_{1/2} = 2 \pm 2$ $x_1 = 4 \quad x_2 = 0$
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Aufgabe (30)

<p><u>x-Ausklammern</u></p> $-\frac{3}{4}x^2 - 3x = 0$ $x(-\frac{3}{4}x - 3) = 0$ $-\frac{3}{4}x - 3 = 0 \quad / +3$ $-\frac{3}{4}x = 3 \quad / : (-\frac{3}{4})$ $x = \frac{3}{-\frac{3}{4}}$ $x_1 = 0$ $x_2 = -4$	<p><u>a-b-c Formel</u></p> $-\frac{3}{4}x^2 - 3x + 0 = 0$ $x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot (-\frac{3}{4}) \cdot 0}}{2 \cdot (-\frac{3}{4})}$ $x_{1/2} = \frac{+3 \pm \sqrt{9}}{-1\frac{1}{2}}$ $x_{1/2} = \frac{3 \pm 3}{-1\frac{1}{2}}$ $x_1 = \frac{3 + 3}{-1\frac{1}{2}} \quad x_2 = \frac{3 - 3}{-1\frac{1}{2}}$ $x_1 = -4 \quad x_2 = 0$	<p><u>p-q Formel</u></p> $-\frac{3}{4}x^2 - 3x + 0 = 0 \quad / : -\frac{3}{4}$ $x^2 + 4x + 0 = 0$ $x_{1/2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0}$ $x_{1/2} = -2 \pm \sqrt{4}$ $x_{1/2} = -2 \pm 2$ $x_1 = 0 \quad x_2 = -4$
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Aufgabe (31)

<p><u>Umformen</u></p> $\frac{5}{9}x^2 - 5 = 0 \quad / +5$ $\frac{5}{9}x^2 = 5 \quad / : \frac{5}{9}$ $x^2 = \frac{5}{\frac{5}{9}}$ $x = \pm\sqrt{9}$ $x_1 = 3 \quad x_2 = -3$	<p><u>a-b-c Formel</u></p> $\frac{5}{9}x^2 + 0x - 5 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{5}{9} \cdot (-5)}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{-0 \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{0 \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{0 + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{0 - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 3 \quad x_2 = -3$	<p><u>p-q Formel</u></p> $\frac{5}{9}x^2 + 0x - 5 = 0 \quad / : \frac{5}{9}$ $x^2 + 0x - 9 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)}$ $x_{1/2} = 0 \pm \sqrt{9}$ $x_{1/2} = 0 \pm 3$ $x_1 = 3 \quad x_2 = -3$
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Aufgabe (32)

x-Ausklammern	a-b-c Formel	p-q Formel
$12x^2 + 12x = 0$ $x(12x + 12) = 0$ $12x + 12 = 0 \quad / -12$ $12x = -12 \quad / : 12$ $x = \frac{-12}{12}$ $x_1 = 0$ $x_2 = -1$	$12x^2 + 12x + 0 = 0$ $x_{1/2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 12 \cdot 0}}{2 \cdot 12}$ $x_{1/2} = \frac{-12 \pm \sqrt{144}}{24}$ $x_{1/2} = \frac{-12 \pm 12}{24}$ $x_1 = \frac{-12 + 12}{24} \quad x_2 = \frac{-12 - 12}{24}$ $x_1 = 0 \quad x_2 = -1$	$12x^2 + 12x + 0 = 0 \quad / : 12$ $x^2 + 1x + 0 = 0$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 0}$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = -\frac{1}{2} \pm \frac{1}{2}$ $x_1 = 0 \quad x_2 = -1$

Aufgabe (33)

a-b-c Formel	p-q Formel
$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{1\frac{23}{25}^2 - 4 \cdot \left(-\frac{6}{25}\right) \cdot 2\frac{4}{25}}}{2 \cdot \left(-\frac{6}{25}\right)}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{5\frac{19}{25}}}{-\frac{12}{25}}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = \frac{-1\frac{23}{25} + 2\frac{2}{5}}{-\frac{12}{25}} \quad x_2 = \frac{-1\frac{23}{25} - 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = -1 \quad x_2 = 9$	$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0 \quad / : -\frac{6}{25}$ $x^2 - 8x - 9 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - (-9)}$ $x_{1/2} = 4 \pm \sqrt{25}$ $x_{1/2} = 4 \pm 5$ $x_1 = 9 \quad x_2 = -1$

Aufgabe (34)

a-b-c Formel	p-q Formel
$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{\left(-2\frac{22}{25}\right)^2 - 4 \cdot \left(-\frac{9}{25}\right) \cdot 3\frac{6}{25}}}{2 \cdot \left(-\frac{9}{25}\right)}$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{12\frac{24}{25}}}{-\frac{18}{25}}$ $x_{1/2} = \frac{2\frac{22}{25} \pm 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = \frac{2\frac{22}{25} + 3\frac{3}{5}}{-\frac{18}{25}} \quad x_2 = \frac{2\frac{22}{25} - 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = -9 \quad x_2 = 1$	$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0 \quad / : -\frac{9}{25}$ $x^2 + 8x - 9 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - (-9)}$ $x_{1/2} = -4 \pm \sqrt{25}$ $x_{1/2} = -4 \pm 5$ $x_1 = 1 \quad x_2 = -9$

Aufgabe (35)

a-b-c Formel

$$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{\frac{1}{4}^2 - 4 \cdot \left(-\frac{1}{8}\right) \cdot 7\frac{7}{8}}}{2 \cdot \left(-\frac{1}{8}\right)}$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{4}}{-\frac{1}{4}}$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm 2}{-\frac{1}{4}}$$

$$x_1 = \frac{-\frac{1}{4} + 2}{-\frac{1}{4}} \quad x_2 = \frac{-\frac{1}{4} - 2}{-\frac{1}{4}}$$

$$x_1 = -7 \quad x_2 = 9$$

p-q Formel

$$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0 \quad / : -\frac{1}{8}$$

$$x^2 - 2x - 63 = 0$$

$$x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-63)}$$

$$x_{1/2} = 1 \pm \sqrt{64}$$

$$x_{1/2} = 1 \pm 8$$

$$x_1 = 9 \quad x_2 = -7$$

Aufgabe (36)

a-b-c Formel

$$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{3\frac{33}{49}^2 - 4 \cdot \frac{20}{49} \cdot 3\frac{13}{49}}}{2 \cdot \frac{20}{49}}$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{8\frac{8}{49}}}{\frac{40}{49}}$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm 2\frac{6}{7}}{\frac{40}{49}}$$

$$x_1 = \frac{-3\frac{33}{49} + 2\frac{6}{7}}{\frac{40}{49}} \quad x_2 = \frac{-3\frac{33}{49} - 2\frac{6}{7}}{\frac{40}{49}}$$

$$x_1 = -1 \quad x_2 = -8$$

p-q Formel

$$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0 \quad / : \frac{20}{49}$$

$$x^2 + 9x + 8 = 0$$

$$x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 8}$$

$$x_{1/2} = -4\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = -4\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = -1 \quad x_2 = -8$$

Aufgabe (37)

a-b-c Formel

$$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{\frac{4}{9}^2 - 4 \cdot \left(-\frac{4}{9}\right) \cdot \frac{8}{9}}}{2 \cdot \left(-\frac{4}{9}\right)}$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{1\frac{7}{9}}}{-\frac{8}{9}}$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm 1\frac{1}{3}}{-\frac{8}{9}}$$

$$x_1 = \frac{-\frac{4}{9} + 1\frac{1}{3}}{-\frac{8}{9}} \quad x_2 = \frac{-\frac{4}{9} - 1\frac{1}{3}}{-\frac{8}{9}}$$

$$x_1 = -1 \quad x_2 = 2$$

p-q Formel

$$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0 \quad / : -\frac{4}{9}$$

$$x^2 - 1x - 2 = 0$$

$$x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-2)}$$

$$x_{1/2} = \frac{1}{2} \pm \sqrt{2\frac{1}{4}}$$

$$x_{1/2} = \frac{1}{2} \pm 1\frac{1}{2}$$

$$x_1 = 2 \quad x_2 = -1$$

Aufgabe (38)

a-b-c Formel

$$\begin{aligned}
 & -2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0 \\
 x_{1/2} &= \frac{+2\frac{2}{9} \pm \sqrt{(-2\frac{2}{9})^2 - 4 \cdot (-2\frac{2}{9}) \cdot 4\frac{4}{9}}}{2 \cdot (-2\frac{2}{9})} \\
 x_{1/2} &= \frac{+2\frac{2}{9} \pm \sqrt{44\frac{4}{9}}}{-4\frac{4}{9}} \\
 x_{1/2} &= \frac{2\frac{2}{9} \pm 6\frac{2}{3}}{-4\frac{4}{9}} \\
 x_1 &= \frac{2\frac{2}{9} + 6\frac{2}{3}}{-4\frac{4}{9}} & x_2 &= \frac{2\frac{2}{9} - 6\frac{2}{3}}{-4\frac{4}{9}} \\
 x_1 &= -2 & x_2 &= 1
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & -2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0 \quad / : -2\frac{2}{9} \\
 x^2 + 1x - 2 &= 0 \\
 x_{1/2} &= -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-2)} \\
 x_{1/2} &= -\frac{1}{2} \pm \sqrt{2\frac{1}{4}} \\
 x_{1/2} &= -\frac{1}{2} \pm 1\frac{1}{2} \\
 x_1 &= 1 & x_2 &= -2
 \end{aligned}$$

Aufgabe (39)

x-Ausklammern

$$\begin{aligned}
 & -\frac{7}{9}x^2 + 4\frac{2}{3}x = 0 \\
 x(-\frac{7}{9}x + 4\frac{2}{3}) &= 0 \\
 -\frac{7}{9}x + 4\frac{2}{3} &= 0 & / -4\frac{2}{3} \\
 -\frac{7}{9}x &= -4\frac{2}{3} & / : (-\frac{7}{9}) \\
 x &= \frac{-4\frac{2}{3}}{-\frac{7}{9}} \\
 x_1 &= 0 \\
 x_2 &= 6
 \end{aligned}$$

a-b-c Formel

$$\begin{aligned}
 & -\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 = 0 \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm \sqrt{4\frac{2}{3}^2 - 4 \cdot (-\frac{7}{9}) \cdot 0}}{2 \cdot (-\frac{7}{9})} \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm \sqrt{21\frac{7}{9}}}{-1\frac{5}{9}} \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm 4\frac{2}{3}}{-1\frac{5}{9}} \\
 x_1 &= \frac{-4\frac{2}{3} + 4\frac{2}{3}}{-1\frac{5}{9}} & x_2 &= \frac{-4\frac{2}{3} - 4\frac{2}{3}}{-1\frac{5}{9}} \\
 x_1 &= 0 & x_2 &= 6
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & -\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 = 0 \quad / : -\frac{7}{9} \\
 x^2 - 6x + 0 &= 0 \\
 x_{1/2} &= -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0} \\
 x_{1/2} &= 3 \pm \sqrt{9} \\
 x_{1/2} &= 3 \pm 3 \\
 x_1 &= 6 & x_2 &= 0
 \end{aligned}$$

Aufgabe (40)

a-b-c Formel

$$\begin{aligned}
 & \frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0 \\
 x_{1/2} &= \frac{+\frac{6}{49} \pm \sqrt{(-\frac{6}{49})^2 - 4 \cdot \frac{3}{49} \cdot (-2\frac{46}{49})}}{2 \cdot \frac{3}{49}} \\
 x_{1/2} &= \frac{+\frac{6}{49} \pm \sqrt{\frac{36}{49}}}{\frac{6}{49}} \\
 x_{1/2} &= \frac{\frac{6}{49} \pm \frac{6}{7}}{\frac{6}{49}} \\
 x_1 &= \frac{\frac{6}{49} + \frac{6}{7}}{\frac{6}{49}} & x_2 &= \frac{\frac{6}{49} - \frac{6}{7}}{\frac{6}{49}} \\
 x_1 &= 8 & x_2 &= -6
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & \frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0 \quad / : \frac{3}{49} \\
 x^2 - 2x - 48 &= 0 \\
 x_{1/2} &= -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-48)} \\
 x_{1/2} &= 1 \pm \sqrt{49} \\
 x_{1/2} &= 1 \pm 7 \\
 x_1 &= 8 & x_2 &= -6
 \end{aligned}$$

Aufgabe (41)

<p>x-Ausklammern</p> <hr/> $\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$ $x\left(\frac{5}{9}x - 3\frac{1}{3}\right) = 0$ $\frac{5}{9}x - 3\frac{1}{3} = 0 \quad / + 3\frac{1}{3}$ $\frac{5}{9}x = 3\frac{1}{3} \quad / : \frac{5}{9}$ $x = \frac{3\frac{1}{3}}{\frac{5}{9}}$ $x_1 = 0$ $x_2 = 6$	<p>a-b-c Formel</p> <hr/> $\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{\left(-3\frac{1}{3}\right)^2 - 4 \cdot \frac{5}{9} \cdot 0}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{3\frac{1}{3} \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{3\frac{1}{3} + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{3\frac{1}{3} - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 6 \quad x_2 = 0$	<p>p-q Formel</p> <hr/> $\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0 \quad / : \frac{5}{9}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$
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Aufgabe (42)

<p>a-b-c Formel</p> <hr/> $-1\frac{1}{4}x^2 - 10x - 15 = 0$ $x_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot \left(-1\frac{1}{4}\right) \cdot (-15)}}{2 \cdot \left(-1\frac{1}{4}\right)}$ $x_{1/2} = \frac{+10 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{10 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{10 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{10 - 5}{-2\frac{1}{2}}$ $x_1 = -6 \quad x_2 = -2$	<p>p-q Formel</p> <hr/> $-1\frac{1}{4}x^2 - 10x - 15 = 0 \quad / : -1\frac{1}{4}$ $x^2 + 8x + 12 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 12}$ $x_{1/2} = -4 \pm \sqrt{4}$ $x_{1/2} = -4 \pm 2$ $x_1 = -2 \quad x_2 = -6$
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Aufgabe (43)

<p>x-Ausklammern</p> <hr/> $4x^2 - 8x = 0$ $x(4x - 8) = 0$ $4x - 8 = 0 \quad / + 8$ $4x = 8 \quad / : 4$ $x = \frac{8}{4}$ $x_1 = 0$ $x_2 = 2$	<p>a-b-c Formel</p> <hr/> $4x^2 - 8x + 0 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 0}}{2 \cdot 4}$ $x_{1/2} = \frac{+8 \pm \sqrt{64}}{8}$ $x_{1/2} = \frac{8 \pm 8}{8}$ $x_1 = \frac{8 + 8}{8} \quad x_2 = \frac{8 - 8}{8}$ $x_1 = 2 \quad x_2 = 0$	<p>p-q Formel</p> <hr/> $4x^2 - 8x + 0 = 0 \quad / : 4$ $x^2 - 2x + 0 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - 0}$ $x_{1/2} = 1 \pm \sqrt{1}$ $x_{1/2} = 1 \pm 1$ $x_1 = 2 \quad x_2 = 0$
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Aufgabe (44)

a-b-c Formel

$$\begin{aligned}
 & -\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0 \\
 x_{1/2} &= \frac{-2\frac{22}{49} \pm \sqrt{2\frac{22}{49}^2 - 4 \cdot \left(-\frac{24}{49}\right) \cdot 2\frac{46}{49}}}{2 \cdot \left(-\frac{24}{49}\right)} \\
 x_{1/2} &= \frac{-2\frac{22}{49} \pm \sqrt{11\frac{37}{49}}}{-\frac{48}{49}} \\
 x_{1/2} &= \frac{-2\frac{22}{49} \pm 3\frac{3}{7}}{-\frac{48}{49}} \\
 x_1 &= \frac{-2\frac{22}{49} + 3\frac{3}{7}}{-\frac{48}{49}} & x_2 &= \frac{-2\frac{22}{49} - 3\frac{3}{7}}{-\frac{48}{49}} \\
 x_1 &= -1 & x_2 &= 6
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & -\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0 \quad / : -\frac{24}{49} \\
 & x^2 - 5x - 6 = 0 \\
 x_{1/2} &= -\frac{-5}{2} \pm \sqrt{\left(\frac{-5}{2}\right)^2 - (-6)} \\
 x_{1/2} &= 2\frac{1}{2} \pm \sqrt{12\frac{1}{4}} \\
 x_{1/2} &= 2\frac{1}{2} \pm 3\frac{1}{2} \\
 x_1 &= 6 & x_2 &= -1
 \end{aligned}$$

Aufgabe (45)

x-Ausklammern

$$\begin{aligned}
 & \frac{8}{27}x^2 + 2\frac{2}{3}x = 0 \\
 x\left(\frac{8}{27}x + 2\frac{2}{3}\right) &= 0 \\
 \frac{8}{27}x + 2\frac{2}{3} &= 0 & / - 2\frac{2}{3} \\
 \frac{8}{27}x &= -2\frac{2}{3} & / : \frac{8}{27} \\
 x &= \frac{-2\frac{2}{3}}{\frac{8}{27}} \\
 x_1 &= 0 \\
 x_2 &= -9
 \end{aligned}$$

a-b-c Formel

$$\begin{aligned}
 & \frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0 \\
 x_{1/2} &= \frac{-2\frac{2}{3} \pm \sqrt{2\frac{2}{3}^2 - 4 \cdot \frac{8}{27} \cdot 0}}{2 \cdot \frac{8}{27}} \\
 x_{1/2} &= \frac{-2\frac{2}{3} \pm \sqrt{7\frac{1}{9}}}{\frac{16}{27}} \\
 x_{1/2} &= \frac{-2\frac{2}{3} \pm 2\frac{2}{3}}{\frac{16}{27}} \\
 x_1 &= \frac{-2\frac{2}{3} + 2\frac{2}{3}}{\frac{16}{27}} & x_2 &= \frac{-2\frac{2}{3} - 2\frac{2}{3}}{\frac{16}{27}} \\
 x_1 &= 0 & x_2 &= -9
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & \frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0 \quad / : \frac{8}{27} \\
 & x^2 + 9x + 0 = 0 \\
 x_{1/2} &= -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0} \\
 x_{1/2} &= -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}} \\
 x_{1/2} &= -4\frac{1}{2} \pm 4\frac{1}{2} \\
 x_1 &= 0 & x_2 &= -9
 \end{aligned}$$

Aufgabe (46)

x-Ausklammern

$$\begin{aligned}
 & \frac{20}{81}x^2 + 2\frac{2}{9}x = 0 \\
 x\left(\frac{20}{81}x + 2\frac{2}{9}\right) &= 0 \\
 \frac{20}{81}x + 2\frac{2}{9} &= 0 & / - 2\frac{2}{9} \\
 \frac{20}{81}x &= -2\frac{2}{9} & / : \frac{20}{81} \\
 x &= \frac{-2\frac{2}{9}}{\frac{20}{81}} \\
 x_1 &= 0 \\
 x_2 &= -9
 \end{aligned}$$

a-b-c Formel

$$\begin{aligned}
 & \frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0 \\
 x_{1/2} &= \frac{-2\frac{2}{9} \pm \sqrt{2\frac{2}{9}^2 - 4 \cdot \frac{20}{81} \cdot 0}}{2 \cdot \frac{20}{81}} \\
 x_{1/2} &= \frac{-2\frac{2}{9} \pm \sqrt{4\frac{76}{81}}}{\frac{40}{81}} \\
 x_{1/2} &= \frac{-2\frac{2}{9} \pm 2\frac{2}{9}}{\frac{40}{81}} \\
 x_1 &= \frac{-2\frac{2}{9} + 2\frac{2}{9}}{\frac{40}{81}} & x_2 &= \frac{-2\frac{2}{9} - 2\frac{2}{9}}{\frac{40}{81}} \\
 x_1 &= 0 & x_2 &= -9
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 & \frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0 \quad / : \frac{20}{81} \\
 & x^2 + 9x + 0 = 0 \\
 x_{1/2} &= -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0} \\
 x_{1/2} &= -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}} \\
 x_{1/2} &= -4\frac{1}{2} \pm 4\frac{1}{2} \\
 x_1 &= 0 & x_2 &= -9
 \end{aligned}$$

Aufgabe (47)

a-b-c Formel

$$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{10\frac{2}{25}^2 - 4 \cdot 1\frac{11}{25} \cdot 8\frac{16}{25}}}{2 \cdot 1\frac{11}{25}}$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{51\frac{21}{25}}}{2\frac{22}{25}}$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm 7\frac{1}{5}}{2\frac{22}{25}}$$

$$x_1 = \frac{-10\frac{2}{25} + 7\frac{1}{5}}{2\frac{22}{25}} \quad x_2 = \frac{-10\frac{2}{25} - 7\frac{1}{5}}{2\frac{22}{25}}$$

$$x_1 = -1 \quad x_2 = -6$$

p-q Formel

$$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0 \quad / : 1\frac{11}{25}$$

$$x^2 + 7x + 6 = 0$$

$$x_{1/2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 6}$$

$$x_{1/2} = -3\frac{1}{2} \pm \sqrt{6\frac{1}{4}}$$

$$x_{1/2} = -3\frac{1}{2} \pm 2\frac{1}{2}$$

$$x_1 = -1 \quad x_2 = -6$$

3 Kubische Gleichungen

Umformen: $ax^3 + b = 0$

$$ax^3 + b = 0$$

$$ax^3 + b = 0 \quad / -b$$

$$ax^3 = -b \quad / : a$$

$$x^3 = \frac{-b}{a}$$

$$x = \sqrt[3]{\frac{-b}{a}}$$

$$\frac{-b}{a} > 0 \quad x = \sqrt[3]{\frac{-b}{a}}$$

$$\frac{-b}{a} < 0 \quad x = -\sqrt[3]{\left|\frac{-b}{a}\right|}$$

$$3x^3 + 24 = 0$$

$$3x^3 + 24 = 0 \quad 3x^3 + 24 = 0 \quad / -24$$

$$3x^3 = -24 \quad / : 3$$

$$x^3 = \frac{-24}{3}$$

$$x = \sqrt[3]{-8}$$

$$x = -2$$

$$-3x^3 + 24 = 0$$

$$-3x^3 + 24 = 0 \quad / -24$$

$$-3x^3 = -24 \quad / : (-3)$$

$$x^3 = \frac{-24}{-3}$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Faktorisieren: $ax^3 + bx = 0$

$$ax^3 + bx = 0$$

$$x(ax^2 + b) = 0$$

$$x_1 = 0 \quad \vee \quad (ax^2 + b) = 0$$

$$-9x^3 + 25x = 0$$

$$x(-9x^2 + 25) = 0$$

$$\Rightarrow x_1 = 0 \quad \wedge \quad -9x^2 + 25 = 0$$

$$-9x^2 + 25 = 0 \quad / -25$$

$$-9x^2 = -25 \quad / : (-9)$$

$$x^2 = \frac{-25}{-9}$$

$$x = \pm \sqrt{2\frac{7}{9}}$$

$$x_2 = 1\frac{2}{3} \quad x_3 = -1\frac{2}{3}$$

Faktorisieren: $ax^3 + bx^2 = 0$

$$ax^3 + bx^2 = 0$$

$$x^2(ax + b) = 0$$

$$x_{1/2} = 0 \quad \vee \quad (ax + b) = 0$$

$$-6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$$

$$x^2(-6\frac{3}{4}x - 13\frac{1}{2}) = 0$$

$$\Rightarrow x_{1/2} = 0 \quad \wedge \quad -6\frac{3}{4}x - 13\frac{1}{2} = 0$$

$$-6\frac{3}{4}x - 13\frac{1}{2} = 0 \quad / +13\frac{1}{2}$$

$$-6\frac{3}{4}x = 13\frac{1}{2} \quad / : (-6\frac{3}{4})$$

$$x = \frac{13\frac{1}{2}}{-6\frac{3}{4}}$$

$$x_3 = -2$$

Polynomdivision

$$ax^3 + bx^2 + d = 0$$

$$ax^3 + cx + d = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

Die ganzzahligen Faktoren von d auf Nullstellen prüfen.

Mit der gefunden Nullstelle die Polynomdivision durchführen.

$$x^3 + 3x^2 - 4 = 0$$

$$x^3 + 3x^2 - 4 = 0$$

$$d = 4 \quad \text{Ganzzahlige Faktoren: } \pm 1, \pm 2, \pm 4$$

$$f(1) = 0$$

$$\text{Nullstelle gefunden: } x_1 = 1$$

$$(x^3 + 3x^2 \quad \quad \quad -4) : (x - 1) = x^2 + 4x + 4$$

$$\begin{array}{r} -(x^3 \quad -x^2) \\ \hline 4x^2 \quad \quad \quad -4 \\ -(4x^2 \quad -4x) \\ \hline \quad \quad \quad 4x \quad -4 \\ \quad \quad \quad -(4x \quad -4) \\ \hline \quad \quad \quad \quad \quad 0 \end{array}$$

$$1x^2 + 4x + 4 = 0$$

$$x_{2/3} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$x_{2/3} = \frac{-4 \pm \sqrt{0}}{2}$$

$$x_{2/3} = \frac{-4 \pm 0}{2}$$

$$x_2 = \frac{-4 + 0}{2} \quad x_3 = \frac{-4 - 0}{2}$$

$$x_2 = -2 \quad x_3 = -2$$

3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ax^3 + bx^2 + cx + d = 0$

Gesucht:

Lösung der Gleichung

$$(1) \quad -2x^3 = 0$$

$$(2) \quad 3x^3 + 24 = 0$$

$$(3) \quad -3x^3 + 24 = 0$$

$$(4) \quad -8x^3 + 27 = 0$$

$$(5) \quad -1x^3 + 4x = 0$$

$$(6) \quad -9x^3 + 25x = 0$$

$$(7) \quad -\frac{1}{4}x^3 + \frac{2}{3}x^2 = 0$$

$$(8) \quad x^3 - 3x^2 = 0$$

$$(9) \quad \frac{1}{2}x^3 + 4 = 0$$

$$(10) \quad -\frac{1}{6}x^3 + 2x = 0$$

$$(11) \quad \frac{1}{2}x^3 - 3x^2 + 5x = 0$$

$$(12) \quad -1x^3 + 3x + 2 = 0$$

$$(13) \quad -1x^3 + 3x^2 - 4 = 0$$

$$(14) \quad 4x^3 + 5x^2 - 6x = 0$$

$$(15) \quad -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

$$(16) \quad x^3 - 4x^2 + 3x = 0$$

$$(17) \quad -\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

$$(18) \quad \frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

$$(19) \quad -5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

$$(20) \quad -6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$$

$$(21) \quad \frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

$$(22) \quad -\frac{27}{28}x^3 - \frac{27}{28}x^2 + 5\frac{11}{14}x = 0$$

$$(23) \quad x^3 + 3x^2 - 4 = 0$$

$$(24) \quad -5\frac{1}{16}x^3 + 10\frac{1}{8}x^2 = 0$$

$$(25) \quad \frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

$$(26) \quad -2x^3 + 12x^2 - 18x = 0$$

$$(27) \quad 40\frac{1}{2}x^3 + 81x^2 + 40\frac{1}{2}x = 0$$

$$(28) \quad 54x^3 - 270x^2 + 432x - 216 = 0$$

$$(29) \quad 1\frac{19}{35}x^3 - 10\frac{4}{5}x^2 + 18\frac{18}{35}x = 0$$

$$(30) \quad -2x^3 + 6x^2 = 0$$

$$(31) \quad -2x^3 + 6x^2 = 0$$

$$(32) \quad 5\frac{2}{5}x^3 + 27x^2 + 32\frac{2}{5}x = 0$$

$$(33) \quad \frac{1}{3}x^3 - 1x^2 - 1\frac{1}{3}x = 0$$

$$(34) \quad -0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

$$(35) \quad -\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

$$(36) \quad -13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

3.2 Lösungen

Aufgabe (1)

$$x^3 = 0 \Rightarrow x = 0$$

$$x_1 = 0; \quad \underline{\text{3-fache Nullstelle}}$$

Aufgabe (2)

$$3x^3 + 24 = 0 \quad / -24$$

$$3x^3 = -24 \quad / :3$$

$$x^3 = \frac{-24}{3}$$

$$x = \sqrt[3]{-8}$$

$$x = -2$$

Polynomdivision: (-2)

$$\begin{array}{r} (3x^3) : (x+2) = 3x^2 - 6x + 12 \\ -(3x^3 + 6x^2) \\ \hline -6x^2 \\ -(-6x^2 - 12x) \\ \hline 12x \\ -(12x + 24) \\ \hline 0 \end{array}$$

$$3x^2 - 6x + 12 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot 3 \cdot 12}}{2 \cdot 3}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{-108}}{6}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = -2; \quad \text{1-fache Nullstelle}}$$

Aufgabe (3)

$$-3x^3 + 24 = 0 \quad / -24$$

$$-3x^3 = -24 \quad / :(-3)$$

$$x^3 = \frac{-24}{-3}$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Polynomdivision: 2

$$\begin{array}{r} (-3x^3) : (x-2) = -3x^2 - 6x - 12 \\ -(-3x^3 + 6x^2) \\ \hline -6x^2 \\ -(-6x^2 + 12x) \\ \hline -12x \\ -(-12x + 24) \\ \hline 0 \end{array}$$

$$-3x^2 - 6x - 12 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot (-3) \cdot (-12)}}{2 \cdot (-3)}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{-108}}{-6}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (4)

$$-8x^3 + 27 = 0 \quad -8x^3 + 27 = 0 \quad / -27$$

$$-8x^3 = -27 \quad / : (-8)$$

$$x^3 = \frac{-27}{-8}$$

$$x = \sqrt[3]{3\frac{3}{8}}$$

$$x = 1\frac{1}{2}$$

Polynomdivision: $1\frac{1}{2}$

$$\begin{array}{r} (-8x^3 \quad +27) : (x - 1\frac{1}{2}) = -8x^2 - 12x - 18 \\ \underline{-(-8x^3 \quad +12x^2)} \\ -12x^2 \\ \underline{-(-12x^2 \quad +18x)} \\ -18x \\ \underline{-(-18x \quad +27)} \\ 0 \end{array}$$

$$-8x^2 - 12x - 18 = 0$$

$$x_{1/2} = \frac{+12 \pm \sqrt{(-12)^2 - 4 \cdot (-8) \cdot (-18)}}{2 \cdot (-8)}$$

$$x_{1/2} = \frac{+12 \pm \sqrt{-432}}{-16}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = 1\frac{1}{2}; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (5)

$$x(-1x^2 + 4) = 0 \Rightarrow x = 0 \quad \wedge \quad -1x^2 + 4 = 0$$

$$-1x^2 + 4 = 0 \quad / -4$$

$$-1x^2 = -4 \quad / : (-1)$$

$$x^2 = \frac{-4}{-1}$$

$$x = \pm\sqrt{4}$$

$$x_1 = 2 \quad x_2 = -2$$

$$\underline{x_1 = -2; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (6)

$$\begin{aligned}
 x(-9x^2 + 25) = 0 &\Rightarrow x = 0 \quad \wedge \quad -9x^2 + 25 = 0 \\
 -9x^2 + 25 = 0 &\quad / -25 \\
 -9x^2 = -25 &\quad / : (-9) \\
 x^2 = \frac{-25}{-9} \\
 x = \pm \sqrt{2\frac{7}{9}} \\
 x_1 = 1\frac{2}{3} \quad x_2 = -1\frac{2}{3} \\
 x_1 = -1\frac{2}{3}; &\quad \text{1-fache Nullstelle} \\
 \hline
 x_2 = 0; &\quad \text{1-fache Nullstelle} \\
 \hline
 x_3 = 1\frac{2}{3}; &\quad \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (7)

$$\begin{aligned}
 x^2(-\frac{1}{4}x + \frac{2}{3}) = 0 &\Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\
 -\frac{1}{4}x + \frac{2}{3} = 0 &\quad / -\frac{2}{3} \\
 -\frac{1}{4}x = -\frac{2}{3} &\quad / : (-\frac{1}{4}) \\
 x = \frac{-\frac{2}{3}}{-\frac{1}{4}} \\
 x = 2\frac{2}{3} \\
 x_1 = 0; &\quad \text{2-fache Nullstelle} \\
 \hline
 x_2 = 2\frac{2}{3}; &\quad \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (8)

$$\begin{aligned}
 x^2(x - 3) = 0 &\Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\
 x - 3 = 0 &\quad / +3 \\
 x = 3 \\
 x_1 = 0; &\quad \text{2-fache Nullstelle} \\
 \hline
 x_2 = 3; &\quad \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (9)

$$\begin{aligned}
 \frac{1}{2}x^3 + 4 = 0 &\Rightarrow \frac{1}{2}x^3 + 4 = 0 \quad / -4 \\
 \frac{1}{2}x^3 = -4 &\quad / : \frac{1}{2} \\
 x^3 = \frac{-4}{\frac{1}{2}} \\
 x = \sqrt[3]{-8} \\
 x = -2 \\
 \text{Polynomdivision:} &\quad (-2)
 \end{aligned}$$

$$\begin{array}{r}
 \left(\frac{1}{2}x^3 + x^2\right) + 4 \quad : (x+2) = \frac{1}{2}x^2 - 1x + 2 \\
 \hline
 -\left(\frac{1}{2}x^3 + x^2\right) \quad -1x^2 \quad +4 \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} -(-1x^2 - 2x) \quad +4 \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} \quad 2x \quad +4 \\
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} \quad - (2x + 4) \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} 0
 \end{array}$$

$$\frac{1}{2}x^2 - 1x + 2 = 0$$

$$x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{2} \cdot 2}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+1 \pm \sqrt{-3}}{1}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = -2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (10)

$$\begin{aligned}
 x\left(-\frac{1}{6}x^2 + 2\right) = 0 &\Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0 \\
 -\frac{1}{6}x^2 + 2 = 0 &\quad / -2 \\
 -\frac{1}{6}x^2 = -2 &\quad / : \left(-\frac{1}{6}\right) \\
 x^2 = \frac{-2}{-\frac{1}{6}} &
 \end{aligned}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,46 \quad x_2 = -3,46$$

$$\underline{x_1 = -3,46; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 3,46; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (11)

$$\begin{aligned}
 x\left(\frac{1}{2}x^2 - 3x + 5\right) = 0 &\Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0 \\
 \frac{1}{2}x^2 - 3x + 5 = 0 &
 \end{aligned}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (12)

$$-1x^3 + 3x + 2 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-1x^3 \quad +3x \quad +2) : (x+1) = -1x^2 + x + 2 \\
 \hline
 -(-1x^3 \quad -1x^2) \\
 \hline
 \quad \quad x^2 \quad +3x \quad +2 \\
 \quad \quad \quad -(x^2 \quad +x) \\
 \hline
 \quad \quad \quad \quad 2x \quad +2 \\
 \quad \quad \quad \quad \quad -(2x \quad +2) \\
 \hline
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -1x^2 + 1x + 2 &= 0 \\
 x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
 x_{1/2} &= \frac{-1 \pm 3}{-2} \\
 x_1 &= \frac{-1 + 3}{-2} & x_2 &= \frac{-1 - 3}{-2} \\
 x_1 &= -1 & x_2 &= 2 \\
 x_1 &= -1; & \text{2-fache Nullstelle} \\
 x_2 &= 2; & \text{1-fache Nullstelle}
 \end{aligned}$$

Aufgabe (13)

$$\begin{array}{r}
 -1x^3 + 3x^2 - 4 = 0 \\
 \text{Nullstelle für Polynomdivision erraten: } -1 \\
 (-1x^3 \quad +3x^2 \quad -4) : (x+1) = -1x^2 + 4x - 4 \\
 \hline
 -(-1x^3 \quad -1x^2) \\
 \hline
 \quad \quad 4x^2 \quad -4 \\
 \quad \quad \quad -(4x^2 \quad +4x) \\
 \hline
 \quad \quad \quad \quad -4x \quad -4 \\
 \quad \quad \quad \quad \quad -(-4x \quad -4) \\
 \hline
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -1x^2 + 4x - 4 &= 0 \\
 x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-4 \pm \sqrt{0}}{-2} \\
 x_{1/2} &= \frac{-4 \pm 0}{-2} \\
 x_1 &= \frac{-4 + 0}{-2} & x_2 &= \frac{-4 - 0}{-2} \\
 x_1 &= 2 & x_2 &= 2 \\
 x_1 &= -1; & \text{1-fache Nullstelle} \\
 x_2 &= 2; & \text{2-fache Nullstelle}
 \end{aligned}$$

Aufgabe (14)

$$x(4x^2 + 5x - 6) = 0 \Rightarrow x = 0 \quad \wedge \quad 4x^2 + 5x - 6 = 0$$

$$4x^2 + 5x - 6 = 0$$

$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$

$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$

$$x_{1/2} = \frac{-5 \pm 11}{8}$$

$$x_1 = \frac{-5 + 11}{8} \quad x_2 = \frac{-5 - 11}{8}$$

$$x_1 = \frac{3}{4} \quad x_2 = -2$$

$$x_1 = -2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = \frac{3}{4}; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (15)

$$-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-\frac{1}{2}x^3 \quad -\frac{1}{2}x^2 \quad +4x \quad +6) : (x+2) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3 \\ -(-\frac{1}{2}x^3 \quad -1x^2) \\ \hline \frac{1}{2}x^2 \quad +4x \quad +6 \\ -(\frac{1}{2}x^2 \quad +x) \\ \hline 3x \quad +6 \\ -(3x \quad +6) \\ \hline 0 \end{array}$$

$$-\frac{1}{2}x^2 + \frac{1}{2}x + 3 = 0$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{2}^2 - 4 \cdot (-\frac{1}{2}) \cdot 3}}{2 \cdot (-\frac{1}{2})}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{6\frac{1}{4}}}{-1}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm 2\frac{1}{2}}{-1}$$

$$x_1 = \frac{-\frac{1}{2} + 2\frac{1}{2}}{-1} \quad x_2 = \frac{-\frac{1}{2} - 2\frac{1}{2}}{-1}$$

$$x_1 = -2 \quad x_2 = 3$$

$$x_1 = -2; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (16)

$$x(x^2 - 4x + 3) = 0 \Rightarrow x = 0 \quad \wedge \quad x^2 - 4x + 3 = 0$$

$$1x^2 - 4x + 3 = 0$$

$$x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+4 \pm \sqrt{4}}{2}$$

$$x_{1/2} = \frac{4 \pm 2}{2}$$

$$x_1 = \frac{4+2}{2} \quad x_2 = \frac{4-2}{2}$$

$$x_1 = 3 \quad x_2 = 1$$

$$x_1 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 1; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 3; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (17)

$$-\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

NumerischeSuche :

$$x_1 = -4; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = -1; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 3; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (18)

$$\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

Nullstelle für Polynomdivision erraten: -3

$$\begin{array}{r} \left(\begin{array}{l} \frac{1}{10}x^3 \\ -\left(\frac{1}{10}x^3\right) \end{array} \right. \quad \left. \begin{array}{l} +\frac{3}{10}x^2 \\ +\frac{3}{10}x^2 \end{array} \right) : (x+3) = \frac{1}{10}x^2 - 5,55 \cdot 10^{-17}x - 1\frac{3}{5} \\ \hline \begin{array}{l} -5,55 \cdot 10^{-17}x^2 \\ -(-5,55 \cdot 10^{-17}x^2) \end{array} \quad \begin{array}{l} -1\frac{3}{5}x \\ -1,67 \cdot 10^{-16}x \end{array} \quad \begin{array}{l} -4\frac{4}{5} \\ -4\frac{4}{5} \end{array} \\ \hline \begin{array}{l} -1\frac{3}{5}x \\ -(-1\frac{3}{5}x) \end{array} \quad \begin{array}{l} -4\frac{4}{5} \\ -4\frac{4}{5} \end{array} \\ \hline 0 \end{array}$$

$$\frac{1}{10}x^2 - 5,55 \cdot 10^{-17}x - 1\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+5,55 \cdot 10^{-17} \pm \sqrt{(-5,55 \cdot 10^{-17})^2 - 4 \cdot \frac{1}{10} \cdot (-1\frac{3}{5})}}{2 \cdot \frac{1}{10}}$$

$$x_{1/2} = \frac{+5,55 \cdot 10^{-17} \pm \sqrt{\frac{16}{25}}}{\frac{1}{5}}$$

$$x_{1/2} = \frac{5,55 \cdot 10^{-17} \pm \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = \frac{5,55 \cdot 10^{-17} + \frac{4}{5}}{\frac{1}{5}} \quad x_2 = \frac{5,55 \cdot 10^{-17} - \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = 4 \quad x_2 = -4$$

$$x_1 = -4; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = -3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 4; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (19)

$$-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5}) : (x+2) = -5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} \\ \underline{-(-5\frac{2}{5}x^3 - 10\frac{4}{5}x^2)} \\ -27x^2 - 75\frac{3}{5}x - 43\frac{1}{5} \\ \underline{-(-27x^2 - 54x)} \\ -21\frac{3}{5}x - 43\frac{1}{5} \\ \underline{-(-21\frac{3}{5}x - 43\frac{1}{5})} \\ 0 \end{array}$$

$$-5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+27 \pm \sqrt{(-27)^2 - 4 \cdot (-5\frac{2}{5}) \cdot (-21\frac{3}{5})}}{2 \cdot (-5\frac{2}{5})}$$

$$x_{1/2} = \frac{+27 \pm \sqrt{262\frac{11}{25}}}{-10\frac{4}{5}}$$

$$x_{1/2} = \frac{27 \pm 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = \frac{27 + 16\frac{1}{5}}{-10\frac{4}{5}} \quad x_2 = \frac{27 - 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = -4 \quad x_2 = -1$$

$$x_1 = -4; \quad 1\text{-fache Nullstelle}$$

$$x_2 = -2; \quad 1\text{-fache Nullstelle}$$

$$x_3 = -1; \quad 1\text{-fache Nullstelle}$$

Aufgabe (20)

$$x^2(-6\frac{3}{4}x - 13\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \wedge \quad -6\frac{3}{4}x - 13\frac{1}{2} = 0$$

$$-6\frac{3}{4}x - 13\frac{1}{2} = 0 \quad / + 13\frac{1}{2}$$

$$-6\frac{3}{4}x = 13\frac{1}{2} \quad / : (-6\frac{3}{4})$$

$$x = \frac{13\frac{1}{2}}{-6\frac{3}{4}}$$

$$x = -2$$

$$x_1 = -2; \quad 1\text{-fache Nullstelle}$$

$$x_2 = 0; \quad 2\text{-fache Nullstelle}$$

Aufgabe (21)

$$\frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

Numerische Suche :

$$x_1 = -3; \quad 1\text{-fache Nullstelle}$$

$$x_2 = -2; \quad 1\text{-fache Nullstelle}$$

$$x_3 = 2; \quad 1\text{-fache Nullstelle}$$

Aufgabe (22)

$$x\left(-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14}\right) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{\left(-\frac{27}{28}\right)^2 - 4 \cdot \left(-\frac{27}{28}\right) \cdot 5\frac{11}{14}}}{2 \cdot \left(-\frac{27}{28}\right)}$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{23,2}}{-1\frac{13}{14}}$$

$$x_{1/2} = \frac{\frac{27}{28} \pm 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = \frac{\frac{27}{28} + 4\frac{23}{28}}{-1\frac{13}{14}} \quad x_2 = \frac{\frac{27}{28} - 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = -3 \quad x_2 = 2$$

$$x_1 = -3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 2; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (23)

$$x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten: 1

$$\begin{array}{r} (x^3 + 3x^2 - 4) : (x - 1) = x^2 + 4x + 4 \\ -(x^3 - 1x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 - 4x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$1x^2 + 4x + 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{2}$$

$$x_{1/2} = \frac{-4 \pm 0}{2}$$

$$x_1 = \frac{-4 + 0}{2} \quad x_2 = \frac{-4 - 0}{2}$$

$$x_1 = -2 \quad x_2 = -2$$

$$x_1 = -2; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 1; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (24)

$$x^2\left(-5\frac{1}{16}x + 10\frac{1}{8}\right) = 0 \Rightarrow x = 0 \quad \wedge \quad -5\frac{1}{16}x + 10\frac{1}{8} = 0$$

$$-5\frac{1}{16}x + 10\frac{1}{8} = 0 \quad / -10\frac{1}{8}$$

$$-5\frac{1}{16}x = -10\frac{1}{8} \quad / : \left(-5\frac{1}{16}\right)$$

$$x = \frac{-10\frac{1}{8}}{-5\frac{1}{16}}$$

$$x = 2$$

$$x_1 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 2; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (25)

$$\frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

NumerischeSuche :

$$x_1 = -3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 4; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (26)

$$x(-2x^2 + 12x - 18) = 0 \Rightarrow x = 0 \quad \wedge \quad -2x^2 + 12x - 18 = 0$$

$$-2x^2 + 12x - 18 = 0$$

$$x_{1/2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot (-2) \cdot (-18)}}{2 \cdot (-2)}$$

$$x_{1/2} = \frac{-12 \pm \sqrt{0}}{-4}$$

$$x_{1/2} = \frac{-12 \pm 0}{-4}$$

$$x_1 = \frac{-12 + 0}{-4} \quad x_2 = \frac{-12 - 0}{-4}$$

$$x_1 = 3 \quad x_2 = 3$$

$$x_1 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (27)

$$x(40\frac{1}{2}x^2 + 81x + 40\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \wedge \quad 40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} = 0$$

$$40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} = 0$$

$$x_{1/2} = \frac{-81 \pm \sqrt{81^2 - 4 \cdot 40\frac{1}{2} \cdot 40\frac{1}{2}}}{2 \cdot 40\frac{1}{2}}$$

$$x_{1/2} = \frac{-81 \pm \sqrt{0}}{81}$$

$$x_{1/2} = \frac{-81 \pm 0}{81}$$

$$x_1 = \frac{-81 + 0}{81} \quad x_2 = \frac{-81 - 0}{81}$$

$$x_1 = -1 \quad x_2 = -1$$

$$x_1 = -1; \quad \underline{2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (28)

$$54x^3 - 270x^2 + 432x - 216 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (54x^3 \quad -270x^2 \quad +432x \quad -216) : (x-1) = 54x^2 - 216x + 216 \\ -(54x^3 \quad -54x^2) \\ \hline \quad -216x^2 \quad +432x \quad -216 \\ \quad -(-216x^2 \quad +216x) \\ \hline \qquad \qquad 216x \quad -216 \\ \qquad \qquad -(216x \quad -216) \\ \hline \qquad \qquad \qquad \qquad \qquad 0 \end{array}$$

$$54x^2 - 216x + 216 = 0$$

$$x_{1/2} = \frac{+216 \pm \sqrt{(-216)^2 - 4 \cdot 54 \cdot 216}}{2 \cdot 54}$$

$$x_{1/2} = \frac{+216 \pm \sqrt{0}}{216}$$

$$x_{1/2} = \frac{108}{216 \pm 0}$$

$$x_1 = \frac{108}{216 + 0} \quad x_2 = \frac{216 - 0}{108}$$

$$x_1 = 2 \quad x_2 = 2$$

$$\underline{x_1 = 1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 2; \quad 2\text{-fache Nullstelle}}$$

Aufgabe (29)

$$x(1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35}) = 0 \Rightarrow x = 0 \quad \wedge \quad 1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{(-10\frac{4}{5})^2 - 4 \cdot 1\frac{19}{35} \cdot 18\frac{18}{35}}}{2 \cdot 1\frac{19}{35}}$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{2,38}}{3\frac{3}{35}}$$

$$x_{1/2} = \frac{10\frac{4}{5} \pm 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = \frac{10\frac{4}{5} + 1\frac{19}{35}}{3\frac{3}{35}} \quad x_2 = \frac{10\frac{4}{5} - 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = 4 \quad x_2 = 3$$

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 4; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (30)

$$\begin{aligned}
 x^2(-2x + 6) = 0 &\Rightarrow x = 0 \quad \wedge \quad -2x + 6 = 0 \\
 -2x + 6 = 0 &\quad / -6 \\
 -2x = -6 &\quad / : (-2) \\
 x = \frac{-6}{-2} \\
 x = 3 \\
 \underline{x_1 = 0; \quad 2\text{-fache Nullstelle}} \\
 \underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (31)

$$\begin{aligned}
 x^2(-2x + 6) = 0 &\Rightarrow x = 0 \quad \wedge \quad -2x + 6 = 0 \\
 -2x + 6 = 0 &\quad / -6 \\
 -2x = -6 &\quad / : (-2) \\
 x = \frac{-6}{-2} \\
 x = 3 \\
 \underline{x_1 = 0; \quad 2\text{-fache Nullstelle}} \\
 \underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (32)

$$\begin{aligned}
 x(5\frac{2}{5}x^2 + 27x + 32\frac{2}{5}) = 0 &\Rightarrow x = 0 \quad \wedge \quad 5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} = 0 \\
 5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} = 0 \\
 x_{1/2} &= \frac{-27 \pm \sqrt{27^2 - 4 \cdot 5\frac{2}{5} \cdot 32\frac{2}{5}}}{2 \cdot 5\frac{2}{5}} \\
 x_{1/2} &= \frac{-27 \pm \sqrt{29\frac{4}{25}}}{10\frac{4}{5}} \\
 x_{1/2} &= \frac{-27 \pm 5\frac{2}{5}}{10\frac{4}{5}} \\
 x_1 = \frac{-27 + 5\frac{2}{5}}{10\frac{4}{5}} \quad x_2 = \frac{-27 - 5\frac{2}{5}}{10\frac{4}{5}} \\
 x_1 = -2 \quad x_2 = -3 \\
 \underline{x_1 = -3; \quad 1\text{-fache Nullstelle}} \\
 \underline{x_2 = -2; \quad 1\text{-fache Nullstelle}} \\
 \underline{x_3 = 0; \quad 1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (33)

$$\begin{aligned}
 x(\frac{1}{3}x^2 - 1x - 1\frac{1}{3}) = 0 &\Rightarrow x = 0 \quad \wedge \quad \frac{1}{3}x^2 - 1x - 1\frac{1}{3} = 0 \\
 \frac{1}{3}x^2 - 1x - 1\frac{1}{3} = 0 \\
 x_{1/2} &= \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{3} \cdot (-1\frac{1}{3})}}{2 \cdot \frac{1}{3}}
 \end{aligned}$$

$$x_{1/2} = \frac{+1 \pm \sqrt{2\frac{7}{9}}}{\frac{2}{3}}$$

$$x_{1/2} = \frac{1 \pm 1\frac{2}{3}}{\frac{2}{3}}$$

$$x_1 = \frac{1 + 1\frac{2}{3}}{\frac{2}{3}} \quad x_2 = \frac{1 - 1\frac{2}{3}}{\frac{2}{3}}$$

$$x_1 = 4 \quad x_2 = -1$$

$$\underline{x_1 = -1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 4; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (34)

$$-0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

NumerischeSuche :

$$\underline{x_1 = -4,02; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = -2; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 4,01; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (35)

$$-\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

NumerischeSuche :

$$\underline{x_1 = -3; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = -1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (36)

$$-13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54) : (x+1) = -13\frac{1}{2}x^2 - 54x - 54 \\ \underline{-(-13\frac{1}{2}x^3 \quad -13\frac{1}{2}x^2)} \\ \quad \quad \quad -54x^2 \quad -108x \quad -54 \\ \quad \quad \quad \underline{-(-54x^2 \quad -54x)} \\ \quad \quad \quad \quad \quad -54x \quad -54 \\ \quad \quad \quad \quad \quad \underline{-(-54x \quad -54)} \\ \quad \quad \quad \quad \quad \quad \quad \quad 0 \end{array}$$

$$-13\frac{1}{2}x^2 - 54x - 54 = 0$$

$$x_{1/2} = \frac{+54 \pm \sqrt{(-54)^2 - 4 \cdot (-13\frac{1}{2}) \cdot (-54)}}{2 \cdot (-13\frac{1}{2})}$$

$$x_{1/2} = \frac{+54 \pm \sqrt{0}}{-27}$$

$$x_{1/2} = \frac{54 \pm 0}{-27}$$

$$x_1 = \frac{54 + 0}{-27} \quad x_2 = \frac{54 - 0}{-27}$$

$$x_1 = -2 \quad x_2 = -2$$

$$x_1 = -2; \quad \text{2-fache Nullstelle}$$

$$x_2 = -1; \quad \text{1-fache Nullstelle}$$

4 Gleichungen höheren Grades

Gerader Exponent: $ax^n + c = 0$

$$ax^n + c = 0 \quad / -c$$

$$ax^n = -c \quad / : a$$

$$x_{1/2} = \pm \sqrt[n]{\frac{-c}{a}}$$

Diskriminante:

$$D = \frac{-c}{a}$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$-2x^4 + 162 = 0 \quad / -162$$

$$-2x^4 = -162 \quad / : (-2)$$

$$x^4 = \frac{-162}{-2}$$

$$x = \pm \sqrt[4]{81}$$

$$x_1 = 3 \quad x_2 = -3$$

Ungerader Exponent: $ax^n + c = 0$

Umformen

$$ax^n + b = 0$$

$$ax^n + b = 0 \quad / -b$$

$$ax^n = -b \quad / : a$$

$$x^n = \frac{-b}{a}$$

$$x = \sqrt[n]{\frac{-b}{a}}$$

$$\frac{-b}{a} > 0 \quad x = \sqrt[n]{\frac{-b}{a}}$$

$$\frac{-b}{a} < 0 \quad x = -\sqrt[n]{\left|\frac{-b}{a}\right|}$$

$$5x^3 + 320 = 0 \quad / -320$$

$$5x^3 = -320 \quad / : 5$$

$$x^3 = -\frac{320}{5}$$

$$x = -\sqrt[3]{64}$$

$$x = -4$$

Biquadratische Gleichung

$$ax^4 + bx^2 + c = 0$$

$$\text{Substitution: } u = x^2 \quad u^2 = x^4$$

$$\text{Quadratische Gleichung: } au^2 + bu + c = 0$$

$$\text{Lösungen: } u_1 \quad u_2$$

$$\text{Resubstitution: } x^2 = u_1 \quad x^2 = u_2$$

$$x^4 - 10x^2 + 9 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 10u + 9 = 0$$

$$u_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+10 \pm \sqrt{64}}{2}$$

$$u_{1/2} = \frac{10 \pm 8}{2}$$

$$u_1 = \frac{10+8}{2} \quad u_2 = \frac{10-8}{2}$$

$$u_1 = 9 \quad u_2 = 1$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x_3 = 1 \quad x_4 = -1$$

4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ax^n + bx^{n-1} \dots = 0$

Gesucht:

Lösung der Gleichung

Grad des Polynoms = Anzahl der Eingaben -1

- (1) $x^4 - 12x^3 + 54x^2 - 108x + 81 = 0$
- (2) $x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$
- (3) $2x^4 + 16x^3 + 48x^2 + 84x + 72 = 0$
- (4) $-6x^4 + 72x^3 - 324x^2 + 648x - 486 = 0$
- (5) $x^4 - 18x^2 + 81 = 0$
- (6) $-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$
- (7) $-\frac{1}{4}x^4 + \frac{3}{3}x^3 = 0$
- (8) $x^4 - 3x^3 = 0$
- (9) $x^4 + x^3 - 9x^2 + 11x - 4 = 0$
- (10) $-\frac{1}{6}x^4 + 2x^2 = 0$
- (11) $-\frac{1}{6}x^4 + 2x^2 = 0$
- (12) $\frac{1}{2}x^4 - 3x^3 + 5x^2 = 0$
- (13) $-x^4 + 3x^2 + 2x = 0$
- (14) $-x^4 + 3x^3 - 4x = 0$
- (15) $-2x^5 = 0$
- (16) $-\frac{1}{4}x^5 + \frac{2}{3}x^4 = 0$
- (17) $x^5 - 3x^4 = 0$
- (18) $x^5 - 10x^3 + 9x = 0$
- (19) $\frac{1}{2}x^5 + 2x^2 = 0$
- (20) $-\frac{1}{6}x^5 + 2x^3 = 0$
- (21) $\frac{1}{2}x^5 - 3x^4 + 5x^3 = 0$
- (22) $-x^5 + 3x^3 + 2x^2 = 0$
- (23) $-x^5 + 3x^4 - 4x^2 = 0$
- (24) $4x^5 + 5x^4 - 6x^3 = 0$

4.2 Lösungen

Aufgabe (1)

$$x^4 - 12x^3 + 54x^2 - 108x + 81 = 0$$

$$x^4 - 12x^3 + 54x^2 - 108x + 81$$

Nullstelle für Polynomdivision erraten: 3

$$\begin{array}{r}
 (x^4 \quad -12x^3 \quad +54x^2 \quad -108x \quad +81) : (x-3) = x^3 - 9x^2 + 27x - 27 \\
 -(x^4 \quad -3x^3) \\
 \hline
 \quad -9x^3 \quad +54x^2 \quad -108x \quad +81 \\
 \quad -(-9x^3 \quad +27x^2) \\
 \hline
 \quad \quad 27x^2 \quad -108x \quad +81 \\
 \quad \quad -(27x^2 \quad -81x) \\
 \hline
 \quad \quad \quad -27x \quad +81 \\
 \quad \quad \quad -(-27x \quad +81) \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$x^3 - 9x^2 + 27x - 27 = 0$$

Nullstelle für Polynomdivision erraten: 3

$$\begin{array}{r}
 (x^3 \quad -9x^2 \quad +27x \quad -27) : (x-3) = x^2 - 6x + 9 \\
 -(x^3 \quad -3x^2) \\
 \hline
 \quad -6x^2 \quad +27x \quad -27 \\
 \quad -(-6x^2 \quad +18x) \\
 \hline
 \quad \quad 9x \quad -27 \\
 \quad \quad -(9x \quad -27) \\
 \hline
 \quad \quad \quad 0
 \end{array}$$

$$1x^2 - 6x + 9 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{0}}{2}$$

$$x_{1/2} = \frac{6 \pm 0}{2}$$

$$x_1 = \frac{6+0}{2} \quad x_2 = \frac{6-0}{2}$$

$$x_1 = 3 \quad x_2 = 3$$

$$x_1 = 3; \quad \underline{4\text{-fache Nullstelle}}$$

Aufgabe (2)

$$x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$$

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (x^4 \quad +4x^3 \quad +6x^2 \quad +4x \quad +1) : (x+1) = x^3 + 3x^2 + 3x + 1 \\
 -(x^4 \quad +x^3) \\
 \hline
 \quad 3x^3 \quad +6x^2 \quad +4x \quad +1 \\
 \quad -(3x^3 \quad +3x^2) \\
 \hline
 \quad \quad 3x^2 \quad +4x \quad +1 \\
 \quad \quad -(3x^2 \quad +3x) \\
 \hline
 \quad \quad \quad x \quad +1 \\
 \quad \quad \quad -(x \quad +1) \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$x^3 + 3x^2 + 3x + 1 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (x^3 + 3x^2 + 3x + 1) : (x + 1) = x^2 + 2x + 1 \\
 -(x^3 + x^2) \\
 \hline
 2x^2 + 3x + 1 \\
 -(2x^2 + 2x) \\
 \hline
 x + 1 \\
 -(x + 1) \\
 \hline
 0
 \end{array}$$

$$\begin{aligned}
 1x^2 + 2x + 1 &= 0 \\
 x_{1/2} &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\
 x_{1/2} &= \frac{-2 \pm \sqrt{0}}{2} \\
 x_{1/2} &= \frac{-2 \pm 0}{2} \\
 x_1 &= \frac{-2 + 0}{2} & x_2 &= \frac{-2 - 0}{2} \\
 x_1 &= -1 & x_2 &= -1 \\
 x_1 &= -1; & & \text{4-fache Nullstelle}
 \end{aligned}$$

Aufgabe (3)

$$2x^4 + 16x^3 + 48x^2 + 84x + 72 = 0$$

$$2x^4 + 16x^3 + 48x^2 + 84x + 72$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r}
 (2x^4 + 16x^3 + 48x^2 + 84x + 72) : (x + 2) = 2x^3 + 12x^2 + 24x + 36 \\
 -(2x^4 + 4x^3) \\
 \hline
 12x^3 + 48x^2 + 84x + 72 \\
 -(12x^3 + 24x^2) \\
 \hline
 24x^2 + 84x + 72 \\
 -(24x^2 + 48x) \\
 \hline
 36x + 72 \\
 -(36x + 72) \\
 \hline
 0
 \end{array}$$

$$2x^3 + 12x^2 + 24x + 36 = 0$$

NumerischeSuche :

$$x_1 = -4, 15; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

Aufgabe (4)

$$-6x^4 + 72x^3 - 324x^2 + 648x - 486 = 0$$

$$-6x^4 + 72x^3 - 324x^2 + 648x - 486$$

Nullstelle für Polynomdivision erraten:3

$$\begin{array}{r}
 (-6x^4 + 72x^3 - 324x^2 + 648x - 486) : (x - 3) = -6x^3 + 54x^2 - 162x + 162 \\
 -(-6x^4 + 18x^3) \\
 \hline
 54x^3 - 324x^2 + 648x - 486 \\
 -(54x^3 - 162x^2) \\
 \hline
 -162x^2 + 648x - 486 \\
 -(-162x^2 + 486x) \\
 \hline
 162x - 486 \\
 -(162x - 486) \\
 \hline
 0
 \end{array}$$

$$-6x^3 + 54x^2 - 162x + 162 = 0$$

Nullstelle für Polynomdivision erraten:3

$$\begin{array}{r}
 (-6x^3 + 54x^2 - 162x + 162) : (x - 3) = -6x^2 + 36x - 54 \\
 -(-6x^3 + 18x^2) \\
 \hline
 36x^2 - 162x + 162 \\
 -(36x^2 - 108x) \\
 \hline
 -54x + 162 \\
 -(-54x + 162) \\
 \hline
 0
 \end{array}$$

$$\begin{aligned}
 -6x^2 + 36x - 54 &= 0 \\
 x_{1/2} &= \frac{-36 \pm \sqrt{36^2 - 4 \cdot (-6) \cdot (-54)}}{2 \cdot (-6)} \\
 x_{1/2} &= \frac{-36 \pm \sqrt{0}}{-12} \\
 x_{1/2} &= \frac{-36 \pm 0}{-12} \\
 x_1 &= \frac{-36 + 0}{-12} & x_2 &= \frac{-36 - 0}{-12} \\
 x_1 &= 3 & x_2 &= 3 \\
 x_1 &= 3; & & \text{4-fache Nullstelle}
 \end{aligned}$$

Aufgabe (5)

$$x^4 - 18x^2 + 81 = 0$$

$$\begin{aligned}
 u &= x^2 & u^2 &= x^4 \\
 1u^2 - 18u + 81 &= 0
 \end{aligned}$$

$$\begin{aligned}
 u_{1/2} &= \frac{+18 \pm \sqrt{(-18)^2 - 4 \cdot 1 \cdot 81}}{2 \cdot 1} \\
 u_{1/2} &= \frac{+18 \pm \sqrt{0}}{2} \\
 u_{1/2} &= \frac{18 \pm 0}{2} \\
 u_1 &= \frac{18 + 0}{2} & u_2 &= \frac{18 - 0}{2} \\
 u_1 &= 9 & u_2 &= 9 \\
 x^2 &= 9 \\
 x &= \pm\sqrt{9} \\
 x_1 &= 3 & x_2 &= -3 \\
 x^2 &= 9 \\
 x &= \pm\sqrt{9} \\
 x_1 &= 3 & x_2 &= -3 \\
 x_1 &= -3; & & \text{2-fache Nullstelle} \\
 x_2 &= 3; & & \text{2-fache Nullstelle}
 \end{aligned}$$

Aufgabe (6)

$$\begin{aligned}
 -\frac{1}{4}x^4 + \frac{2}{3}x^3 &= 0 \\
 x^3(-\frac{1}{4}x + \frac{2}{3}) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\
 -\frac{1}{4}x + \frac{2}{3} &= 0 & / -\frac{2}{3} \\
 -\frac{1}{4}x &= -\frac{2}{3} & / : (-\frac{1}{4}) \\
 x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\
 x &= 2\frac{2}{3} \\
 x_1 &= 0; & & \text{3-fache Nullstelle}
 \end{aligned}$$

$$\underline{x_2 = 2\frac{2}{3}; \quad 1\text{-fache Nullstelle}$$

Aufgabe (7)

$$\begin{aligned} -\frac{1}{4}x^4 + \frac{2}{3}x^3 &= 0 \\ x^3(-\frac{1}{4}x + \frac{2}{3}) = 0 &\Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\ -\frac{1}{4}x + \frac{2}{3} &= 0 \quad / -\frac{2}{3} \\ -\frac{1}{4}x &= -\frac{2}{3} \quad / : (-\frac{1}{4}) \end{aligned}$$

$$x = \frac{-\frac{2}{3}}{-\frac{1}{4}}$$

$$x = 2\frac{2}{3}$$

$$\underline{x_1 = 0; \quad 3\text{-fache Nullstelle}$$

$$\underline{x_2 = 2\frac{2}{3}; \quad 1\text{-fache Nullstelle}$$

Aufgabe (8)

$$\begin{aligned} x^4 - 3x^3 &= 0 \\ x^3(x - 3) = 0 &\Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\ x - 3 &= 0 \quad / +3 \\ x &= 3 \end{aligned}$$

$$\underline{x_1 = 0; \quad 3\text{-fache Nullstelle}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}$$

Aufgabe (9)

$$x^4 + x^3 - 9x^2 + 11x - 4 = 0$$

$$x^4 + x^3 - 9x^2 + 11x - 4$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^4 + x^3 - 9x^2 + 11x - 4) : (x - 1) = x^3 + 2x^2 - 7x + 4 \\ \underline{-(x^4 - x^3)} \\ 2x^3 - 9x^2 + 11x - 4 \\ \underline{-(2x^3 - 2x^2)} \\ -7x^2 + 11x - 4 \\ \underline{-(-7x^2 + 7x)} \\ 4x - 4 \\ \underline{-(4x - 4)} \\ 0 \end{array}$$

$$x^3 + 2x^2 - 7x + 4 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^3 + 2x^2 - 7x + 4) : (x - 1) = x^2 + 3x - 4 \\ \underline{-(x^3 - x^2)} \\ 3x^2 - 7x + 4 \\ \underline{-(3x^2 - 3x)} \\ -4x + 4 \\ \underline{-(-4x + 4)} \\ 0 \end{array}$$

$$1x^2 + 3x - 4 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{25}}{2}$$

$$x_{1/2} = \frac{-3 \pm 5}{2}$$

$$x_1 = \frac{-3+5}{2} \quad x_2 = \frac{-3-5}{2}$$

$$x_1 = 1 \quad x_2 = -4$$

$$x_1 = -4; \quad \text{1-fache Nullstelle}$$

$$x_2 = 1; \quad \text{3-fache Nullstelle}$$

Aufgabe (10)

$$-\frac{1}{6}x^4 + 2x^2 = 0$$

$$x^2(-\frac{1}{6}x^2 + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0$$

$$-\frac{1}{6}x^2 + 2 = 0 \quad / -2$$

$$-\frac{1}{6}x^2 = -2 \quad / : (-\frac{1}{6})$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,46 \quad x_2 = -3,46$$

$$x_1 = -3,46; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{2-fache Nullstelle}$$

$$x_3 = 3,46; \quad \text{1-fache Nullstelle}$$

Aufgabe (11)

$$-\frac{1}{6}x^4 + 2x^2 = 0$$

$$x^2(-\frac{1}{6}x^2 + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0$$

$$-\frac{1}{6}x^2 + 2 = 0 \quad / -2$$

$$-\frac{1}{6}x^2 = -2 \quad / : (-\frac{1}{6})$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,46 \quad x_2 = -3,46$$

$$x_1 = -3,46; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{2-fache Nullstelle}$$

$$x_3 = 3,46; \quad \text{1-fache Nullstelle}$$

Aufgabe (12)

$$\frac{1}{2}x^4 - 3x^3 + 5x^2 = 0$$

$$x^2(\frac{1}{2}x^2 - 3x + 5) = 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0$$

$$\frac{1}{2}x^2 - 3x + 5 = 0$$

$$x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1}$$

Diskriminante negativ keine Lösung

$$x_1 = 0; \quad \text{2-fache Nullstelle}$$

Aufgabe (13)

$$-x^4 + 3x^2 + 2x = 0$$

$$x(-x^3 + 3x + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x + 2 = 0$$

$$-x^3 + 3x + 2 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x \quad +2) : (x+1) = -x^2 + x + 2 \\
 \underline{-(-x^3 \quad -x^2)} \\
 \quad \quad x^2 \quad +3x \quad +2 \\
 \quad \quad \underline{-(x^2 \quad +x)} \\
 \quad \quad \quad \quad 2x \quad +2 \\
 \quad \quad \quad \quad \underline{-(2x \quad +2)} \\
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + x + 2 &= 0 \\
 x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
 x_{1/2} &= \frac{-1 \pm 3}{-2} \\
 x_1 &= \frac{-1 + 3}{-2} & x_2 &= \frac{-1 - 3}{-2} \\
 x_1 &= -1 & x_2 &= 2 \\
 x_1 &= -1; \quad \underline{2\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{1\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (14)

$$\begin{aligned}
 -x^4 + 3x^3 - 4x &= 0 \\
 x(-x^3 + 3x^2 - 4) &= 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x^2 - 4 = 0 \\
 -x^3 + 3x^2 - 4 &= 0
 \end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x^2 \quad -4) : (x+1) = -x^2 + 4x - 4 \\
 \underline{-(-x^3 \quad -x^2)} \\
 \quad \quad 4x^2 \quad -4 \\
 \quad \quad \underline{-(4x^2 \quad +4x)} \\
 \quad \quad \quad \quad -4x \quad -4 \\
 \quad \quad \quad \quad \underline{-(-4x \quad -4)} \\
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + 4x - 4 &= 0 \\
 x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-4 \pm \sqrt{0}}{-2} \\
 x_{1/2} &= \frac{-4 \pm 0}{-2} \\
 x_1 &= \frac{-4 + 0}{-2} & x_2 &= \frac{-4 - 0}{-2} \\
 x_1 &= 2 & x_2 &= 2 \\
 x_1 &= -1; \quad \underline{1\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{1\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{2\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (15)

$$\begin{aligned}
 -2x^5 &= 0 \\
 x^5 &= 0 \Rightarrow x = 0 \\
 x_1 &= 0; \quad \underline{5\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (16)

$$\begin{aligned}
-\frac{1}{4}x^5 + \frac{2}{3}x^4 &= 0 \\
x^4\left(-\frac{1}{4}x + \frac{2}{3}\right) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\
-\frac{1}{4}x + \frac{2}{3} &= 0 \quad / -\frac{2}{3} \\
-\frac{1}{4}x &= -\frac{2}{3} \quad / : \left(-\frac{1}{4}\right) \\
x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\
x &= 2\frac{2}{3} \\
x_1 &= 0; \quad \underline{\text{4-fache Nullstelle}} \\
x_2 &= 2\frac{2}{3}; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (17)

$$\begin{aligned}
x^5 - 3x^4 &= 0 \\
x^4(x - 3) &= 0 \Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\
x - 3 &= 0 \quad / + 3 \\
x &= 3 \\
x_1 &= 0; \quad \underline{\text{4-fache Nullstelle}} \\
x_2 &= 3; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (18)

$$\begin{aligned}
x^5 - 10x^3 + 9x &= 0 \\
x(x^4 - 10x^2 + 9) &= 0 \Rightarrow x = 0 \quad \wedge \quad x^4 - 10x^2 + 9 = 0 \\
u &= x^2 \quad u^2 = x^4 \\
1u^2 - 10u + 9 &= 0 \\
u_{1/2} &= \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} \\
u_{1/2} &= \frac{+10 \pm \sqrt{64}}{2} \\
u_{1/2} &= \frac{10 \pm 8}{2} \\
u_1 &= \frac{10+8}{2} \quad u_2 = \frac{10-8}{2} \\
u_1 &= 9 \quad u_2 = 1 \\
x^2 &= 9 \\
x &= \pm\sqrt{9} \\
x_1 &= 3 \quad x_2 = -3 \\
x^2 &= 1 \\
x &= \pm\sqrt{1} \\
x_1 &= 1 \quad x_2 = -1 \\
x_1 &= -3; \quad \underline{\text{1-fache Nullstelle}} \\
x_2 &= -1; \quad \underline{\text{1-fache Nullstelle}} \\
x_3 &= 0; \quad \underline{\text{1-fache Nullstelle}} \\
x_4 &= 1; \quad \underline{\text{1-fache Nullstelle}} \\
x_5 &= 3; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (19)

$$\begin{aligned}
\frac{1}{2}x^5 + 2x^2 &= 0 \\
x^2\left(\frac{1}{2}x^3 + 2\right) &= 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^3 + 2 = 0 \\
\frac{1}{2}x^3 + 2 &= 0 \quad \frac{1}{2}x^3 + 2 = 0 \quad / - 2 \\
\frac{1}{2}x^3 &= -2 \quad / : \frac{1}{2}
\end{aligned}$$

$$x^3 = \frac{-2}{\frac{1}{2}}$$

$$x = \sqrt[3]{-4}$$

$$x = -1,59$$

Polynomdivision: $(-1,59)$

$$\begin{array}{r} (\frac{1}{2}x^3) : (x + 1,59) = \frac{1}{2}x^2 - 0,794x + 1,26 \\ -(\frac{1}{2}x^3) \\ \hline \phantom{(\frac{1}{2}x^3} -0,794x^2 \\ -(-0,794x^2) \\ \hline \phantom{(\frac{1}{2}x^3} 1,26x \\ \phantom{(\frac{1}{2}x^3} -(1,26x) \\ \hline \phantom{(\frac{1}{2}x^3} 4,44 \cdot 10^{-16} \end{array}$$

$$\frac{1}{2}x^2 - 0,794x + 1,26 = 0$$

$$x_{1/2} = \frac{+0,794 \pm \sqrt{(-0,794)^2 - 4 \cdot \frac{1}{2} \cdot 1,26}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+0,794 \pm \sqrt{-1,89}}{1}$$

Diskriminante negativ keine Lösung

$$x_1 = -1,59; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (20)

$$-\frac{1}{6}x^5 + 2x^3 = 0$$

$$x^3(-\frac{1}{6}x^2 + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0$$

$$-\frac{1}{6}x^2 + 2 = 0 \quad / -2$$

$$-\frac{1}{6}x^2 = -2 \quad / : (-\frac{1}{6})$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,46 \quad x_2 = -3,46$$

$$x_1 = -3,46; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{3\text{-fache Nullstelle}}$$

$$x_3 = 3,46; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (21)

$$\frac{1}{2}x^5 - 3x^4 + 5x^3 = 0$$

$$x^3(\frac{1}{2}x^2 - 3x + 5) = 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0$$

$$\frac{1}{2}x^2 - 3x + 5 = 0$$

$$x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1}$$

Diskriminante negativ keine Lösung

$$x_1 = 0; \quad \underline{3\text{-fache Nullstelle}}$$

Aufgabe (22)

$$-x^5 + 3x^3 + 2x^2 = 0$$

$$x^2(-x^3 + 3x + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x + 2 = 0$$

$$-x^3 + 3x + 2 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x \quad +2) : (x+1) = -x^2 + x + 2 \\
 -(-x^3 \quad -x^2) \\
 \hline
 \quad \quad x^2 \quad +3x \quad +2 \\
 \quad \quad - (x^2 \quad +x) \\
 \hline
 \quad \quad \quad \quad 2x \quad +2 \\
 \quad \quad \quad \quad - (2x \quad +2) \\
 \hline
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + x + 2 &= 0 \\
 x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
 x_{1/2} &= \frac{-1 \pm 3}{-2} \\
 x_1 &= \frac{-1 + 3}{-2} & x_2 &= \frac{-1 - 3}{-2} \\
 x_1 &= -1 & x_2 &= 2 \\
 x_1 &= -1; \quad \underline{2\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{2\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (23)

$$\begin{aligned}
 -x^5 + 3x^4 - 4x^2 &= 0 \\
 x^2(-x^3 + 3x^2 - 4) &= 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x^2 - 4 = 0 \\
 -x^3 + 3x^2 - 4 &= 0
 \end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x^2 \quad -4) : (x+1) = -x^2 + 4x - 4 \\
 -(-x^3 \quad -x^2) \\
 \hline
 \quad \quad 4x^2 \quad -4 \\
 \quad \quad - (4x^2 \quad +4x) \\
 \hline
 \quad \quad \quad \quad -4x \quad -4 \\
 \quad \quad \quad \quad - (-4x \quad -4) \\
 \hline
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + 4x - 4 &= 0 \\
 x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-4 \pm \sqrt{0}}{-2} \\
 x_{1/2} &= \frac{-4 \pm 0}{-2} \\
 x_{1/2} &= \frac{-4 + 0}{-2} & x_2 &= \frac{-4 - 0}{-2} \\
 x_1 &= 2 & x_2 &= 2 \\
 x_1 &= -1; \quad \underline{1\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{2\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{2\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (24)

$$\begin{aligned}
 4x^5 + 5x^4 - 6x^3 &= 0 \\
 x^3(4x^2 + 5x - 6) &= 0 \Rightarrow x = 0 \quad \wedge \quad 4x^2 + 5x - 6 = 0
 \end{aligned}$$

$$4x^2 + 5x - 6 = 0$$

$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$

$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$

$$x_{1/2} = \frac{-5 \pm 11}{8}$$

$$x_1 = \frac{-5 + 11}{8} \quad x_2 = \frac{-5 - 11}{8}$$

$$x_1 = \frac{3}{4} \quad x_2 = -2$$

$$x_1 = -2; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{3-fache Nullstelle}$$

$$x_3 = \frac{3}{4}; \quad \text{1-fache Nullstelle}$$